International Journal of Current Research in Science and Technology
Volume 3, Issue 9 (2017), 1–11.
ISSN: 2394-5745
Available Online: http://ijcrst.in/

International Journal of Current Research in Science and Technology

(Λ, μ)-Closed Sets and the Related Notions

Research Article

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Abstract: We introduce and study the notions of (Λ, μ)-continuity, (Λ, μ)-irresoluteness, (Λ, μ)-compactness and (Λ, μ)-connectedness.

MSC: 54A05.

Keywords: μ-open set, $\wedge_\mu^*$-set, g_μ-open set Λ_μ-set, (Λ, μ)-continuity and quasi-(Λ, μ)-irresolute.

1. Introduction and Preliminaries

It is observed that a large number of papers are devoted to the study of generalized open like sets of a topological space containing the class of open sets and possessing properties more or less similar to those of open sets. This paper deals with the notions of Λ_μ-sets and (Λ, μ)-closed sets which are defined by utilizing the notions of μ-open and μ-closed sets. We also introduce and characterize some new low separation axioms. Moreover, we introduce and study the notions of (Λ, μ)-continuity, (Λ, μ)-irresoluteness, (Λ, μ)-compactness and (Λ, μ)-connectedness.

Definition 1.1 ([3]). *Let X be a nonempty set and λ be a collection of subsets of X. Then λ is called a generalized topology (briefly GT) on X iff $\phi \in \lambda$ and $G_1 \in \lambda$ for $i \in I \neq \phi$ implies $G = \bigcup_{i\in I} G_i \in \lambda$. We call the pair (X, λ) a generalized topological space (briefly GTS) on X.*

The elements of λ are called λ-open sets and the complements are called λ-closed sets. The generalized closure of a subset S of X, denoted by c_λ(S), is the intersection of λ-closed sets including S. And the interior of S, denoted by i_λ(S), is the union of λ-open sets contained in S.

Definition 1.2 ([1]). *Let (X, μ) and (Y, λ) be GTS's. Then a function $f : (X, \mu) \to (Y, \lambda)$ is said to be μ-α-irresolute if the inverse image of every λ-α-open set in Y is an μ-α-open set in X.*

Definition 1.3 ([7]). *Let (X, μ) and (Y, λ) be GTS's. Then a function $f : (X, \mu) \to (Y, \lambda)$ is said to be (μ, λ)-open if the image of each μ-open set in X is an λ-open set in Y.*

Definition 1.4 ([8]). *Let A be a subset of a GTS (X, μ). A subset $\Lambda_\mu(A)$ is defined as follows:*

$$\Lambda_\mu(A) = \begin{cases} \cap\{G : G \in \mu, A \subseteq A\} & \text{if there exists } \ G \in \mu \ \text{ and } \ A \subseteq G, \\ X, & \text{otherwise} \end{cases}$$

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Lemma 1.5 ([8]). *For subsets A, B and $A_i (i \in I)$ of a GTS (X, μ), the following properties hold:*

(1). $A \subseteq \Lambda_\mu(A)$;

(2). If $A \subseteq B$, then $\Lambda_\mu(A) \subseteq \Lambda_\mu(B)$;

(3). $\Lambda_\mu(\Lambda_\mu(A)) = \Lambda_\mu(A)$;

(4). $\Lambda_\mu(\cap\{A_i \mid i \in I\}) \subseteq \cap\{\Lambda_\mu(A_i) \mid i \in I\}$;

(5). $\Lambda_\mu(\cup\{A_i \mid i \in I\}) = \cup\{\Lambda_\mu(A_i) \mid i \in I\}$.

Definition 1.6 ([8]). *A subset A of a GTS (X, μ) is called a Λ_μ-set if $A = \Lambda_\mu(A)$.*

Lemma 1.7 ([8]). *For subsets A and A_i $(i \in I)$ of a GTS (X, μ), the following hold:*

(1). $\Lambda_\mu(A)$ is a Λ_μ-set.

(2). If A is μ-open, then A is a Λ_μ-set.

(3). If A_i is a Λ_μ-set for each $i \in I$, then $\cap_{i\in I} A_i$ is a Λ_μ-set.

(4). If A_i is a Λ_μ-set for each $i \in I$, then $\cup_{i\in I} A_i$ is a Λ_μ-set.

Definition 1.8 ([8]). *A subset A of a GTS (X, μ) is called (Λ, μ)-closed if $A = T \cap C$, where T is a Λ_μ-set and C is a μ-closed set. The complement of a (Λ, μ)-closed set is called a (Λ, μ)-open set. We shall denote the collection of all (Λ, μ)-open sets (resp. (Λ, μ)-closed sets) in a GTS (X, μ) by Λ_μ-$O(X, \mu)$ (resp. Λ_μ-$C(X, \mu)$).*

Theorem 1.9 ([8]). *Let A be (Λ, μ)-closed subset of a GTS (X, μ). Then, we have*

(1). $A = T \cap c_\mu(A)$, where T is a Λ_μ-set;

(2). $A = \Lambda_\mu(A) \cap c_\mu(A)$.

Lemma 1.10 ([8]). *For a GTS (X, μ),*

(1). Every Λ_μ-set (resp. μ-closed set) is (Λ, μ)-closed;

(2). Λ_μ-$C(X, \mu)$ (resp. Λ_μ-$O(X, \mu)$) is closed under arbitrary intersection (resp. union).

Definition 1.11 ([5]). *A subset A of a GTS (X, μ) is called a generalized μ-closed set (briefly. g_μ-closed) if $c_\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is μ-open in (X, μ). A subset A is said to be g_μ-open if $X - A$ is g_μ-closed.*

Definition 1.12 ([9]). *A GTS (X, μ) is called an μ-R_0 space if for each μ-open set U and each $x \in U$, $c_\mu(\{x\}) \subseteq U$.*

Definition 1.13 ([10]). *A GTS (X, μ) is said to be μ-T_1 if for any distinct pair of points x and y in X, there is an μ-open set U in X containing x but not y and an μ-open set V in X containing y but not x.*

Definition 1.14 ([8]). *Let (X, μ) be a GTS and $A \subseteq X$. A point $x \in X$ is called a (Λ, μ)-cluster point of A if for every (Λ, μ)-open set U of X containing x we have $A \cap U = \phi$. The set of all (Λ, μ)-cluster points is called the (Λ, μ)-closure set of A and is denoted by $A^{(\Lambda,\mu)}$.*

Lemma 1.15 ([8]). *Let A and B be subsets of a GTS (X, μ). For the (Λ, μ)-closure, the following properties hold.*

(1). $A \subseteq A^{(\Lambda,\mu)}$ and $(A^{(\Lambda,\mu)})^{(\Lambda,\mu)} = A^{(\Lambda,\mu)}$.

(2). $A^{(\Lambda,\mu)} = \cap\{F : A \subseteq F$ and F is $(\Lambda,\ \mu)$-closed$\}$.

(3). If $A \subseteq B$, then $A^{(\Lambda,\mu)} \subseteq B^{(\Lambda,\mu)}$.

(4). A is $(\Lambda,\ \mu)$-closed if and only if $A = A^{(\Lambda,\mu)}$.

(5). $A^{(\Lambda,\mu)}$ is $(\Lambda,\ \mu)$-closed.

Definition 1.16 ([8]).

(1). A subset A of a GTS $(X,\ \mu)$ is called a Λ_μ-D set if there are two $(\Lambda,\ \mu)$-open sets U, V in X such that $U \neq X$ and $A = U - V$. Observe that a $(\Lambda,\ \mu)$-open set $A \neq X$ is Λ_μ-D set since $A = U$ and $V = \phi$.

(2). A GTS $(X,\ \mu)$ is Λ_μ-D_0 (resp. Λ_μ-D_1) if for x, $y \in X$ such that $x \neq y$ there exists a Λ_μ-D set of X containing x but not y or (resp. and) a Λ_μ-D set containing y but not x.

(3). A GTS $(X,\ \mu)$ is Λ_μ-D_2 if for x, $y \in X$ such that $x \neq y$ there exist disjoint Λ_μ-D sets G_1 and G_2 such that $x \in G_1$ and $y \in G_2$.

Definition 1.17 ([8]). *A GTS $(X,\ \mu)$ satisfies $(\Lambda,\ \mu)$-property if for any distinct pair of points in X, there is a $(\Lambda,\ \mu)$-open set containing one of the points but not the other.*

Remark 1.18 ([7]).

(1). If $(X,\ \mu)$ satisfies $(\Lambda,\ \mu)$-property, then $(X,\ \mu)$ is Λ_μ-D_0.

(2). If $(X,\ \mu)$ is Λ_μ-D_i, then $(X,\ \mu)$ is Λ_μ-D_{i-1}, $i = 1,\ 2$.

Theorem 1.19 ([8]). *For a GTS $(X,\ \mu)$ the following statements are true.*

(1). $(X,\ \mu)$ is Λ_μ-D_0 if and only if it satisfies $(\Lambda,\ \mu)$-property.

(2). $(X,\ \mu)$ is Λ_μ-D_1 if and only if it is Λ_μ-D_2.

Definition 1.20 ([4]). *A GTS $(X,\ \mu)$ is said to be μ-compact if every μ-open cover of X has a finite μ-open subcover.*

2. $(\Lambda,\ \mu)$-Closed Sets

Definition 2.1. *A GTS $(X,\ \mu)$ is said to be μ-Alexandroff space if every point has a minimal neighbourhood or equivalently, has a unique minimal base.*

Theorem 2.2. *For a GTS $(X,\ \mu)$, we put $\mu^{\Lambda_\mu} = \{A$: A is a Λ_μ-set of $X\}$. Then the pair $(X,\ \mu^{\Lambda_\mu})$ is an μ-Alexandroff space.*

Theorem 2.3. *Let A_i $(i \in I)$ be a subset of a GTS $(X,\ \mu)$.*

(1). If A_i is $(\Lambda,\ \mu)$-closed for each $i \in I$, then $\cap\{A_i \mid i \in I\}$ is $(\Lambda,\ \mu)$-closed.

(2). If A_i is $(\Lambda,\ \mu)$-open for each $i \in I$, then $\cup\{A_i| \ i \in I\}$ is $(\Lambda,\ \mu)$-open.

Proof.

(1). Suppose that A_i is (Λ, μ)-closed for each $i \in I$. Then, for each i, there exist a Λ_μ-set T_i and an μ-closed set C_i such that $A_i = T_i \cap C_i$. We have $\cap_{i\in I} A_i = \cap_{i\in I}(T_i \cap C_i) = (\cap_{i\in I} T_i) \cap (\cap_{i\in I} C_i)$. By Lemma 2.7, $\cap_{i\in I} T_i$ is a Λ_μ-set and $\cap_{i\in I} C_i$ is a μ-closed. This shows that $\cap_{i\in I} A_i$ is (Λ, μ)-closed.

(2). Let A_i be (Λ, μ)-open for each $i \in I$. Then $X - A_i$ is (Λ, μ)-closed and $X - \cup_{i\in I} A_i = \cap_{i\in I}(X - A_i)$. Therefore, by (1) $\cup_{i\in I} A_i$ is (Λ, μ)-open. □

Definition 2.4. *Let A be a subset of a GTS (X, μ). A set* $\wedge_\mu^*(A)$ *is defined as follows* $\wedge_\mu^*(A) = \cup\{B : B \,.\, \mu_c,\, B \subseteq A\}$.

Definition 2.5. *A subset A of a GTS (X, μ) is called a* $\wedge_\mu^*$*-set if* $A = \wedge_\mu^*(A)$. *We obtain the following two lemmas which are similar to Lemma 2.5 and Lemma 2.7.*

Lemma 2.6. *For subsets A, B and* $A_i (i \in I)$ *of a GTS (X, μ), the following properties hold:*

(1). $\wedge_\mu^*(A) \subseteq A$.

(2). If $A \subseteq B$, *then* $\wedge_\mu^*(A) \subseteq \wedge_\mu^*(B)$.

(3). If A is μ-closed, then $\wedge_\mu^*(A) = A$.

(4). $\wedge_\mu^*(\cap\{A_i : i \in I\}) = \cap\{\wedge_\mu^*(A_i) : i \in I\}$.

(5). $\cup\{\wedge_\mu^*(A_i) : i \in I\} \subseteq \wedge_\mu^*(\cup\{A_i : i \in I\})$.

(6). $\Lambda_\mu(X - A) = X - \wedge_\mu^*(A)$ *and* $\wedge_\mu^*(X - A) = X - \Lambda_\mu(A)$.

Lemma 2.7. *For subsets A, B and* $A_i (i \in I)$ *of a GTS (X, μ), the following properties hold:*

(1). $\wedge_\mu^*(A)$ *is a* $\wedge_\mu^*$*-set.*

(2). If A is μ-closed, then A is a $\wedge_\mu^*$*-set.*

(3). If A_i *is a* $\wedge_\mu^*$*-set for each* $i \in I$, *then* $\cup\{A_i : i \in I\}$ *and* $\cap\{A_i : i \in I\}$ *are* $\wedge_\mu^*$*-sets.*

The following two lemmas are obtained easily from the definitions.

Lemma 2.8. *For a subset A of a GTS (X, μ), the following properties hold:*

(1). A is g_μ*-closed if and only if* $c_\mu(A) \subseteq \Lambda_\mu(A)$.

(2). A is μ-closed if and only if A is g_μ*-closed and* (Λ, μ)*-closed.*

Lemma 2.9. *For a subset A of a GTS (X, μ), the following properties hold:*

(1). A is g_μ*-open if and only if* $\wedge_\mu^*(A) \subseteq i_\mu(A)$.

(2). A is μ-open if and only if A is g_μ*-open and* (Λ, μ)*-open.*

Theorem 2.10. *Let A be a* (Λ, μ)*-open subset of a GTS (X, μ). Then, we have*

(1). $A = T \cup C$, *where T is a* $\wedge_\mu^*$*-set and C is μ-open;*

(2). $A = T \cup i_\mu(A)$, *where T is a* $\wedge_\mu^*$*-set;*

(3). $A = \wedge_\mu^*(A) \cup i_\mu(A)$.

Proof.

(1). Suppose that A is (Λ, μ)-open. Then X − A is (Λ, μ)-closed and X − A = K ∩ D, where K is a Λ_μ-set and D is a μ-closed set. Hence, we have A = (X − K) ∪ (X − D), where X − K is a $\wedge_\mu^*$-set and X − D is μ-open set.

(2). Since A is an (Λ, μ)-open we have A = T ∪ C, where T is an $\wedge_\mu^*$-set and C is μ-open. Also C ⊆ A and C is μ-open, C ⊆ i_μ(A) and hence A = T ∪ C ⊆ T ∪ i_μ(A) ⊆ A. Therefore, we obtain A = T ∪ i_μ(A).

(3). Since A is an (Λ, μ)-open we have A = T ∪ i_μ(A), where T is a $\wedge_\mu^*$-set. Also T ⊆ A, we have $\wedge_\mu^*$(A) ⊇ $\wedge_\mu^*$(T) and hence A ⊇ $\wedge_\mu^*$(A) ∪ i_μ(A) ⊇ $\wedge_\mu^*$(T) ∪ i_μ(A) = T ∪ i_μ(A) = A. Therefore, we obtain A = $\wedge_\mu^*$(A) ∪ i_μ(A). □

Theorem 2.11. *Let (X, μ) be a μ-R_0 space. A singleton $\{x\}$ is (Λ, μ)-closed if and only if $\{x\}$ is μ-closed.*

Proof. Necessity. Suppose that {x} is (Λ, μ)-closed. Then, by Theorem 2.9, {x} = Λ_μ({x}) ∩ c_μ({x}). For any μ-open set U containing x, c_μ({x}) ⊆ U and hence c_μ({x}) ⊆ Λ_μ({x}). Therefore, we have {x} = Λ_μ({x}) ∩ c_μ({x}) ⊇ c_μ({x}). This shows that {x} is μ-closed.

Sufficiency. Suppose that {x} is μ-closed. Since {x} ⊆ Λ_μ({x}), we have Λ_μ({x}) ∩ c_μ({x}) = Λ_μ({x}) ∩ {x} = {x}. This shows that {x} is (Λ, μ)-closed. □

Theorem 2.12. *A GTS (X, μ) is μ-T_1 if and only if for each x . X, the singleton $\{x\}$ is a Λ_μ-set.*

Proof. Necessity. Suppose that y ∈ Λ_μ({x}) for some point y distinct from x. Then y ∈ ∩{V_x | x ∈ V_x and V_x is μ-open} and hence y ∈ V_x for every μ-open set V_x containing x. This contradicts that (X, μ) is an μ-T_1.

Sufficiency. Suppose that {x} is a Λ_μ-set for each x ∈ X. Let x and y be any distinct points. Then y ∉ Λ_μ({x}) and there exists an μ-open set V_x such that x ∈ V_x and y ∉ V_x. Similarly, x ∉ Λ_μ({y}) and there exists an μ-open set V_y such that y ∈ V_y and x ∉ V_y. This shows that (X, μ) is μ-T_1. □

Theorem 2.13. *A GTS (X, μ) is μ-T_1 if and only if (X, μ^{Λ_μ}) is the discrete space.*

Proof. Necessity. Suppose that (X, μ) is μ-T_1. Let x be any point of X. By Theorem 3.12, {x} is a Λ_μ-set and {x} ∈ μ^{Λ_μ}. For any subset A of X, by Lemma 2.7, Λ_μ(A) ∈ μ^{Λ_μ}. This shows that (X, μ^{Λ_μ}) is discrete.

Sufficiency. For each x ∈ X, {x} ∈ μ^{Λ_μ} and hence {x} is Λ_μ-set. By Theorem 3.12, (X, μ) is μ-T_1. □

Theorem 2.14. *If a function $f : (X, \mu) \to (Y, \lambda)$ is μ-α-irresolute, then $f : (X, \mu^{\Lambda_\mu}) \to (Y, \lambda^{\Lambda\lambda})$ is (μ, λ)-continuous.*

Proof. Let V be any Λ_λ-set of (Y, λ), i.e. V ∈ $\lambda^{\Lambda\lambda}$. Then V = Λ_λ(V) = ∩{W : V ⊆ W and W is λ-α-open in (Y, λ)}. Since f is λ-α-irresolute, f^{-1}(W) is μ-α-open in (X, μ) for each W. Hence we have f^{-1}(V) = ∩{f^{-1}(W) : f^{-1}(V) ⊆ f^{-1}(W) and W is λ-α-open in (Y, λ)} ⊇ ∩{U : f^{-1}(V) ⊆ U and U is μ-open in (X, μ)} = Λ_μ(f^{-1}(V)). On the other hand, by the definition f^{-1}(V) ⊆ Λ_μ(f^{-1}(V)). Therefore, we obtain f^{-1}(V) = Λ_μ(f^{-1}(V)). Hence, f^{-1}(V) ∈ μ^{Λ_μ} and f : (X, μ) → (Y, λ) is (μ, λ)-continuous. □

3. (Λ, μ)-Continuous Functions

Definition 3.1. *Let (X, μ) be a GTS, $x \in X$ and $\{x_s, s \in S\}$ be a net of X. We say that the net $\{x_s, s \in S\}$ (Λ, μ)-converges to x if for each (Λ, μ)-open set U containing x there exists an element $s_0 \in S$ such that $s \leq s_0$ implies $x_s \in U$.*

Lemma 3.2. *Let (X, μ) be a GTS and $A \subseteq X$. A point $x \in A^{(\Lambda,\mu)}$ if and only if there exists a net $\{x_s, s \in S\}$ of A which (Λ, μ)-converges to x.*

Definition 3.3. *Let (X, μ) be a GTS, $F = \{F_i : i \in I\}$ be a filterbase of X and $x \in X$. We say that the filter base F(Λ, μ)-converges to x if for each (Λ, μ)-open set U containing x there is a member $F_i \in F$ such that $F_i \subseteq U$.*

Definition 3.4. *A function $f : (X, \mu) \to (Y, \lambda)$ is called (Λ, μ)-continuous if $f^{-1}(V)$ is a (Λ, μ)-open subset of X for every λ-open subset V of Y.*

Theorem 3.5. *For a function $f : (X, \mu) \to (Y, \lambda)$, the following statements are equivalent:*

(1). f is (Λ, μ)-continuous;

(2). For each $x \in X$ and for each open set V of Y containing $f(x)$ there exists a (Λ, μ)-open set U of X containing x and $f(U) \subseteq V$;

(3). For each $x \in X$ and each filterbase F which (Λ, μ)-converges to x, $f(F)$ converges to $f(x)$.

(4). For each $x \in X$ and each net $\{x_s, s \in S\}$ in X which (Λ, μ)-converges to x, the net $\{f(x_s), s \in S\}$ of Y converges to $f(x) \in Y$.

Definition 3.6. *A function $f : (X, \mu) \to (Y, \lambda)$ is called (Λ, μ)-irresolute if $f^{-1}(V)$ is a (Λ, μ)-open subset of X for every (Λ, λ)-open subset V of Y. Now we have the following result with its proof is obvious.*

Theorem 3.7. *For a function $f : (X, \mu) \to (Y, \lambda)$, the following statements are equivalent.*

(1). f is (Λ, μ)-irresolute;

(2). $f^{-1}(B)$ is a (Λ, μ)-closed subset of X for every (Λ, λ)-closed subset B of Y;

(3). For each $x \in X$ and for each (Λ, λ)-open set V of Y containing $f(x)$ there exists a (Λ, μ)-open set U of X containing x and $f(U) \subseteq V$;

(4). $f(A^{(\Lambda,\lambda)}) \subseteq [f(A)]^{(\Lambda,\lambda)}$ for each subset A of X;

(5). $[f^{-1}(B)]^{(\Lambda,\lambda)} \subseteq f^{-1}(B^{(\Lambda,\lambda)})$ for each subset B of Y;

(6). For each $x \in X$ and each filterbase F which (Λ, μ)-converges to x, $f(F)$ (Λ, λ)-converges to $f(x)$;

(7). For each $x \in X$ and each net $\{x_s, s \in S\}$ in X which (Λ, μ)-converges to x, we have that the net $\{f(x_s), s \in S\}$ of Y (Λ, λ)-converges to $f(x) \in Y$.

Definition 3.8. *A function $f : (X, \mu) \to (Y, \lambda)$ is called quasi-(Λ, μ)-irresolute if $f^{-1}(V)$ is a (Λ, μ)-open subset of X for every λ-α-open subset V of Y.*

Theorem 3.9. *For a function $f : (X, \mu) \to (Y, \lambda)$, the following statements are equivalent.*

(1). f is quasi-(Λ, μ)-irresolute;

(2). For each $x \in X$ and for each λ-open set V of Y containing $f(x)$ there exists a (Λ, μ)-open set U of X containing x and $f(U) \subseteq V$;

(3). For each $x \in X$ and each filterbase F which (Λ, μ)-converges to x, $f(F)$ λ-converges to $f(x)$ (that is, for each μ-open set U containing $f(x)$ there is a member $F_i \in F$ such that $F_i \subseteq U$);

(4). For each $x \in X$ and each net $\{x_s, s \in S\}$ in X which (Λ, μ)-converges to x, the net $\{f(x_s), s \in S\}$ of Y λ-converges to $f(x) \in Y$ (i.e. for each μ-open set U containing $f(x)$ there exists an element $s_0 \in S$ such that $s \geq s_0$ implies $f(x_s) \in U$).

Theorem 3.10. *For a function $f : (X, \mu) \to (Y, \lambda)$, the following statements are true.*

(1). If the function f is (Λ, μ)-irresolute, then the function f is (Λ, μ)-continuous and quasi-(Λ, μ)-irresolute.

(2). If the function f is quasi-(Λ, μ)-irresolute, then the function f is (Λ, μ)-continuous.

(3). If the function f is μ-irresolute, then the function f is quasi-(Λ, μ)-irresolute.

(4). If the function f is μ-continuous, then the function f is (Λ, μ)-continuous.

Example 3.11. *Let (X, μ) be a GTS such that $X = \{a, b, c\}$ with $\mu = \{\phi\ X, \{a\}, \{a, b\}, \{a, c\}\}$. Also, the family of all $\wedge_\mu^*$-sets is $\{\phi, X, \{c\}, \{b\}, \{b, c\}\}$ and the family of all (Λ, μ)-open sets is $\{\phi, X, \{a, b\}, \{a, c\}, \{b\}, \{c\}, \{a\}, \{b, c\}\}$. We consider the function $f : X \to X$ defined by $f(c) = a$ and $f(a) = f(b) = c$. We have*

(1). f is (Λ, μ)-irresolute, quasi-(Λ, μ)-irresolute and (Λ, μ)-continuous,

(2). f is not μ-irresolute, since if $x = c$ and $\{a\}$ is the μ-open neighbourhood of $f(c) = a$ in X, then for every μ-open neighbourhood of c in X we have $f(U) \nsubseteq \{a\}$, and

(3). f is not (α, μ)-continuous and the proof is similar to that of (2).

Example 3.12. *Let (X, μ) be a GTS such that $X = \{a, b, c, d\}$ and $\mu = \{\phi, X, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}\}$. Also, the family of all $\wedge_\mu^*$-sets is $\{\phi, X, \{c, d\}, \{a, c, d\}, \{a, d\}, \{a, c\}, \{d\}, \{c\}\}$ and the family of all (Λ, μ)-open sets is $\{\phi, X, \{a, b\}, \{b\}, \{c\}, \{d\}, \{b, c, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{a, d\}, \{a, c\}, \{c, d\}\}$. We consider the function $f : X \to X$ defined as follows: $f(a) = d$, $f(b) = c$, $f(c) = d$ and $f(d) = a$. The following hold.*

(1). f is not (Λ, μ)-irresolute at the point a since if $\{d\}$ is the (Λ, μ)-open neighbourhood of $f(a) = d$ in X, then $f(U) \nsubseteq \{d\}$ for every (Λ, μ)-open neighbourhood of a in X and

(2). f is (Λ, μ)-continuous.

4. Λ_μ-D Sets and Associated Separation Axioms

Definition 4.1. *A GTS (X, μ) is called Λ_μ-T_1 if for any distinct pair of points x and y in X, there is a (Λ, μ)-open set U in X containing x but not y and a (Λ, μ)-open set V in X containing y but not x.*

Definition 4.2. *A GTS (X, μ) is called Λ_μ-T_2 if for any distinct pair of points x and y in X, there exist (Λ, μ)-open sets U and V in X containing x and y, respectively, such that $U \cap V = \phi$.*

Definition 4.3. *A GTS (X, μ) is called μ-α-R_0 if for each μ-α-open set U and each $x \in U$, $C_\alpha(x) \subseteq U$.*

Definition 4.4. *A GTS (X, μ) is called sober $(\mu$-$\alpha)$- R_0 if $\cap_{x\in X} c_\mu(\{x\}) = \phi$.*

Example 4.5. *Let (X, μ) be a GTS such that $X = \{a, b, c\}$ and $\mu = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$. Clearly, the singletons $\{a\}$, $\{b\}$ and $\{c\}$ are Λ_μ-D sets. Now we have*

(1). (X, μ) is Λ_μ-T_i, $i = 0, 1, 2$,

(2). (X, μ) is Λ_μ-D_i, $i = 0, 1, 2$, and

(3). (X, μ) is not μ-R_0.

Example 4.6. *Let (X, μ) be a GTS such that $X = \{a, b, c, d\}$ and $\mu = \{\phi, X, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}\}$. The singletons $\{a\}$, $\{b\}$, $\{c\}$ and $\{d\}$ are Λ_μ-D sets. We have*

(1). (X, μ) is not μ-T_i, i = 1, 2, but satisfies (Λ, μ)-property,

(2). (X, μ) is Λ_μ-T_i, i = 1, 2, and

(3). (X, μ) is Λ_μ-D_i, i = 0, 1, 2.

Example 4.7. *Let (X, μ) be a GTS such that $X = \{a, b, c, d\}$ and $\mu = \{\phi, X, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$. The family of $\wedge_\mu^*$-sets is $\{\phi, X, \{c, d\}, \{d\}, \{c\}\}$ and the family of (Λ, μ)-open sets is $\{\phi, X, \{a, b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, b\}, \{c, d\}, \{d\}, \{c\}\}$. So, we have*

(1). (X, μ) is not Λ_μ-D_i, i = 0, 1, 2,

(2). (X, μ) is not μ-R_0 and μ-R_1 and

(3). (X, μ) is sober (μ-α)-R_0 (i.e., $\cap_{x\in X}\ c_\mu(\{x\}) = \phi$).

Theorem 4.8. *A GTS (X, μ) satisfies (Λ, μ)-property if and only if for each pair of distinct points x, y of X, $\{x\}^{(\Lambda,\mu)} \neq \{y\}^{(\Lambda,\mu)}$.*

Proof. Sufficiency. Suppose that x, y $\in$ X, x $\neq$ y and $\{x\}^{(\Lambda,\mu)} \neq \{y\}^{(\Lambda,\mu)}$. Let z be a point of X such that $z \in \{x\}^{(\Lambda,\mu)}$ but $z \notin \{y\}^{(\Lambda,\mu)}$. We claim that $x \notin \{y\}^{(\Lambda,\mu)}$. For, if $x \in \{y\}^{(\Lambda,\mu)}$ then $\{x\}^{(\Lambda,\mu)} \subseteq \{y\}^{(\Lambda,\mu)}$. This contradicts the fact that z $\notin \{y\}^{(\Lambda,\mu)}$. Consequently x belongs to the (Λ, μ)-open set $[\{y\}^{(\Lambda,\mu)}]^c$ to which y does not belong.
Necessity. Let (X, μ) satisfies (Λ, μ)-property and x, y be any two distinct points of X. There exists a (Λ, μ)-open set G containing x or y, say x but not y. Then G^c is a (Λ, μ)-closed set which does not contain x but contains y. Since $\{y\}^{(\Lambda,\mu)}$ is the smallest (Λ, μ)-closed set containing y (Lemma 2.15(2)), $\{y\}^{(\Lambda,\mu)} \subseteq G^c$ and so $x \notin \{y\}^{(\Lambda,\mu)}$. Therefore $\{x\}^{(\Lambda,\mu)} \neq \{y\}^{(\Lambda,\mu)}$. □

Theorem 4.9. *A GTS (X, μ) is Λ_μ-T_1 if and only if the singletons are (Λ, μ)-closed sets.*

Proof. Suppose (X, μ) is Λ_μ-T_1 and x be any point of X. Let $y \in \{x\}^c$. Then x $\neq$ y and so there exists a (Λ, μ)-open set U_y such that $y \in U_y$ but $x \notin U_y$. Consequently $y \in U_y \subseteq \{x\}^c$ i.e., $\{x\}^c = \cup\{U_y \mid y \in \{x\}^c\}$ which is (Λ, μ)-open.
To prove the converse, suppose {p} is (Λ, μ)-closed for every p $\in$ X. Let x, y $\in$ X with x $\neq$ y. Now x $\neq$ y implies $y \in \{x\}^c$. Hence $\{x\}^c$ is a (Λ, μ)-open set containing y but not containing x. Similarly $\{y\}^c$ is a (Λ, μ)-open set containing x but not containing y. This means that X is a Λ_μ-T_1 space. □

Theorem 4.10. *If f : (X, μ) $\to$ (Y, λ) is a (Λ, μ)-irresolute surjective function and E is a Λ_μ-D set in Y, then the inverse image of E is a Λ_μ-D set in X .*

Proof. Let E be a Λ_μ-D set in Y. Then there are (Λ, μ)-open sets U_1 and U_2 in Y such that $S = U_1 - U_2$ and $U_1 \neq Y$. By the (Λ, μ)-irresoluteness of f, $f^{-1}(U_1)$ and $f^{-1}(U_2)$ are (Λ, μ)-open in X. Since $U_1 \neq Y$, we have $f^{-1}(U_1) \neq X$. Hence $f^{-1}(E) = f^{-1}(U_1) - f^{-1}(U_2)$ is a Λ_μ-D set in X. □

Theorem 4.11. *If (Y, λ) is Λ_λ-D_1 and f : (X, μ) $\to$ (Y, λ) is (Λ, μ)-irresolute and bijective, then (X, μ) is Λ_λ-D_1.*

Proof. Suppose that Y is a Λ_λ-D_1 space. Let x and y be any pair of distinct points in X. Since f is injective and Y is Λ_λ-D_1, there exist Λ_λ-D sets G_x and G_y of Y containing f(x) and f(y) respectively, such that $f(y) \notin G_x$ and $f(x) \notin G_y$. By Theorem 5.10, $f^{-1}(G_x)$ and $f^{-1}(G_y)$ are Λ_λ-D sets in X containing x and y respectively. This implies that X is a Λ_λ-D_1 space. □

Theorem 4.12. *A GTS (X, μ) is Λ_μ-D_1 if and only if for each pair of distinct points x, y $\in$ X, there exists a (Λ, μ)-irresolute surjective function f : (X, μ) $\to$ (Y, λ), where Y is a Λ_λ-D_1 space such that f(x) and f(y) are distinct.*

Proof. Necessity. For every pair of distinct points of X, it suffices to take the identity function on X.
Sufficiency. Let x and y be any pair of distinct points in X. By hypothesis, there exists a (Λ, μ)-irresolute, surjective function f of a space X onto a Λ_λ-D_1 space Y such that f(x) $\neq$ f(y). Therefore, there exist disjoint Λ_μ-D sets G_x and G_y in Y such that f(x) $\in$ G_x and f(y) $\in$ G_y. Since f is (Λ, μ)-irresolute and surjective, by Theorem 5.10, $f^{-1}(G_x)$ and $f^{-1}(G_y)$ are disjoint Λ_μ-D sets in X containing x and y, respectively. Hence by Theorem 2.21(2) X is Λ_μ-D_1 space. □

5. (Λ, μ)-Compactness and (Λ, μ)-Connectedness

Definition 5.1. *A GTS (X, μ) is said to be (Λ, μ)-compact if every cover of X by (Λ, μ)-open sets of (X, μ) has a finite subcover.*

Theorem 5.2. *A GTS (X, μ) is (Λ, μ)-compact if and only if for every family $\{A_i : i \in I\}$ of (Λ, μ)-closed sets in X satisfying $\cap\{A_i : i \in I\} = \phi$, there is a finite subfamily $A_{i1},\dots,A_{in}$ with $\cap\{A_{ik} : k = 1, \dots, n\} = \phi$.*

Theorem 5.3. *For a GTS (X, μ), the following properties hold.*

(1). If (X, μ^{Λ_μ}) is compact, then (X, μ) is μ-compact.

(2). If (X, μ) is (Λ, μ)-compact, then (X, μ) is μ-compact.

(3). If (X, μ) is (Λ, μ)-compact, then (X, $\wedge_\mu^*$) is compact.

Proof.

(1). Let $\{V_\mu : \mu \in \nabla\}$ be any μ-open cover of X. By Lemma 2.7, every μ-open V_μ is a Λ_μ-set for each $\mu \in \nabla$. Moreover, by the compactness of (X, μ^{Λ_μ}) there exists a finite subset ∇_0 of ∇ such that X = $\cup\{V_\mu \mid \mu \in \nabla_0\}$. This shows that (X, μ) is μ-compact.

(2). Let $\{F_\mu \mid \mu \in \nabla\}$ be a family of μ-closed sets of (X, μ) such that $\cap\{F\mu \mid \mu \in \nabla\} = \phi$. Every μ-closed is (Λ, μ)-closed for each $\mu \in \nabla$. By Theorem 6.2, there exists a finite subset ∇_0 of ∇ such that $\cap\{F_\mu \mid \mu \in \nabla_0\} = \phi$. It follows from [[2], Theorem 2.17] that (X, μ) is μ-compact.

(3). Let $\{V_\mu \mid \mu \in \nabla\}$ be a cover of X by $\wedge_\mu^*$-sets of (X, μ). Since $V_\mu = V_\mu \cup \phi$ and the empty set is μ-open, by Lemma 2.7 each $V\mu$ is (Λ, μ)-open in (X, μ). Since (X, μ) is (Λ, μ)-compact, there exists a finite subset ∇_0 of ∇ such that X = $\cup\{V_\mu \mid \mu \in \nabla_0\}$. This shows that (X, $\wedge_\mu^*$) is compact. □

Corollary 5.4. *If (X, μ) is (Λ, μ)-compact, then (X, μ) is compact.*

The following example shows that the converse of Corollary 6.4 does not hold.

Example 5.5. *Let I be an infinite space and let (X, μ) be a GTS such that X = $\{a\} \cup \{a_i : i \in I\}$ and $\mu = \{\phi, X, \{a\}\}$. Clearly, the space (X, μ) is compact but it is not (Λ, μ)-compact.*

Theorem 5.6. *If f : (X, μ) $\to$ (Y, λ) is a (Λ, μ)-irresolute surjection and (X, μ) is a (Λ, μ)-compact space, then (Y, λ) is (Λ, λ)-compact.*

Proof. Let $\{V_\lambda \mid \lambda \in \nabla\}$ be any cover of Y by (Λ, λ)-open sets of (Y, λ). Since f is (Λ, μ)-irresolute, by Theorem 5.8 $\{f^{-1}(V_\mu) \mid \mu \in \nabla\}$ is a cover of X by (Λ, μ)-open sets of (X, μ). Thus, there exists a finite subset ∇_0 of ∇ such that $X = \cup\{f^{-1}(V_\mu) \mid \mu \in \nabla_0\}$. Since f is surjective, we obtain $Y = f(X) = \cup\{V_\lambda \mid \lambda \in \nabla_0\}$. This shows that (Y, λ) is (Λ, λ)-compact. □

Definition 5.7. *A GTS (X, μ) is called (Λ, μ)-connected (resp. μ-α-connected) if X cannot be written as a disjoint union of two non-empty (Λ, μ)-open (resp. μ-α-open) sets.*

The proof of the following theorem is straightforward and therefore is omitted.

Theorem 5.8. *Every (Λ, μ)-connected space is μ-α-connected space.*

The following example shows that μ-connectedness does not imply (Λ, μ)-connectedness.

Example 5.9. *Let (X, μ) be a GTS such that $X = \{a, b, c\}$ and $\mu = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$. We have*

(1). (X, μ) is μ-connected and μ-α-connected, and

(2). (X, μ) is not (Λ, μ)-connected.

Theorem 5.10. *For a GTS (X, μ), the following statements are equivalent.*

(1). (X, μ) is (Λ, μ)-connected;

(2). The only subsets of X, which are both (Λ, μ)-open and (Λ, μ)-closed are the empty set ϕ and X.

Theorem 5.11. *If a GTS (X, μ) is (Λ, μ)-connected, then (X, μ^{Λ_μ}) is connected.*

Proof. Suppose that (X, μ^{Λ_μ}) is not connected. There exist nonempty Λ_μ-sets G, H of (X, μ) such that $G \cap H = \phi$ and $G \cup H = X$. By Lemma 2.10, G and H are (Λ, μ)-closed sets. This shows that (X, μ) is not (Λ, μ)-connected. □

Theorem 5.12. *If $f : (X, \mu) \to (Y, \lambda)$ is a (Λ, μ)-irresolute surjection and (X, μ) is (Λ, μ)-connected, then (Y, λ) is (Λ, λ)-connected.*

Proof. Suppose that (Y, λ) is not (Λ, λ)-connected. There exist nonempty (Λ, λ)-open sets G, H of Y such that $G \cap H = \phi$ and $G \cup H = Y$. Then we have $f^{-1}(G) \cap f^{-1}(H) = \phi$ and $f^{-1}(G) \cup f^{-1}(H) = X$. Moreover, $f^{-1}(G)$ and $f^{-1}(H)$ are nonempty (Λ, μ)-open sets of (X, μ). This shows that (X, μ) is not (Λ, μ)- connected. Therefore, (Y, λ) is (Λ, λ)-connected. □

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