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λ-Mildly Normal Spaces and some Functions in Generalized Topological Spaces

Research Article

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Abstract: In this paper we introduce the notion of regular g_λ-closed (rg_λ-closed) sets and by using rg_λ-closed sets we obtain a characterization of λ-mildly normal spaces and use it to improve the preservation theorems of λ-mildly normal spaces.

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1. Introduction and Preliminaries

In [2, 3], Csàszàr founded the theory of generalized topological spaces, and studied the extremely elementary character of these classes. Especially he introduced the notions of continuous functions on generalized topological spaces, and investigated characterizations of generalized continuous functions (= (λ, μ)-continuous functions in [5]). In [9, 10, 11], Min introduced the notions of weak (λ, μ)-continuity, almost (λ, μ)-continuity, (α, μ)-continuity, (σ, μ)-continuity, (π, μ)-continuity and (β, μ)-continuity on generalized topological spaces. In this paper we introduce the notion of regular g_λ-closed (rg_λ-closed) sets and by using rg_λ-closed sets we obtain a characterization of λ-mildly normal spaces and use it to improve the preservation theorems of λ-mildly normal spaces.

Definition 1.1 ([3]). *Let X be a nonempty set and μ be a collection of subsets of X. Then μ is called a generalized topology (briefly GT) on X if $\phi \in \mu$ and $G_i \in \mu$ for $i \in I \neq \phi$ implies $G = \cup_{i\in I} G_i \in \mu$. We call the pair (X, μ) a generalized topological space (briefly GTS) on X. The elements of μ are called μ-open sets and their complements are called μ-closed sets. The generalized closure of a subset S of X, denoted by $c_\mu(S)$, is the intersection of μ-closed sets including S. And the interior of S, denoted by $i_\mu(S)$, is the union of μ-open sets contained in S.*

Definition 1.2. *Let (X, λ) be a generalized topological space and $A \subseteq X$. Then A is said to be*

(1). λ-semi-open [2] if $A \subseteq c_\lambda(i_\lambda(A))$,

(2). λ-preopen [2] if $A \subseteq i_\lambda(c_\lambda(A))$,

(3). λ-α-open [2] if $A \subseteq i_\lambda(c_\lambda(i_\lambda(A)))$,

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(4). λ-β-open [2] if $A \subseteq c_\lambda(i_\lambda(c_\lambda(A)))$,

(5). λ-regular open [10] if $A = i_\lambda(c_\lambda(A))$.

The complement of λ-semi-open (resp., λ-preopen, λ-α-open, λ-β-open, λ-b-open, λ-regular open) is said to be λ-semi-closed (resp., λ-preclosed, λ-α-closed, λ-β-closed, λ-b-closed, λ-regular closed).
Let us denote by $\sigma(\lambda_X)$ *(briefly* σ_X *or* σ*) the class of all λ-semi-open sets on X, by* $\pi(\lambda_X)$ *(briefly* π_X *or* π*) the class of all λ-preopen sets on X, by* $\alpha(\lambda_X)$ *(briefly* α_X *or* α*) the class of all λ-α-open sets on X, by* $\beta(\lambda_X)$ *(briefly* β_X *or* β*) the class of all λ-β-open sets on X, by* $\rho(\lambda_X)$ *(briefly* ρ_X *or* ρ*) the class of all λ-regular open sets on X.*

Definition 1.3 ([4]). *Let (X, λ) and (Y, μ) be GTS's. Then a function* $f : (X, \lambda) \to (Y, \mu)$ *is said to be (λ, μ)-continuous if for each μ-open set U in Y,* $f^{-1}(U)$ *is λ-open in X.*

Definition 1.4 ([7]). *A subset A of a GTS (X, λ) is called a generalized λ-closed set (briefly* g_λ*-closed) if* $c_\lambda(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is λ-open in (X, λ). A subset A is said to be* g_λ*-open if* $X - A$ *is* g_λ*-closed.*

Definition 1.5 ([6]). *A GTS (X, λ) is said to be λ-compact if every λ-open cover of X has a finite λ-open subcover.*

Definition 1.6. *Let (X, λ) and (Y, μ) be GTS's. Then a function* $f : (X, \lambda) \to (Y, \mu)$ *is said to be*

(1). (g_λ, μ)*-continuous [11] if for each μ-open set U in Y,* $f^{-1}(U)$ *is* g_λ*-open in X,*

(2). almost (λ, μ)-continuous [8] if for each μ-regular open set U in Y, $f^{-1}(U)$ *is λ-open in X,*

Definition 1.7 ([1]). *Let (X, λ) be a GTS. Then the space X is said to be almost λ-regular if for each* $F \in \lambda\text{-}RC(X)$ *and each point* $x \in X - F$*, there exists disjoint λ-open sets U and V such that* $x \in U$ *and* $F \subseteq V$*.*

Definition 1.8 ([12]). *A GTS (X, λ) is said to be λ-*T_1 *if for any distinct pair of points x and y in X, there is an λ-open set U in X containing x but not y and an λ-open set V in X containing y but not x.*

Definition 1.9 ([12]). *Let (X, λ) be a strong GTS. Then X is called an λ-*T_3 *space if it satisfies the following λ-*T_3*-separation condition, if* $x \notin F$ *where F is λ-closed, then there exists* $U_x \in \lambda_X$ *and* $U_F \in \lambda$ *such that* $F \subseteq U_F$ *and* $U_x \cap U_F = \phi$*.*

Definition 1.10 ([12]). *A λ-*T_1*-space is called a λ-regular space if it is a λ-*T_3*-space.*

Definition 1.11 ([9]). *Let (X, λ) and (Y, μ) be GTS's. Then a function* $f : (X, \lambda) \to (Y, \mu)$ *is said to be (λ, μ)-open if the image of each λ-open set in X is an μ-open set of Y.*

2. Regular g_λ-Closed Sets

Definition 2.1. *A subset A of a GTS (X, λ) is called a regular generalized λ-closed set (briefly* rg_λ*-closed) if* $c_\lambda(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is λ- regular open in (X, λ). A subset A is said to be* rg_λ*-open if* $X - A$ *is* rg_λ*-closed.*

Remark 2.2. *We have the following implications for properties of subsets:*

$$\lambda\text{-regular closed} \to \lambda\text{-closed} \to g_\lambda\text{-closed} \to rg_\lambda\text{-closed}$$

where none of these implications is reversible as shown by the following Examples 3.3.

Example 2.3. *Let (X, λ) be a GTS such that*

(1). $X = \{a, b, c\}$ and $\lambda = \{\phi, \{a\}, \{a, b\}\}$. Then $\{b, c\}$ is λ-closed but not λ-regular closed and also $\{a, c\}$ is g_λ-closed but not λ-closed.

(2). $X = \{a, b, c\}$ and $\lambda = \{\phi, \{a\}, \{a, b\}, \{a, c\}, X\}$. Then $\{a, b\}$ is rg_λ-closed but not g_λ-closed.

Definition 2.4. *Let (X, λ) be a generalized topological space. A set $S \subseteq X$ is said to be quasi H-λ-closed relative to X if for every cover $\{U_\alpha : \alpha \in A\}$ of S by λ-open sets of X, there exists a finite subfamily $A_0 \subseteq A$ such that $S \subseteq \cup\{c_\lambda(U_\alpha) : \alpha \in A_0\}$. If X is quasi H-$\lambda$-closed relative to X, then it is called quasi H-λ-closed.*

Proposition 2.5. *If a space X is almost λ-regular and a subset A of X is quasi H-λ-closed relative to X, then A is rg_λ-closed.*

Proof. Let U be any λ-regular open set of X containing A. For each $x \in A$, there exists an λ-open set V(x) such that $x \in V(x) \subseteq c_\lambda(V(x)) \subseteq U$. Since $\{V(x) : x \in A\}$ is an λ-open cover of A, there exists a finite subset A_0 of A such that $A \subseteq \{c_\lambda(V(x)) : x \in A_0\}$. Therefore, we obtain $A \subseteq c_\lambda(A) \subseteq \cup\{c_\lambda(V(x)) : x \in A_0\} \subseteq U$. This shows that A is rg_λ-closed. □

Corollary 2.6. *If a space X is λ-regular and A is a λ-compact set of X, then A is rg_λ-closed.*

Proof. Every λ-regular space is almost λ-regular and every λ-compact set of X is quasi H-λ-closed relative to X. Therefore it is an immediate consequence of Proposition 3.5. □

Corollary 2.7.

(1). Every λ-compact subset of a λ-regular space is g_λ-closed.

(2). Every λ-compact subset of a λ-regular space is rg_λ-closed.

3. Characterization of Mildly Normal Spaces

Definition 3.1. *A GTS (X, λ) is said to be λ-mildly normal if for every pair of disjoint $H, K \in \lambda$-RC(X), there exist disjoint λ-open sets U, V of X such that $H \subseteq U$ and $K \subseteq V$.*

Lemma 3.2. *A subset A of a GTS (X, λ) is rg_λ-open if and only if $F \subseteq i_\lambda(A)$ whenever $F \in \lambda$-RC(X) and $F \subseteq A$.*

Theorem 3.3. *The following are equivalent for a GTS (X, λ):*

(1). X is λ-mildly normal;

(2). for any disjoint $H, K \in \lambda$-RC(X), there exist disjoint g_λ-open sets U, V such that $H \subseteq U$ and $K \subseteq V$;

(3). for any disjoint $H, K \in \lambda$-RC(X), there exist disjoint rg_λ-open sets U, V such that $H \subseteq U$ and $K \subseteq V$;

(4). for any disjoint $H \in \lambda$-RC(X) and any $V \in \lambda$-RO(X) containing H, there exists a rg_λ-open set U of X such that $H \subseteq U \subseteq c_\lambda(U) \subseteq V$.

Proof. It is obvious that (1) implies (2) and (2) implies (3).
(3) ⇒ (4) : Let $H \in \lambda$-RC(X) and $H \subseteq V \in \lambda$-RO(X). There exist disjoint rg_λ-open sets U, W such that $H \subseteq U$ and $X - V \subseteq W$. By Lemma 4.2, we have $X - V \subseteq i_\lambda(W)$ and $U \cap i_\lambda(W) = \phi$. Therefore, we obtain $c_\lambda(U) \cap i_\lambda(W) =$ and hence $H \subseteq U \subseteq c_\lambda(U) \subseteq X - i_\lambda(W) \subseteq V$.
(4) ⇒ (1) : Let H, K be disjoint λ-regular closed sets of X. Then $H \subseteq X - K \in \lambda$-RO(X) and there exists a rg_λ-open set G of X such that $H \subseteq G \subseteq c_\lambda(G) \subseteq X - K$. Put $U = i_\lambda(G)$ and $V = X - c_\lambda(G)$. Then U and V are disjoint λ-open sets of X such that $H \subseteq U$ and $K \subseteq V$. Therefore X is λ-mildly normal. □

4. Some Functions

Definition 4.1. *A function $f : (X, \lambda) \to (Y, \mu)$ is said to be almost (g_λ, μ)-continuous (resp almost (rg_λ, μ)-continuous) if $f^{-1}(R)$ is g_λ-closed (resp. rg_λ-closed) for every $R \in \lambda$-$RC(Y)$. We shall recall the definitions of some functions used in the sequel.*

Definition 4.2. *A function $f : (X, \lambda) \to (Y, \mu)$ is said to be*

(1). (rg_λ, μ)-continuous if $f^{-1}(F)$ is rg_λ-closed for every μ-closed set F of Y;

(2). R_λ-map if for each μ-regular open set U in Y, $f^{-1}(U)$ is λ-regular open in X,

(3). Completely (λ, μ)-continuous or (λ, μ)-regular continuous if $f^{-1}(V) \in \lambda$-$RO(X)$ for every μ-open set V of Y.

(4). rg_λ-irresolute if $f^{-1}(F)$ is rg_λ-closed in X for every rg_μ-closed set F of Y.

From the definitions stated above, we obtain the following diagram:

DIAGRAM I

complete (λ, μ)-continuity $\longrightarrow$ R_λ-map

$\uparrow$ $\downarrow$

(λ, μ)-continuity $\longrightarrow$ almost (λ, μ)-continuity

$\uparrow$ $\downarrow$

(g_λ, μ)-continuity $\longrightarrow$ almost (g_λ, μ)-continuity

$\uparrow$ $\downarrow$

(rg_λ, μ)-continuity $\longrightarrow$ almost (rg_λ, μ)-continuity

Remark 4.3. *None of the implications in DIAGRAM I is reversible as shown by the following Examples.*

Example 4.4. *Let $X = \{a, b, c, d\}$, $\lambda = \{\phi, \{a\}\}$ and $\mu = \{\phi, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, b, c\}\}$. Then the identify function $f : (X, \lambda) \to (X, \mu)$ is (λ, μ)-continuous and almost (λ, μ)-continuous. But it is neither completely (λ, μ)-continuous nor an R_λ-map.*

Example 4.5.

(1). Let $X = \{a, b, c\}$, $\lambda = \{\phi, \{a\}\}$ and $\mu = \{\phi, \{a\}, \{b\}, \{a, b\}\}$. Then the identify function $f : (X, \lambda) \to (X, \mu)$ is (g_λ, μ)-continuous but not almost (λ, μ)-continuous. Therefore, (λ, μ)-continuity (resp. almost (λ, μ)-continuity) is strictly stronger than (g_λ, μ)-continuity (resp. almost (g_λ, μ)-continuity).

(2). Let $X = \{a, b, c\}$, $\lambda = \{\phi, \{a\}, \{a, b\}\}$ and $\mu = \{\phi, \{a\}, \{a, c\}\}$. Then the identify function $f : (X, \lambda) \to (X, \mu)$ is an R_λ-map and (rg_λ, μ)-continuous. But it is not (g_λ, μ)-continuous.

(3). Let $X = \{a, b, c\}$, $\lambda = \{\phi, \{a\}, \{c\}, \{a, c\}\}$ and $\mu = \{\phi, \{a\}, \{b\}, \{a, b\}\}$. Then the identify function $f : (X, \lambda) \to (X, \mu)$ is almost (rg_λ, μ)-continuous. But it is neither almost (g_λ, μ)-continuous nor (rg_λ, μ)-continuous.

Definition 4.6. *A GTS X is said to be λ-regular $T_{1/2}$ if every rg_λ-closed set of X is λ-regular closed.*

Proposition 4.7. *If a function $f : (X, \lambda) \to (Y, \mu)$ is (rg_λ, μ)-continuous and X is λ-regular $T_{1/2}$, then f is completely (λ, μ)-continuous.*

Proof. Let F be any μ-closed set of Y. Since f is (rg$_\lambda$, μ)-continuous, f^{-1}(F) is rg$_\lambda$-closed in X and hence f^{-1}(F) $\in \lambda$-RC(X). Therefore, f is completely (λ, μ)-continuous. □

Remark 4.8. *Every rg$_\lambda$-irresolute function is (rg$_\lambda$, μ)-continuous but not conversely as shown by the following example.*

Example 4.9. *Let $X = \{a, b, c\}$ $\lambda = \{\phi, \{a\}, \{a, b\}\}$ and $\mu = \{\phi, \{a\}\}$. Then the identity function $f : (X, \lambda) \to (X, \mu)$ is (λ, μ)-continuous and hence (g$_\lambda$, μ)-continuous but not rg$_\lambda$-irresolute.*

Corollary 4.10. *If $f : (X, \lambda) \to (Y, \mu)$ is rg$_\lambda$-irresolute and X is λ-regular $T_{1/2}$, then f is λ-regular irresolute.*

Definition 4.11. *A function $f : (X, \lambda) \to (Y, \mu)$ is said to be*

(1). (λ, μ)-regular closed (resp. (g$_\lambda$, μ)-closed, (rg$_\lambda$, μ)-closed) if f(F) is μ-regular closed (resp. g$_\mu$-closed, rg$_\mu$-closed) in Y for every λ-closed set F of X;

(2). (λ, μ)-rc-preserving (resp. almost (λ, μ)-closed, almost (g$_\lambda$, μ)-closed, almost (rg$_\lambda$, μ)-closed) if f(F) is μ-regular closed (resp. μ-closed, g$_\mu$-closed, rg$_\mu$-closed) in Y for every F $\in \lambda$-RC(X).

From the definitions stated above, we obtain the following diagram:

DIAGRAM II

(λ, μ)-regular closed $\longrightarrow$ (λ, μ)-rc-preserving

$\uparrow$ $\qquad\qquad$ $\downarrow$

(λ, μ)-closed $\longrightarrow$ almost (λ, μ)-closed

$\uparrow$ $\qquad\qquad$ $\downarrow$

(g$_\lambda$, μ)-closed $\longrightarrow$ almost (g$_\lambda$, μ)-closed

$\uparrow$ $\qquad\qquad$ $\downarrow$

(rg$_\lambda$, μ)-closed $\longrightarrow$ almost (rg$_\lambda$, μ)-closed

Remark 4.12. *The following Example and the inverse function $f^{-1} : (X, \mu) \to (X, \lambda)$ in Examples 5.5 enable us to realize that none of the implications in DIAGRAM II is reversible.*

Example 4.13. *Let $X = \{a, b, c\}$, $\lambda = \{\phi, \{a\}, \{a, b\}\}$ and $\mu = \{\phi, \{a\},\{b\},\{a, b\},\{a, c\}\}$. Then the identity function f : $(X, \lambda) \to (X, \mu)$ is (λ, μ)-closed but not (λ, μ)-rc-preserving.*

Proposition 4.14. *Let $f : (X, \lambda) \to (Y, \mu)$ be a function. Then,*

(1). if f is (rg$_\lambda$, μ)-continuous (λ, μ)-rc-preserving, then it is rg$_\lambda$-irresolute;

(2). if f is an R$_\lambda$-map and rg$_\lambda$-closed, then f(A) is rg$_\mu$-closed in Y for every rg$_\lambda$-closed set A of X.

Proof.

(1). Let B be any rg$_\mu$-closed set of Y and U $\in \lambda$-RO(X) containing f^{-1}(B). Put V = Y − f(X − U), then we have B $\subseteq$ V, f^{-1}(V) $\subseteq$ U and V $\in \mu$-RO(Y) since f is (λ, μ)-rc-preserving. Hence we obtain c$_\mu$(B) $\subseteq$ V and hence f^{-1}(c$_\mu$(B)) $\subseteq$ U. By the (rg$_\lambda$, μ)-continuity of f, we have c$_\lambda$(f^{-1} (B)) $\subseteq$ c$_\lambda$(f^{-1}(c$_\mu$(B))) $\subseteq$ U. This shows that f^{-1}(B) is (rg$_\lambda$, μ)-closed in X. Therefore f is rg$_\lambda$-irresolute.

(2). Let A be any rg$_\lambda$-closed set of X and V $\in \mu$-RO(Y) containing f(A). Since f is an R$_\lambda$-map, f^{-1}(V) $\in \lambda$-RO(X) and A f^{-1}(V). Therefore, we have c$_\lambda$(A) $\subseteq$ f^{-1}(V) and hence f(c$_\lambda$(A)) $\subseteq$ V. Since f is (rg$_\lambda$, μ)-closed, f(c$_\lambda$(A)) is rg$_\mu$-closed in Y and hence we obtain c$_\lambda$(f(A)) $\subseteq$ c$_\lambda$(f(c$_\lambda$(A))) $\subseteq$ V. This shows that f(A) is rg$_\mu$-closed in Y. □

Corollary 4.15. *Let f : (X, λ) → (Y, μ) be a function. Then,*

(1). if f is (λ, μ)-continuous and (λ, μ)-regular closed, then $f^{-1}(B)$ is rg_λ-closed in X for every rg_μ-closed set B of Y.

(2). if f is R_λ-map and (λ, μ)-closed, then f(A) is rg_μ-closed in Y for every rg_λ-closed set A of X.

Proposition 4.16. *A surjection f : (X, λ) → (Y, μ) is almost (rg_λ, μ)-closed (resp. almost (g_λ, μ)-closed) if and only if for each subset S of Y and each U ∈ λ-RO(X) containing $f^{-1}(S)$ there exists an rg_μ-open (resp. g_μ-open) set V of Y such that S ⊆ V and $f^{-1}(V)$ ⊆ U.*

Proof. We prove only the first case, the proof of the second being entirely analogous.
Necessity. Suppose that f is almost (rg_λ, μ)-closed. Let S be a subset of Y and U ∈ λ-RO(X) containing $f^{-1}(S)$. Put V = Y − f(X − U), then V is an rg_μ-open set of Y such that S ⊆ V and $f^{-1}(V)$ ⊆ U.
Sufficiency. Let F be any λ-regular closed set of X. Then $f^{-1}(Y − f(F))$ ⊆ X − F and X − F ∈ λ-RO(X). There exists an rg_μ-open set V of Y such that Y − f(F) ⊆ V and $f^{-1}(V)$ ⊆ X − F. Therefore, we have f(F) ⊆ Y − V and F ⊆ $f^{-1}(Y − V)$. Hence, we obtain f(F) = Y − V and f(F) is rg_μ-closed in Y. This shows that f is almost (rg_λ, μ)-closed. □

5. Preservation Theorems

In this section we investigate preservation theorems concerning λ-mildly normal spaces.

Theorem 5.1. *If f : (X, λ) → (Y, μ) is an almost (rg_λ, μ)-continuous (λ, μ)-rc-preserving (resp. almost (λ, μ)-closed) injection and Y is μ-mildly normal (resp. μ-normal), then X is λ-mildly normal.*

Proof. Let A and B be any disjoint λ-regular closed sets of X. Since f is an (λ, μ)-rc-preserving (resp. almost (λ, μ)-closed) injection, f(A) and f(B) are disjoint μ-regular closed (resp. μ-closed) sets of Y. By the μ-mild normality (resp. μ-normality) of Y, there exists disjoint μ-open sets U and V of Y such that f(A) ⊆ U and f(B) ⊆ V. Now, put G = $i_\mu(c_\mu(U))$ and H = $i_\mu(c_\mu(V))$, then G and H are disjoint μ-regular open sets such that f(A) ⊆ G and f(B) ⊆ H. Since f is almost (rg_λ, μ)-continuous, $f^{-1}(G)$ and $f^{-1}(H)$ are disjoint rg_λ-open sets containing A and B, respectively. It follows from Theorem 4.3 that X is λ-mildly normal. □

Theorem 5.2. *If f : (X, λ) → (Y, μ) is a completely (λ, μ)-continuous, almost (g_λ, μ)-closed surjection and X is λ-mildly normal, then Y is μ-normal.*

Proof. Let A and B be any disjoint μ-closed sets of Y. Then $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint λ-regular closed sets of X. Since X is λ-mildly normal, there exists disjoint λ-open sets U and V of X such that $f^{-1}(A)$ ⊆ U and $f^{-1}(B)$ ⊆ V. Let G = $i_\lambda(c_\lambda(U))$ and H = $i_\lambda(c_\lambda(V))$, then G and H are disjoint λ-regular open sets such that $f^{-1}(A)$ ⊆ G and $f^{-1}(B)$ ⊆ H. By Proposition 5.16, there exists g_μ-open sets K and L of Y such that A ⊆ K, B ⊆ L, $f^{-1}(K)$ ⊆ G and $f^{-1}(L)$ ⊆ H. Since G and H are disjoint, so are K and L. Since K and L are g_μ-open, we obtain A ⊆ $i_\mu(K)$, B ⊆ $i_\mu(L)$ and $i_\mu(K)$ ∈ $i_\mu(L)$ = φ. This shows that Y is μ-normal. □

Corollary 5.3. *If f : (X, λ) → (Y, μ) is a completely (λ, μ)-continuous (λ, μ)-closed surjection and X is λ-mildly normal, then Y is μ-normal.*

Theorem 5.4. *If f : (X, λ) → (Y, μ) be an R_λ-map (resp. almost (λ, μ)-continuous) and almost (rg_λ, μ)-closed surjection. If X is λ-mildly normal (resp. λ-normal), then Y is μ-mildly normal.*

Proof. Let A and B be any disjoint μ-regular closed sets of Y. Then $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint λ-regular closed (resp. λ-closed) sets of X. Since X is λ-mildly normal (resp. λ-normal), there exists disjoint λ-open sets U and V of X such that $f^{-1}(A) \subseteq U$ and $f^{-1}(B) \subseteq V$. Let $G = i_\lambda(c_\lambda(U))$ and $H = i_\lambda(c_\lambda(V))$, then G and H are disjoint λ-regular open sets such that $f^{-1}(A) \subseteq G$ and $f^{-1}(B) \subseteq H$. By Proposition 5.16, there exists rg_μ-open sets K and L of Y such that $A \subseteq K$, $B \subseteq L$, $f^{-1}(K) \subseteq G$ and $f^{-1}(L) \subseteq H$. Since G and H are disjoint, so are K and L. It follows from Theorem 4.3 that Y is μ-mildly normal. □

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