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$(1,2)^{\star}$-ψ-closed Sets

Research Article

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Abstract: In this paper, we introduce a new class of sets namely $(1,2)^{\star}$-ψ-closed sets in bitopological spaces. This class lies between the class of $\tau_{1,2}$-closed sets and the class of $(1,2)^{\star}$-$\hat{g}$-closed sets. The notion of $(1,2)^{\star}$-ψ-interior is defined and some of its basic properties are studied. Also we introduce the concept of $(1,2)^{\star}$-ψ-closure in bitopological spaces using the notion of $(1,2)^{\star}$-ψ-closed sets, and we obtain some related results. For any A $\subseteq$ X, it is proved that the complement of $(1,2)^{\star}$-ψ-interior of A is the $(1,2)^{\star}$-ψ-closure of the complement of A.

MSC: 54E55.

Keywords: $(1,2)^{\star}$-ψ-closed set, $\tau_{1,2}$-closed set, $(1,2)^{\star}$-$\hat{g}$-closed set, $(1,2)^{\star}$-αsg-closed set, $(1,2)^{\star}$-sg-open set.

1. Introduction

Levine [4] introduced generalized closed sets in general topology as a generalization of closed sets. This concept was found to be useful and many results in general topology were improved. Many researchers like Veerakumar [16] introduced $\hat{g}$-closed sets in topological spaces. Sheik John [15] introduced ω-closed sets in topological spaces. After the advent of these notions, many topologists introduced various types of generalized closed sets and studied their fundamental properties. Quite Recently, Ravi and Ganesan [5] introduced and studied ψ-closed sets in general topology as another generalization of closed sets and proved that the class of ψ-closed sets properly lies between the class of closed sets and the class of ω-closed sets. Ravi et al [12], Ravi and Thivagar [9] and Duszynski et al [1] introduced $(1,2)^{\star}$-g-closed sets, $(1,2)^{\star}$-sg-closed sets and $(1,2)^{\star}$-$\hat{g}$-closed sets respectively. In this paper, we introduce a new class of sets namely $(1,2)^{\star}$-ψ-closed sets in bitopological spaces. This class lies between the class of $\tau_{1,2}$-closed sets and the class of $(1,2)^{\star}$-$\hat{g}$-closed sets. The notion of $(1,2)^{\star}$-ψ-interior is defined and some of its basic properties are studied. Also we introduce the concept of $(1,2)^{\star}$-ψ-closure in bitopological spaces using the notion of $(1,2)^{\star}$-ψ-closed sets, and we obtain some related results. For any A $\subseteq$ X, it is proved that the complement of $(1,2)^{\star}$-ψ-interior of A is the $(1,2)^{\star}$-ψ-closure of the complement of A.

2. Preliminaries

Throughout this paper, (X, τ_1, τ_2) (briefly, X) will denote bitopological space.

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Definition 2.1. *Let S be a subset of X. Then S is said to be $\tau_{1,2}$-open [10] if $S = A \cup B$ where $A \in \tau_1$ and $B \in \tau_2$. The complement of $\tau_{1,2}$-open set is called $\tau_{1,2}$-closed. Notice that $\tau_{1,2}$-open sets need not necessarily form a topology.*

Definition 2.2 ([10])**.** *Let S be a subset of a bitopological space X. Then*

(1). the $\tau_{1,2}$-interior of S, denoted by $\tau_{1,2}$-int(S), is defined as $\cup \{F : F \subseteq S$ and F is $\tau_{1,2}$-open$\}$.

(2). the $\tau_{1,2}$-closure of S, denoted by $\tau_{1,2}$-cl(S), is defined as $\cap \{F : S \subseteq F$ and F is $\tau_{1,2}$-closed$\}$.

Definition 2.3. *A subset A of a bitopological space X is called*

(1). $(1,2)^{\star}$-semi-open [9] if $A \subseteq \tau_{1,2}$-cl($\tau_{1,2}$-int(A));

(2). $(1,2)^{\star}$-α-open [3] if $A \subseteq \tau_{1,2}$-int($\tau_{1,2}$-cl($\tau_{1,2}$-int(A)));

(3). $(1,2)^{\star}$-β-open [13] if $A \subseteq \tau_{1,2}$-cl($\tau_{1,2}$-int($\tau_{1,2}$-cl(A))).

The complements of the above mentioned open sets are called their respective closed sets.

The $(1,2)^{\star}$-semi-closure [14] (resp. $(1,2)^{\star}$-α-closure [14], $(1,2)^{\star}$-sp-closure [13]) of a subset A of X, denoted by $(1,2)^{\star}$-scl(A) (resp. $(1,2)^{\star}$-αcl(A), $(1,2)^{\star}$-spcl(A)), is defined to be the intersection of all $(1,2)^{\star}$-semi-closed (resp. $(1,2)^{\star}$-α-closed, $(1,2)^{\star}$-β-closed) sets of (X, τ_1, τ_2) containing A. It is known that $(1,2)^{\star}$-scl(A) (resp. $(1,2)^{\star}$-αcl(A), $(1,2)^{\star}$-spcl(A)) is a $(1,2)^{\star}$-semi-closed (resp. $(1,2)^{\star}$-α-closed, $(1,2)^{\star}$-β-closed) set.

Definition 2.4. *A subset A of a bitopological space (X, τ_1, τ_2) is called*

(1). $(1,2)^{\star}$-g-closed [12] if $\tau_{1,2}$-cl(A) $\subseteq U$ whenever $A \subseteq U$ and U is $\tau_{1,2}$-open in X. The complement of $(1,2)^{\star}$-g-closed set is called $(1,2)^{\star}$-g-open;

(2). $(1,2)^{\star}$-sg-closed [9] if $(1,2)^{\star}$-scl(A) $\subseteq U$ whenever $A \subseteq U$ and U is $(1,2)^{\star}$-semi-open in X. The complement of $(1,2)^{\star}$-sg-closed set is called $(1,2)^{\star}$-sg-open;

(3). $(1,2)^{\star}$-gs-closed [9] if $(1,2)^{\star}$-scl(A) $\subseteq U$ whenever $A \subseteq U$ and U is $\tau_{1,2}$-open in X. The complement of $(1,2)^{\star}$-gs-closed set is called $(1,2)^{\star}$-gs-open;

(4). $(1,2)^{\star}$-αg-closed [11] if $(1,2)^{\star}$-αcl(A) $\subseteq U$ whenever $A \subseteq U$ and U is $\tau_{1,2}$-open in X. The complement of $(1,2)^{\star}$-αg-closed set is called $(1,2)^{\star}$-αg-open;

(5). $(1,2)^{\star}$-$\hat{g}$-closed [1] or $(1,2)^{\star}$-ω-closed [2] if $\tau_{1,2}$-cl(A) $\subseteq U$ whenever $A \subseteq U$ and U is $(1,2)^{\star}$-semi-open in X. The complement of $(1,2)^{\star}$-$\hat{g}$-closed (resp. $(1,2)^{\star}$-ω-closed) set is called $(1,2)^{\star}$-$\hat{g}$-open (resp. $(1,2)^{\star}$-ω-open);

(6). $(1,2)^{\star}$-$\ddot{g}_{\alpha}$-closed [6] if $(1,2)^{\star}$-αcl(A) $\subseteq U$ whenever $A \subseteq U$ and U is $(1,2)^{\star}$-sg-open in X. The complement of $(1,2)^{\star}$-$\ddot{g}_{\alpha}$-closed set is called $(1,2)^{\star}$-$\ddot{g}_{\alpha}$-open;

(7). $(1,2)^{\star}$-gsp-closed [13] if $(1,2)^{\star}$-spcl(A) $\subseteq U$ whenever $A \subseteq U$ and U is $\tau_{1,2}$-open in X. The complement of $(1,2)^{\star}$-gsp-closed set is called $(1,2)^{\star}$-gsp-open;

(8). $(1,2)^{\star}$-ψ-closed [7] if $(1,2)^{\star}$scl(A) $\subseteq U$ whenever $A \subseteq U$ and U is $(1,2)^{\star}$-sg-open in X. The complement of $(1,2)^{\star}$-ψ-closed set is called $(1,2)^{\star}$-ψ-open.

Remark 2.5. *The collection of all $(1,2)^\star$-$\ddot{g}_\alpha$-closed (resp. $(1,2)^\star$-gsp-closed, $(1,2)^\star$-$\hat{g}$-closed, $(1,2)^\star$-g-closed, $(1,2)^\star$-gs-closed, $(1,2)^\star$-αg-closed, $(1,2)^\star$-sg-closed, $(1,2)^\star$-ψ-closed, $(1,2)^\star$-α-closed, $(1,2)^\star$-semi-closed) sets is denoted by $(1,2)^\star$-$\ddot{G}_\alpha C(X)$ (resp. $(1,2)^\star$-$GSPC(X)$, $(1,2)^\star$-$\hat{G}C(X)$, $(1,2)^\star$-$GC(X)$, $(1,2)^\star$-$GSC(X)$, $(1,2)^\star$-$\alpha GC(X)$, $(1,2)^\star$-$SGC(X)$, $(1,2)^\star$-$\psi C(X)$, $(1,2)^\star$-$\alpha C(X)$, $(1,2)^\star$-$SC(X)$). We denote the power set of X by P(X).*

Remark 2.6.

(1). Every $\tau_{1,2}$-closed set is $(1,2)^\star$-semi-closed but not conversely [9].

(2). Every $\tau_{1,2}$-closed set is $(1,2)^\star$-α-closed but not conversely [14].

(3). Every $(1,2)^\star$-semi-closed set is $(1,2)^\star$-ψ-closed but not conversely [7].

(4). Every $(1,2)^\star$-semi-closed set is $(1,2)^\star$-sg-closed but not conversely [9].

(5). Every $(1,2)^\star$-$\hat{g}$-closed set is $(1,2)^\star$-g-closed but not conversely [1].

(6). Every $(1,2)^\star$-sg-closed set is $(1,2)^\star$-gs-closed but not conversely [11].

(7). Every $(1,2)^\star$-g-closed set is $(1,2)^\star$-αg-closed but not conversely [8].

(8). Every $(1,2)^\star$-g-closed set is $(1,2)^\star$-gs-closed but not conversely [11].

3. $(1,2)^\star$-ψ-closed Sets

We introduce the following definition.

Definition 3.1. *A subset A of a bitopological space X is called $(1,2)^\star$-αgs-closed if $(1,2)^\star$-$\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $(1,2)^\star$-semi-open in X. The complement of $(1,2)^\star$-αsg-closed set is called $(1,2)^\star$-αsg-open. The collection of all $(1,2)^\star$-αgs-closed sets in X is denoted by $(1,2)^\star$-$\alpha GSC(X)$.*

Proposition 3.2. *Every $\tau_{1,2}$-closed set is $(1,2)^\star$-ψ-closed.*

Proof. If A is a $\tau_{1,2}$-closed subset of X and G is any $(1,2)^\star$-sg-open set containing A, then G $\supseteq$ A = $(1,2)^\star$-scl(A). Hence A is $(1,2)^\star$-ψ-closed in X. □

The converse of Proposition 3.2 need not be true as seen from the following example.

Example 3.3. *Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a, b\}\}$ and $\tau_2 = \{\emptyset, X, \{b, c\}\}$. Then the sets in $\{\emptyset, X, \{a, b\}, \{b, c\}\}$ are called $\tau_{1,2}$-open and the sets in $\{\emptyset, X, \{a\}, \{c\}\}$ are called $\tau_{1,2}$-closed. Then $(1,2)^\star$-$\psi C(X) = \{\emptyset, \{a\}, \{c\}, \{a, c\}, X\}$. Clearly, the set $\{a, c\}$ is $(1,2)^\star$-ψ-closed but it is not a $\tau_{1,2}$-closed set in X.*

Proposition 3.4. *Every $(1,2)^\star$-$\ddot{g}_\alpha$-closed set is $(1,2)^\star$-ψ-closed.*

Proof. If A is a $(1,2)^\star$-$\ddot{g}_\alpha$-closed subset of X and G is any $(1,2)^\star$-sg-open set containing A, then G $\supseteq$ $(1,2)^\star$-αcl(A) $\supseteq$ $(1,2)^\star$-scl(A). Hence A is $(1,2)^\star$- ψ-closed in X. □

The converse of Proposition 3.4 need not be true as seen from the following example.

Example 3.5. *Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, X, \{c\}\}$. Then the sets in $\{\emptyset, X, \{a\}, \{c\}, \{a, c\}\}$ are called $\tau_{1,2}$-open and the sets in $\{\emptyset, X, \{b\}, \{a, b\}, \{b, c\}\}$ are called $\tau_{1,2}$-closed. Then $(1,2)^\star$-$\psi C(X) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, X\}$ and $(1,2)^\star$-$\ddot{G}_\alpha C(X) = \{\emptyset, \{b\}, \{a, b\}, \{b, c\}, X\}$. Clearly, the set $\{a\}$ is $(1,2)^\star$-ψ-closed but not an $(1,2)^\star$-$\ddot{g}_\alpha$-closed set in X.*

Proposition 3.6. *Every $(1,2)^{\star}$-ψ-closed set is $(1,2)^{\star}$-$\hat{g}$-closed.*

Proof. Suppose that A $\subseteq$ G and G is $(1,2)^{\star}$-semi-open in X. Since every $(1,2)^{\star}$-semi-open set is $(1,2)^{\star}$-sg-open and A is $(1,2)^{\star}$-ψ-closed, therefore $(1,2)^{\star}$-scl(A) $\subseteq$ G. Hence A is $(1,2)^{\star}$-$\hat{g}$-closed in X. □

The converse of Proposition 3.6 need not be true as seen from the following example.

Example 3.7. *Let $X = \{a, b, c, d\}$, $\tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, X, \{b, c\}\}$. Then the sets in $\{\emptyset, X, \{a\}, \{b, c\}\}$ are called $\tau_{1,2}$-open and the sets in $\{\emptyset, X, \{a\}, \{b, c\}\}$ are called $\tau_{1,2}$-closed. Clearly, the set $\{b\}$ is $(1,2)^{\star}$-$\hat{g}$-closed but not a $(1,2)^{\star}$-ψ-closed set in X.*

Proposition 3.8. *Every $(1,2)^{\star}$-α-closed set is $(1,2)^{\star}$-$\ddot{g}_{\alpha}$-closed.*

Proof. If A is an $(1,2)^{\star}$-α-closed subset of X and G is any $(1,2)^{\star}$-sg-open set containing A, we have $(1,2)^{\star}$-αcl(A) = A $\subseteq$ G. Hence A is $(1,2)^{\star}$-$\ddot{g}_{\alpha}$-closed in X. □

The converse of Proposition 3.8 need not be true as seen from the following example.

Example 3.9. *Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a, b\}\}$ and $\tau_2 = \{\emptyset, X\}$. Then the sets in $\{\emptyset, X, \{a, b\}\}$ are called $\tau_{1,2}$-open and the sets in $\{\emptyset, X, \{c\}\}$ are called $\tau_{1,2}$-closed. Then $(1,2)^{\star}$-$\alpha C(X) = \{\emptyset, \{c\}, X\}$ and $(1,2)^{\star}$-$\ddot{G}_{\alpha} C(X) = \{\emptyset, \{c\}, \{a, c\}, \{b, c\}, X\}$. Clearly, the set $\{a, c\}$ is $(1,2)^{\star}$-$\ddot{g}_{\alpha}$-closed but not an $(1,2)^{\star}$-α-closed set in X.*

Proposition 3.10. *Every $(1,2)^{\star}$-$\hat{g}$-closed set is $(1,2)^{\star}$-αgs-closed.*

Proof. If A is a $(1,2)^{\star}$-$\hat{g}$-closed subset of X and G is any $(1,2)^{\star}$-semi-open set containing A, then G $\supseteq$ $\tau_{1,2}$-cl(A) $\supseteq$ $(1,2)^{\star}$-αcl(A). Hence A is $(1,2)^{\star}$-αgs-closed in X. □

The converse of Proposition 3.10 need not be true as seen from the following example.

Example 3.11. *Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, X\}$. Then the sets in $\{\emptyset, X, \{a\}\}$ are called $\tau_{1,2}$-open and the sets in $\{\emptyset, X, \{b, c\}\}$ are called $\tau_{1,2}$-closed. Then $(1,2)^{\star}$-$\hat{G}C(X) = \{\emptyset, \{b, c\}, X\}$ and $(1,2)^{\star}$-$\alpha GSC(X) = \{\emptyset, \{b\}, \{c\}, \{b, c\}, X\}$. Clearly, the set $\{b\}$ is $(1,2)^{\star}$-αgs-closed but not a $(1,2)^{\star}$-$\hat{g}$-closed set in X.*

Proposition 3.12. *Every $(1,2)^{\star}$-ψ-closed set is $(1,2)^{\star}$-g-closed.*

Proof. If A is a $(1,2)^{\star}$-ψ-closed subset of X and G is any $\tau_{1,2}$-open set containing A, since every $\tau_{1,2}$-open set is $(1,2)^{\star}$-sg-open, we have G $\supseteq$ $(1,2)^{\star}$-scl(A) $\supseteq$ $\tau_{1,2}$-cl(A). Hence A is $(1,2)^{\star}$-g-closed in X. □

The converse of Proposition 3.12 need not be true as seen from the following example.

Example 3.13. *Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, X, \{b, c\}\}$. Then the sets in $\{\emptyset, X, \{a\}, \{b, c\}\}$ are called $\tau_{1,2}$-open as well as $\tau_{1,2}$-closed. Then $(1,2)^{\star}$-$\psi C(X) = \{\emptyset, \{a\}, \{b, c\}, X\}$ and $(1,2)^{\star}$-$GC(X) = P(X)$. Clearly, the set $\{a, b\}$ is $(1,2)^{\star}$-g-closed but not $(1,2)^{\star}$-ψ-closed set in X.*

Proposition 3.14. *Every $(1,2)^{\star}$-ψ-closed set is $(1,2)^{\star}$-αgs-closed.*

Proof. If A is a $(1,2)^{\star}$-ψ-closed subset of X and G is any $(1,2)^{\star}$-semi-open set containing A, since every $(1,2)^{\star}$-semi-open set is $(1,2)^{\star}$-sg-open, we have G $\supseteq$ $(1,2)^{\star}$-scl(A) $\supseteq$ $(1,2)^{\star}$-αcl(A). Hence A is $(1,2)^{\star}$-αgs-closed in X. □

The converse of Proposition 3.14 need not be true as seen from the following example.

Example 3.15. *In Example 3.13, we obtain* $(1,2)^\star$*-*$\psi C(X) = \{\emptyset,\ \{a\},\ \{b,\ c\},\ X\}$ *and* $(1,2)^\star$*-*$\alpha GSC(X) = P(X)$*. Clearly, the set* $\{a,\ c\}$ *is an* $(1,2)^\star$*-αgs-closed but not a* $(1,2)^\star$*-ψ-closed set in X.*

Proposition 3.16. *Every* $(1,2)^\star$*-ψ-closed set is* $(1,2)^\star$*-αg-closed.*

Proof. If A is a $(1,2)^\star$-ψ-closed subset of X and G is any $\tau_{1,2}$-open set containing A, since every $\tau_{1,2}$-open set is $(1,2)^\star$-sg-open, we have G $\supseteq$ $(1,2)^\star$-scl(A) $\supseteq$ $(1,2)^\star$-αcl(A). Hence A is $(1,2)^\star$-αg-closed in X. □

The converse of Proposition 3.16 need not be true as seen from the following example.

Example 3.17. *Let* $X = \{a,\ b,\ c\}$*,* $\tau_1 = \{\emptyset,\ X,\ \{a,\ b\}\}$ *and* $\tau_2 = \{\emptyset,\ X,\ \{c\}\}$*. Then the sets in* $\{\emptyset,\ X,\ \{c\},\ \{a,\ b\}\}$ *are called* $\tau_{1,2}$*-open and the sets in* $\{\emptyset,\ X,\ \{c\},\ \{a,\ b\}\}$ *are called* $\tau_{1,2}$*-closed. Then* $(1,2)^\star$*-*$\psi C(X) = \{\emptyset,\ \{c\},\ \{a,\ b\},\ X\}$ *and* $(1,2)^\star$*-*$\alpha GC(X) = P(X)$*. Clearly, the set* $\{a,\ c\}$ *is* $(1,2)^\star$*-αg-closed but not a* $(1,2)^\star$*-ψ-closed set in X.*

Proposition 3.18. *Every* $(1,2)^\star$*-ψ-closed set is* $(1,2)^\star$*-gs-closed.*

Proof. If A is a $(1,2)^\star$-ψ-closed subset of X and G is any $\tau_{1,2}$-open set containing A, since every $\tau_{1,2}$-open set is $(1,2)^\star$-sg-open, we have G $\supseteq$ $(1,2)^\star$-scl(A). Hence A is $(1,2)^\star$-gs-closed in X. □

The converse of Proposition 3.18 need not be true as seen from the following example.

Example 3.19. *Let* $X = \{a,\ b,\ c\}$*,* $\tau_1 = \{\emptyset,\ X,\ \{a\}\}$ *and* $\tau_2 = \{\emptyset,\ X\}$*. Then the sets in* $\{\emptyset,\ X,\ \{a\}\}$ *are called* $\tau_{1,2}$*-open and the sets in* $\{\emptyset,\ X,\ \{b,\ c\}\}$ *are called* $\tau_{1,2}$*-closed. Then* $(1,2)^\star$*-*$\psi C(X) = \{\emptyset,\ \{b\},\ \{c\},\ \{b,\ c\},\ X\}$ *and* $(1,2)^\star$*-*$GSC(X) = \{\emptyset,\ \{b\},\ \{c\},\ \{a,\ b\},\ \{a,\ c\},\ \{b,\ c\},\ X\}$*. Clearly, the set* $\{a,\ b\}$ *is* $(1,2)^\star$*-gs-closed but not a* $(1,2)^\star$*-ψ-closed set in X.*

Proposition 3.20. *Every* $(1,2)^\star$*-ψ-closed set is* $(1,2)^\star$*-sg-closed.*

Proof. If A is a $(1,2)^\star$-ψ-closed subset of X and G is any $(1,2)^\star$-semi-open set containing A, since every $(1,2)^\star$-semi-open set is $(1,2)^\star$-sg-open, we have G $\supseteq$ $(1,2)^\star$-scl(A). Hence A is $(1,2)^\star$-sg-closed in X. □

The converse of Proposition 3.20 need not be true as seen from the following example.

Example 3.21. *Let* $X = \{a,\ b,\ c\}$*,* $\tau_1 = \{\emptyset,\ X,\ \{b\}\}$ *and* $\tau_2 = \{\emptyset,\ X,\ \{a,\ c\}\}$*. Then the sets in* $\{\emptyset,\ X,\ \{b\},\ \{a,\ c\}\}$ *are called* $\tau_{1,2}$*-open and the sets in* $\{\emptyset,\ X,\ \{b\},\ \{a,\ c\}\}$ *are called* $\tau_{1,2}$*-closed. Then* $(1,2)^\star$*-*$\psi C(X) = \{\emptyset,\ \{b\},\ \{a,\ c\},\ X\}$ *and* $(1,2)^\star$*-SGC(X) =P(X). Clearly, the set* $\{a\}$ *is* $(1,2)^\star$*-sg-closed but not a* $(1,2)^\star$*-ψ-closed set in X.*

Proposition 3.22. *Every* $(1,2)^\star$*-*$\ddot{g}_\alpha$*-closed set is* $(1,2)^\star$*-αgs-closed.*

Proof. If A is an $(1,2)^\star$-$\ddot{g}_\alpha$-closed subset of X and G is any $(1,2)^\star$-semi-open set containing A, since every $(1,2)^\star$-semi-open set is $(1,2)^\star$-sg-open, we have $(1,2)^\star$-αcl(A) $\subseteq$ G. Hence A is $(1,2)^\star$-αgs-closed in X. □

The converse of Proposition 3.22 need not be true as seen from the following example.

Example 3.23. *In Example 3.13, we have* $(1,2)^\star$*-*$\ddot{G}_\alpha C(X) = \{\emptyset,\ \{a\},\ \{b,\ c\},\ X\}$ *and* $(1,2)^\star$*-*$\alpha GSC(X) = P(X)$*. Clearly, the set* $\{a,\ b\}$ *is* $(1,2)^\star$*-αgs-closed but not an* $(1,2)^\star$*-*$\ddot{g}_\alpha$*-closed set in X.*

Proposition 3.24. *Every* $(1,2)^\star$*-αgs-closed set is* $(1,2)^\star$*-αg-closed.*

Proof. If A is an $(1,2)^\star$-αgs-closed subset of X and G is any $\tau_{1,2}$-open set containing A, since every $\tau_{1,2}$-open set is $(1,2)^\star$-semi-open, we have $(1,2)^\star$-αcl(A) $\subseteq$ G. Hence A is $(1,2)^\star$-αg-closed in X. □

The converse of Proposition 3.24 need not be true as seen from the following example.

Example 3.25. *Let $X = \{a,\ b,\ c\}$, $\tau_1 = \{\emptyset,\ X,\ \{a\}\}$ and $\tau_2 = \{\emptyset,\ X\}$. Then the sets in $\{\emptyset,\ X,\ \{a\}\}$ are called $\tau_{1,2}$-open and the sets in $\{\emptyset,\ X,\ \{b,\ c\}\}$ are called $\tau_{1,2}$-closed. We have $(1,2)^\star$-$\alpha GC(X) = \{\emptyset,\ \{b\},\ \{c\},\ \{a,\ b\},\ \{a,\ c\},\ \{b,\ c\},\ X\}$ and $(1,2)^\star$-$\alpha GSC(X) = \{\emptyset,\ \{b\},\ \{c\},\ \{b,\ c\},\ X\}$. Clearly, the set $\{a,\ c\}$ is $(1,2)^\star$-αg-closed but not an $(1,2)^\star$-αgs-closed set in X.*

Proposition 3.26. *Every $(1,2)^\star$-ψ-closed set is $(1,2)^\star$-gsp-closed.*

Proof. If A is a $(1,2)^\star$-ψ-closed subset of X and G is any $\tau_{1,2}$-open set containing A, since every $\tau_{1,2}$-open set is $(1,2)^\star$-sg-open, we have G $\supseteq$ $(1,2)^\star$-scl(A) $\supseteq$ $(1,2)^\star$-spcl(A). Hence A is $(1,2)^\star$-gsp-closed in X. □

The converse of Proposition 3.26 need not be true as seen from the following example.

Example 3.27. *Let $X = \{a,\ b,\ c\}$, $\tau_1 = \{\emptyset,\ X,\ \{b\}\}$ and $\tau_2 = \{\emptyset,\ X,\ \{a,\ b\}\}$. Then the sets in $\{\emptyset,\ X,\ \{b\},\ \{a,\ b\}\}$ are called $\tau_{1,2}$-open and the sets in $\{\emptyset,\ X,\ \{c\},\ \{a,\ c\}\}$ are called $\tau_{1,2}$-closed. We have $(1,2)^\star$-$\psi C(X) = \{\emptyset,\ \{a\},\ \{c\},\ \{a,\ c\},\ X\}$ and $(1,2)^\star$-$GSPC(X) = \{\emptyset,\ \{a\},\ \{c\},\ \{a,\ c\},\ \{b,\ c\},\ X\}$. Clearly, the set $\{b,\ c\}$ is $(1,2)^\star$-gsp-closed but not a $(1,2)^\star$-ψ-closed set in X.*

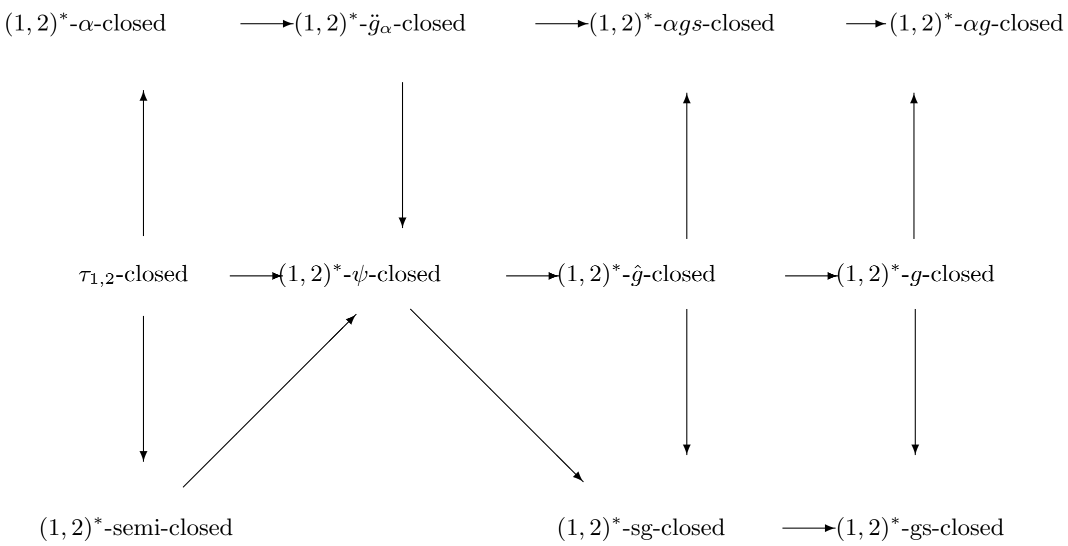
Proposition 3.28. *Every $(1,2)^\star$-$\hat{g}$-closed set is $(1,2)^\star$-sg-closed.*

Proof. If A is a $(1,2)^\star$-$\hat{g}$-closed subset of X and G is any $(1,2)^\star$-semi-open set containing A, then G $\supseteq$ $\tau_{1,2}$-cl(A) $\supseteq$ $(1,2)^\star$-scl(A). Hence A is $(1,2)^\star$-sg-closed in X. □

The converse of Proposition 3.28 need not be true as seen from the following example.

Example 3.29. *Let $X = \{a,\ b,\ c\}$, $\tau_1 = \{\emptyset,\ X,\ \{b\}\}$ and $\tau_2 = \{\emptyset,\ X,\ \{a,\ b\}\}$. Then the sets in $\{\emptyset,\ X,\ \{b\},\ \{a,\ b\}\}$ are called $\tau_{1,2}$-open and the sets in $\{\emptyset,\ X,\ \{c\},\ \{a,\ c\}\}$ are called $\tau_{1,2}$-closed. We have $(1,2)^\star$-$\hat{G}C(X) = \{\emptyset,\ X,\ \{c\},\ \{a,\ c\}\}$ and $(1,2)^\star$-$SGC(X) = \{\emptyset,\ X,\ \{a\},\ \{c\},\ \{d\},\ \{a,\ c\}\}$. Clearly, the set $\{a\}$ is $(1,2)^\star$-sg-closed but not a $(1,2)^\star$-$\hat{g}$-closed set in X.*

Remark 3.30. *From the above Propositions, Examples and Remark, we obtain the following diagram, where $A \rightarrow B$ (resp. $A \nleftrightarrow B$) represents A implies B but not conversely.*



4. Properties of $(1,2)^{\star}$-ψ-closed Sets

Example 4.1. *The intersection of all $(1,2)^{\star}$-sg-open subsets of X containing A is called the $(1,2)^{\star}$-sg-kernel of A and denoted by $(1,2)^{\star}$-sg-ker(A).*

Lemma 4.2. *A subset A of a bitopological space X is $(1,2)^{\star}$-ψ-closed if and only if $(1,2)^{\star}$-scl(A) $\subseteq$ $(1,2)^{\star}$-sg-ker(A).*

Proof. Suppose that A is $(1,2)^{\star}$-ψ-closed. Then $(1,2)^{\star}$-scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $(1,2)^{\star}$-sg-open. Let x $\in$ $(1,2)^{\star}$-scl(A). If x $\in$ $(1,2)^{\star}$-sg-ker(A), then there is a $(1,2)^{\star}$-sg-open set U containing A such that x $\notin$ U. Since U is a $(1,2)^{\star}$-sg-open set containing A, we have x $\notin$ $(1,2)^{\star}$-scl(A) and this is a contradiction.
Conversely, let $(1,2)^{\star}$-scl(A) $\subseteq$ $(1,2)^{\star}$-sg-ker(A). If U is any $(1,2)^{\star}$-sg-open set containing A, then $(1,2)^{\star}$-scl(A) $\subseteq$ $(1,2)^{\star}$-sg-ker(A) $\subseteq$ U. Therefore, A is $(1,2)^{\star}$-ψ-closed. □

Remark 4.3. *Union of any two $(1,2)^{\star}$-ψ-closed sets need not be $(1,2)^{\star}$-ψ-closed as seen from the following example.*

Example 4.4. *Let X = {a, b, c}, τ_1 = { ∅, X, {a, b}} and τ_2 = {∅, X, {b}, {c}, {b, c}, {a, c}}. Then the sets in {∅, X, {b}, {c}, {a, b}, {a, c}, {b, c}} are called $\tau_{1,2}$-open and the sets in {∅, X, {a}, {b}, {c}, {a, b}, {a, c}} are called $\tau_{1,2}$-closed. Then $(1,2)^{\star}$-ψC(X) = {∅, X, {a}, {b}, {c}, {a, b}, {a, c}}. Clearly, the sets {b} and {c} are $(1,2)^{\star}$-ψ-closed but their union {b, c} is not a $(1,2)^{\star}$-ψ-closed set in X.*

Proposition 4.5. *If a set A is $(1,2)^{\star}$-ψ-closed in X then $(1,2)^{\star}$-scl(A) − A contains no nonempty $\tau_{1,2}$-closed set in X.*

Proof. Suppose that A is $(1,2)^{\star}$-ψ-closed. Let F be a $\tau_{1,2}$-closed subset of $(1,2)^{\star}$-scl(A) − A. Then A $\subseteq$ F^c. But A is $(1,2)^{\star}$-ψ-closed, therefore $(1,2)^{\star}$-scl(A) $\subseteq$ F^c. Consequently, F $\subseteq$ $(\tau_{1,2}\text{-cl(A)})^c$. We already have F $\subseteq$ $(1,2)^{\star}$-scl(A). Thus F $\subseteq$ $(1,2)^{\star}$-scl(A) $\cap$ $((1,2)^{\star}\text{-scl(A)})^c$ and F is empty. □

The converse of Proposition 4.5 need not be true as seen from the following example.

Example 4.6. *Let X = {a, b, c}, τ_1 = {∅, X, {a}} and τ_2 = {∅, X}. Then the sets in {∅, X, {a}} are called $\tau_{1,2}$-open and the sets in {∅, X, {b, c}} are called $\tau_{1,2}$-closed. Then $(1,2)^{\star}$-ψC(X) = {∅, {b}, {c}, {b, c}, X}. If A = {b}, then $(1,2)^{\star}$-scl(A) − A = {c} does not contain any nonempty $\tau_{1,2}$-closed set. But A is not a $(1,2)^{\star}$-ψ-closed set in X.*

Theorem 4.7. *If a set A is $(1,2)^{\star}$-ψ-closed in X then $(1,2)^{\star}$-scl(A) − A contains no nonempty $(1,2)^{\star}$-sg-closed set.*

Proof. Suppose that A is $(1,2)^{\star}$-ψ-closed. Let S be a $(1,2)^{\star}$-sg-closed subset of $(1,2)^{\star}$-scl(A) − A. Then A $\subseteq$ S^c. Since A is $(1,2)^{\star}$-ψ-closed, we have $(1,2)^{\star}$-scl(A) $\subseteq$ S^c. Consequently, S $\subseteq$ $((1,2)^{\star}\text{-scl(A)})^c$. Hence, S $\subseteq$ $(1,2)^{\star}$-scl(A) $\cap$ $((1,2)^{\star}\text{-scl(A)})^c$ = ∅. Therefore S is empty. □

Theorem 4.8. *If A is $(1,2)^{\star}$-ψ-closed in X and A $\subseteq$ B $\subseteq$ $(1,2)^{\star}$-scl(A), then B is $(1,2)^{\star}$-ψ-closed in X.*

Proof. Let B $\subseteq$ U where U is $(1,2)^{\star}$-sg-open set in X. Then A $\subseteq$ U. Since A is $(1,2)^{\star}$-ψ-closed, $(1,2)^{\star}$-scl(A) $\subseteq$ U. Since B $\subseteq$ $(1,2)^{\star}$-scl(A), $(1,2)^{\star}$-scl(B) $\subseteq$ $(1,2)^{\star}$-scl(A). Therefore $(1,2)^{\star}$-scl(B) $\subseteq$ U and B is $(1,2)^{\star}$-ψ-closed in X. □

Proposition 4.9. *If A is $(1,2)^{\star}$-sg-open and $(1,2)^{\star}$-ψ-closed in X, then A is $(1,2)^{\star}$-semi-closed in X.*

Proof. Since A is $(1,2)^{\star}$-sg-open and $(1,2)^{\star}$-ψ-closed, $(1,2)^{\star}$-scl(A) $\subseteq$ A and hence A is $(1,2)^{\star}$-semi-closed in X. □

Proposition 4.10. *For each x $\in$ X, either {x} is $(1,2)^{\star}$-sg-closed or $\{x\}^c$ is $(1,2)^{\star}$-ψ-closed in X.*

Proof. Suppose that {x} is not $(1,2)^{\star}$-sg-closed in X. Then $\{x\}^c$ is not $(1,2)^{\star}$-sg-open and the only $(1,2)^{\star}$-sg-open set containing $\{x\}^c$ is the space X itself. Therefore $(1,2)^{\star}$-scl($\{x\}^c$) $\subseteq$ X and so $\{x\}^c$ is $(1,2)^{\star}$-ψ-closed in X. □

5. $(1,2)^{\star}$-ψ-interior

We introduce the following definition.

Definition 5.1. *For any $A \subseteq X$, $(1,2)^{\star}$-ψ-int(A) is defined as the union of all $(1,2)^{\star}$-ψ-open sets contained in A. That is $(1,2)^{\star}$-ψ-int(A) $= \cup\{G : G \subseteq A$ and G is $(1,2)^{\star}$-ψ-open$\}$.*

Lemma 5.2. *For any $A \subseteq X$, $\tau_{1,2}$-int(A) $\subseteq (1,2)^{\star}$-ψ-int(A) $\subseteq A$.*

The following two Propositions are easy consequences from definitions.

Proposition 5.3. *For any $A \subseteq X$,*

(1). $(1,2)^{\star}$-ψ-int(A) is the largest $(1,2)^{\star}$-ψ-open set contained in A.

(2). A is $(1,2)^{\star}$-ψ-open if and only if $(1,2)^{\star}$-ψ-int(A) $= A$.

Proposition 5.4. *For any subsets A and B of X,*

(1). $(1,2)^{\star}$-ψ-int($A \cap B$) $\subseteq (1,2)^{\star}$-ψ-int(A) $\cap (1,2)^{\star}$- ψ-int(B).

(2). $(1,2)^{\star}$-ψ-int($A \cup B$) $\supseteq (1,2)^{\star}$-ψ-int(A) $\cup (1,2)^{\star}$- ψ-int(B).

(3). If $A \subseteq B$, then $(1,2)^{\star}$-ψ-int(A) $\subseteq (1,2)^{\star}$-ψ-int(B).

(4). $(1,2)^{\star}$-ψ-int(X) $= X$ and $(1,2)^{\star}$-ψ-int($\emptyset$) $= \emptyset$.

6. $(1,2)^{\star}$-ψ-closure

Definition 6.1. *For every set $A \subseteq X$, we define the $(1,2)^{\star}$-ψ-closure of A to be the intersection of all $(1,2)^{\star}$-ψ-closed sets containing A. In symbols, $(1,2)^{\star}$-ψ-cl(A) $= \cap \{F : A \subseteq F \in (1,2)^{\star}$-$\psi C(X)\}$.*

Lemma 6.2. *For any $A \subseteq X$, $A \subseteq (1,2)^{\star}$-$\psi$-cl(A) $\subseteq \tau_{1,2}$-cl(A).*

Proof. It follows from the fact that every $\tau_{1,2}$-closed set is $(1,2)^{\star}$-ψ-closed. □

Remark 6.3. *Both containment relations in Lemma 6.2 may be proper as seen from the following example.*

Example 6.4. *Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X\}$ and $\tau_2 = \{\emptyset, X, \{a, b\}\}$. Then the sets in $\{\emptyset, X, \{a, b\}\}$ are called $\tau_{1,2}$-open and the sets in $\{\emptyset, X, \{c\}\}$ are called $\tau_{1,2}$-closed. Let $A = \{a\}$. Then $(1,2)^{\star}$-ψ-cl(A) $= \{a, c\}$ and so $A \subset (1,2)^{\star}$-ψ-cl(A) $\subset \tau_{1,2}$-cl(A).*

Lemma 6.5. *For any $A \subseteq X$, $(1,2)^{\star}$-ω-cl(A) $\subseteq (1,2)^{\star}$-ψ-cl(A), where $(1,2)^{\star}$-ω-cl(A) is given by $(1,2)^{\star}$-ω-cl(A) $= \cap \{F : A \subseteq F \in (1,2)^{\star}$-$\hat{G}C(X)\}$.*

Remark 6.6. *Containment relation in the above Lemma 6.5 may be proper as seen from the following example.*

Example 6.7. *Let $X = \{a, b, c, d\}$, $\tau_1 = \{\emptyset, X, \{a\}, \{a, b, c\}\}$ and $\tau_2 = \{\emptyset, X, \{b, c\}\}$. Then the sets in $\{\emptyset, X, \{a\}, \{b, c\}, \{a, b, c\}\}$ are called $\tau_{1,2}$-open and the sets in $\{\emptyset, X, \{d\}, \{a, d\}, \{b, c, d\}\}$ are called $\tau_{1,2}$-closed. Then $(1,2)^{\star}$-$\psi C(X) = \{\emptyset, \{a\}, \{d\}, \{a, d\}, \{b, c\}, \{b, c, d\}, X\}$ and $(1,2)^{\star}$-$\hat{G}C(X) = \{\emptyset, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$. Let $A = \{c, d\}$. Then $(1,2)^{\star}$-ψ-cl(A) $= \{b, c, d\}$ and $(1,2)^{\star}$-ω-cl(A) $= \{c, d\}$. So, $(1,2)^{\star}$-ω-cl(A) $\subset (1,2)^{\star}$-ψ-cl(A).*

Lemma 6.8. *For an $x \in X$, $x \in (1,2)^{\star}$-ψ-cl(A) if and only if $V \cap A \neq \emptyset$ for every $(1,2)^{\star}$-ψ-open set V containing x.*

Proof. Let x ∈ $(1,2)^\star$-ψ-cl(A) for any x ∈ X. To prove V ∩ A ≠ ∅ for every $(1,2)^\star$-ψ-open set V containing x. Prove the result by contradiction. Suppose there exists a $(1,2)^\star$-ψ-open set V containing x such that V ∩ A = ∅. Then A ⊂ V^c and V^c is $(1,2)^\star$-ψ-closed. We have $(1,2)^\star$-ψ-cl(A) ⊂ V^c. This shows that x ∈ $(1,2)^\star$-ψ-cl(A) which is a contradiction. Hence V ∩ A ≠ ∅ for every $(1,2)^\star$-ψ-open set V containing x.

Conversely, let V ∩ A ≠ ∅ for every $(1,2)^\star$-ψ-open set V containing x. To prove x ∈ $(1,2)^\star$-ψ-cl(A). We prove the result by contradiction. Suppose x ∈ $(1,2)^\star$-ψ-cl(A). Then there exists a $(1,2)^\star$-ψ-closed set F containing A such that x ∉ F. Then x ∈ F^c and F^c is $(1,2)^\star$-ψ-open. Also F^c ∩ A = ∅, which is a contradiction to the hypothesis. Hence x ∈ $(1,2)^\star$-ψ-cl(A). □

Proposition 6.9. *For any* $A \subseteq X$*,*

(1). $(1,2)^\star$-ψ-*cl(A) is the smallest* $(1,2)^\star$-ψ-*closed set containing A.*

(2). A is $(1,2)^\star$-ψ-*closed if and only if* $(1,2)^\star$-ψ-*cl(A) = A.*

Proposition 6.10. *For any two subsets A and B of X,*

(1). If $A \subseteq B$*, then* $(1,2)^\star$-ψ-*cl(A)* ⊆ $(1,2)^\star$-ψ-*cl(B).*

(2). $(1,2)^\star$-ψ-*cl(A ∩ B)* ⊆ $(1,2)^\star$-ψ-*cl(A)* ∩ $(1,2)^\star$-ψ-*cl(B).*

Theorem 6.11. *Let A be any subset of a bitopological space X. Then*

(1). $((1,2)^\star$-ψ-*int(A))*^{c} = $(1,2)^\star$-ψ-*cl(*A^c*).*

(2). $(1,2)^\star$-ψ-*int(A)* = *(*$(1,2)^\star$-ψ-*cl(*A^c*))*^{c}.

(3). $(1,2)^\star$-ψ-*cl(A)* = *(*$(1,2)^\star$-ψ-*int(*A^c*))*^{c}.

Proof.

(1). Let x ∈ $((1,2)^\star$-ψ-int(A))^c. Then x ∈ $(1,2)^\star$-ψ-int(A). That is, every $(1,2)^\star$-ψ-open set U containing x is such that U ⊈ A. That is, every $(1,2)^\star$-ψ-open set U containing x is such that U ∩ A^c ≠ ∅. By Lemma 6.8, x ∈ $(1,2)^\star$-ψ-cl(A^c) and therefore $((1,2)^\star$-ψ-int(A))^c ⊆ $(1,2)^\star$-ψ-cl(A^c).

Conversely, let x ∈ $(1,2)^\star$-ψ-cl(A^c). Then by Lemma 6.8, every $(1,2)^\star$-ψ-open open set U containing x is such that U ∩ A^c ≠ ∅. That is, every $(1,2)^\star$-ψ-open set U containing x is such that U ⊈ A. This implies by Definition 5.1, x ∉ $(1,2)^\star$-ψ-int(A). That is, x ∈ $((1,2)^\star$-ψ-int(A))^c and so $(1,2)^\star$-ψ-cl(A^c) ⊆ $((1,2)^\star$-ψ-int(A))^c. Thus $((1,2)^\star$- ψ-int(A))^c = $(1,2)^\star$-ψ-cl(A^c).

(2). Follows by taking complements in (1).

(3). Follows by replacing A by A^c in (1).

□

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