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Totally Regular Property of Beta and Gamma Product of Fuzzy Graphs

Research Article

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Abstract: In this paper, the total degree of a vertex in fuzzy graphs formed by the operations of Beta product and Gamma product of two fuzzy graphs in terms of the total degree of vertices in the given fuzzy graphs for some particular cases are obtained. Using them, their totally regular property is studied.

MSC: 03E72, 05C72.

Keywords: Total degree of a vertex, Complement of fuzzy graph, Complete graph.

1. Introduction

Fuzzy graph theory was introduced by Azriel Rosenfeld in 1975 [9]. Bhattacharya [1] gave some remarks on fuzzy graphs. Some operations on fuzzy graphs were introduced by Mordeson.J.N. and Peng.C.S. [2]. Zadeh 1965 [3] introduce a mathematical frame work to describe the phenomena of uncertainty in real life situation has been suggested. Research on the theory of fuzzy sets has been witnessing an exponential growth; both within mathematics and in its applications. This ranges from traditional mathematical subjects like logic topology, algebra, analysis etc. to pattern recognition, information theory, artificial intelligence, operations research, neural networks and planning etc. Yeh and Bang [4] have also introduced various concepts in connectedness in fuzzy graphs. The operations of union, join, Cartesian product and composition on two fuzzy graphs were defined by Mordeson.J.N. and Peng.C.S [2]. Sunitha. M. S and Vijayakumar. A discussed about the complement of the operations of union, join, Cartesian product and composition on two fuzzy graphs. The degree of a vertex in some fuzzy graphs and the Regular property of fuzzy graphs which are obtained from two given fuzzy graphs using the operations union, join, Cartesian product and composition was discussed by Nagoorgani. A and Radha. K. [7]. The degree of a vertex in Alpha, Beta, Gamma Product of fuzzy graphs and the Regular property of fuzzy graphs which are obtained from two given fuzzy graphs using the operations Beta and Gamma Product was discussed by Nagoorgani. A and Fathima Kani. B. [11]. In this paper we study about the total degree of vertex in Beta Product and Gamma Product of fuzzy graphs and the Totally Regular property of Beta product and Gamma Product of two fuzzy graphs. First we go through some basic definitions which can be found in [1-10].

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2. Basic Definitions

Throughout this paper, V is assumed to be finite

Definition 2.1 ([1]). *A fuzzy subset of a set V is a mapping σ from V to [0, 1]. A fuzzy graph G is a pair of functions* $G : (\sigma, \mu)$ *where* σ *is a fuzzy subset of a non empty set V and* μ *is a symmetric fuzzy relation on* σ*, (i.e.)* $\mu(uv) = \sigma(u) \wedge \sigma(v)$*. The underlying crisp graph of* $G : (\sigma, \mu)$ *is denoted by* $G^* : (V, E)$ *where* $E \subseteq V \times V$*.*

Definition 2.2 ([1]). *If* $\mu(uv) = \sigma(u) \wedge \sigma(v)$ *for all* $u, v \in V$*, then G is called a complete fuzzy graph. The Complement* $\overline{G^*}$ *of a graph* G^* *also has V(G) as its vertices set, but two vertices are adjacent in* G^* *if and only if they are not adjacent in* G^**. The degree* $d_{G^*}(v)$ *of a vertex v in* G^* *is the number of edges incident with v. We have* $d_{\overline{G^*}}(v) + d_{G^*}(v) = p - 1$ *where p is the number of vertices in G.*

Definition 2.3 ([5]). *Let* $G : (\sigma, \mu)$ *be a fuzzy graph. The degree of a vertex u in G is defined by*

$$d_G(u) = \sum_{u \neq v} \mu(uv) = \sum_{uv \in E} \mu(uv).$$

Definition 2.4 ([7]). *Let* $G : (\sigma, \mu)$ *be a fuzzy graph on* G^**. The total degree of a vertex* $u \in V$ *is defined by*

$$td_G(u) = \sum_{u \neq v} \mu(uv) + s(u) = d_G(u) + \sigma(u)$$

If each vertex of G has the same total degree k, then G is said to be a totally regular fuzzy graph of total degree k or a k-totally regular fuzzy graph.

Notation 2.5 ([4]). *The relation* $\sigma_1 = \mu_2$ *means that* $\sigma_1(u) = \mu_2(e)$*,* $\forall\ u \in V_1$ *and* $\forall\ e \in E_2$ *where* σ_1 *is a fuzzy subset of* V_1 *and* μ_2 *is a fuzzy subset of* E_2*.*

Lemma 2.6 ([4]). *If* $G_1 : (s_1, \mu_1)$ *and* $G_2 : (\sigma_2, \mu_2)$ *are two fuzzy graphs such that* $\sigma_1 = \mu_2$*, then* $\sigma_2 = \mu_1$*.*

Lemma 2.7 ([8]). *The* $\beta-$*product of two fuzzy graphs* G_1 *and* G_2 *is defined as a fuzzy graph* $G_1 \times_\beta G_2 = ((\sigma_1 \times_\beta \sigma_2), (\mu_1 \times_\beta \mu_2))$ *on* $G^* : (V, E)$ *where* $V = V_1 \times V_2$ *and* $E = \{((u_1, u_2), (v_1, v_2))/u_1v_1 \in E_1; u_2v_2 \notin E_2\ \ (or)\ \ u_1v_1 \notin E_1; u_2v_2 \in E_2\ \ (or)\ \ u_1v_1 \in E_1; u_2v_2 \in E_2\}$ *with* $(\sigma_1 \times_\beta \sigma_2)(u_1, u_2) = \sigma_1(u_1) \wedge \sigma_2(u_2)\ \ \forall\ \ (u_1, u_2) \in V_1 \times V_2$

$$(\mu_1 \times_\beta \mu_2)((u_1, u_2)(v_1, v_2)) = \begin{cases} \mu_1(u_1v_1) \wedge \mu_2(u_2v_2), & if\ \ u_1v_1 \in E_1, u_2v_2 \in E_2 \\ \sigma_2(u_2) \wedge \sigma_2(v_2) \wedge \mu_1(u_1v_1), & if\ \ u_1v_1 \in E_1, u_2v_2 \notin E_2 \\ \sigma_1(u_1) \wedge \sigma_1(v_1) \wedge \mu_2(u_2v_2), & if\ \ u_1v_1 \notin E_1, u_2v_2 \in E_2 \end{cases}$$

Definition 2.8 ([8]). *The* β*-product of two fuzzy graphs* G_1 *and* G_2 *is defined as a fuzzy graph* $G_1 \times_\beta G_2 = ((\sigma_1 \times_\beta \sigma_2), (\mu_1 \times_\beta \mu_2))$ *on* $G^* : (V, E)$ *where* $V = V_1 \times V_2$ *and* $E = \{((u_1, u_2), (v_1, v_2))/u_1v_1 \in E_1,\ u_2v_2 \notin E_2\ \ (or)\ \ u_1v_1 \notin E_1u_2v_2 \in E_2\ \ (or)\ \ u_1v_1 \in E_1, u_2v_2 \in E_2\}$ *with* $(\sigma_1 \times_\beta \sigma_2)(u_1, u_2) = \sigma_1(u_1) \wedge \sigma_2(u_2)(u_1, u_2)\ \ \forall\ \ V_1 \times V_2$

$$(\mu_1 \times_\beta \mu_2)((u_1, u_2)(v_1, v_2)) = \begin{cases} \mu_1(u_1v_1) \wedge \mu_2(u_2v_2), & if\ u_1v_1 \in E_1,\ u_2v_2 \in E_2 \\ \sigma_2(u_2) \wedge \sigma_2(v_2) \wedge \mu_1(u_1v_1), & if\ u_1v_1 \in E_1, u_2v_2 \in E_2 \\ \sigma_1(u_1) \wedge \sigma_1(v_1) \wedge \mu_2(u_2v_2), & if\ u_1v_1 \in E_1, u_2v_2 \in E_2 \end{cases}$$

Definition 2.9 ([8]). *The γ-product of two fuzzy graphs G_1 and G_2 is defined as a fuzzy graph $G_1 \times_\gamma G_2 = ((\sigma_1 \times_\gamma \sigma_2), (\mu_1 \times_\gamma \mu_2))$ on $G^* : (V, E)$ where $V = V_1 \times_\gamma V_2$ and $E = \{((u_1, u_2), (v_1, v_2))/u_1 = v_1 u_2 v_2 \in E_2 \ (or) \ u_2 = v_2, u_1 v_1 \in E_1 \ (or) \ u_1 v_1 \notin E_1, \ (or) \ u_2 v_2 \in E_2 \ (or) \ u_2 v_2 \notin E_2, u_1 v_1 \in E_1 \ (or) \ u_1 v_1 \in E_1, u_2 v_2 \in E_2\}$ with $\sigma_1 \times_\gamma \sigma_2 = \sigma_1(u_1) \wedge \sigma_2(u_2)(u_1, u_2) \in V_1 \times_\gamma V_2$*

$$(\mu_1 \times_\gamma \mu_2)((u_1,u_2),(v_1,v_2)) = \begin{cases} \sigma_1(u_1) \wedge \mu_2(u_2 v_2), & \text{if } u_1 = v_1, u_2 v_2 \in E_2 \\ \sigma_2(u_2) \wedge \mu_1(u_1 v_1), & \text{if } u_2 = v_2, u_1 v_1 \in E_1 \\ \mu_1(u_1 v_1) \wedge \mu_2(u_2 v_2), & \text{if } u_1 v_1 \in E_1, u_2 v_2 \in E_2 \\ \sigma_2(u_2) \wedge \sigma_2(v_2) \wedge \mu_1(u_1 v_1), & \text{if } u_2 v_2 \in E_2, u_1 v_1 \in E_1 \\ \sigma_1(u_1) \wedge \sigma_1(v_1) \wedge \mu_2(u_2 v_2), & \text{if } u_1 v_1 \in E_1, u_2 v_2 \in E_2. \end{cases}$$

Note : Throughout this paper $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ denote two fuzzy graphs with underlying crisp graphs $G_1^* : (V_1, E_1)$ and $G_2^* : (V_2, E_2)$ with $|V_i| = p_i$, $i = 1, 2$. Also $d^*_{G_i}(u_i)$ denotes the degree of u_i in G_i.

Lemma 2.10 ([4]). *If $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ are two fuzzy graphs such that $\sigma_1 \leq \mu_2$, then $\sigma_2 \geq \mu_1$. The relation $\sigma_1 \geq \sigma_2$ means that $\sigma_1(u) \geq \sigma_2(v)$, for every $u \in V_1$ and for every $v \in V_2$, where σ_i is a fuzzy subset of V_i, $i = 1, 2$.*

3. Total Degree of a Vertex in Beta Product of Fuzzy Graphs

For any $(u_1, u_2) \in V_1 \times V_2$

$$\begin{aligned} td_{G_1 \times_\beta G_2}(u_1,u_2) &= \sum_{(u_1,u_2)(v_1,v_2) \in E} ((\mu_1 \times_\beta \mu_2)(u_1,u_2)(v_1,v_2)) + (\sigma_1 \times_\beta \sigma_2)(u_1,u_2) \\ &= \sum_{\substack{u_1 v_1 \in E_1, \\ u_2 v_2 \in E_2}} \mu_1(u_1 v_1) \wedge \mu_2(u_2 v_2) + \sum_{\substack{u_1 v_1 \in E_1, \\ u_2 v_2 \in E_2}} \mu_1(u_1 v_1) \wedge \sigma_2(u_2) \wedge \sigma_2(v_2) \\ &\quad + \sum_{\substack{u_1 v_1 \notin E_1, \\ u_2 v_2 \in E_2}} \mu_2(u_2 v_2) \wedge \sigma_1(u_1) \wedge \sigma_1(v_1) + (\sigma_1 \wedge \sigma_2)(u_1,u_2) \end{aligned} \tag{1}$$

Theorem 3.1. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs. If $\sigma_1 \geq \mu_2$, $\sigma_2 \geq \mu_1$ and $\mu_1 \leq \mu_2$, then $td_{G_1 \times_\beta G_2}(u_1, u_2) = [p_2 - 1]d_{G_1}(u_1) + d_{\bar{G}_1^*}(u_1) d_{G_2}(u_2) + \sigma_1(u_1) \wedge \sigma_2(u_2)$.*

Proof. Suppose that $\sigma_1 \geq \mu_2$, $\sigma_2 \geq \mu_1$ and $\mu_1 \leq \mu_2$. Then from (1) for any vertex $(u_1, u_2) \in V_1 \times V_2$

$$\begin{aligned} td_{G_1 \times_\beta G_2}(u_1,u_2) &= \sum_{\substack{u_1 v_1 \in E_1, \\ u_2 v_2 \in E_2}} \mu_1(u_1 v_1) + \sum_{\substack{u_1 v_1 \in E_1, \\ u_2 v_2 \notin E_2}} \mu_1(u_1 v_1) + \sum_{\substack{u_1 v_1 \notin E_1, \\ u_2 v_2 \in E_2}} \mu_2(u_2 v_2) + \sigma_1(u_1) \wedge \sigma_2(u_2) \\ &= d_{G_1}(u_1) d_{G_2^*}(u_2) + d_{\bar{G}_2^*}(u_2) d_{G_1}(u_1) + d_{\bar{G}_1^*}(u_1) d_{G_2}(u_2) + \sigma_1(u_1) \wedge \sigma_2(u_2) \\ &= [d_{G_2^*}(u_2) + d_{\bar{G}_2^*}(u_2)] d_{G_1}(u_1) + d_{\bar{G}_1^*}(u_1) d_{G_2}(u_2) + \sigma_1(u_1) \wedge \sigma_2(u_2) \\ &= [p_2 - 1] d_{G_1}(u_1) + d_{\bar{G}_1^*}(u_1) d_{G_2}(u_2) + \sigma_1(u_1) \wedge \sigma_2(u_2) \end{aligned}$$

□

Corollary 3.2. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs. If $\sigma_1 \geq \mu_2$, $\sigma_2 \geq \mu_1$ and $\mu_1 \leq \mu_2$, then $td_{G_1 \times_\beta G_2}(u_1, u_2) = [p_2 - 1][td_{G_1}(u_1) - \sigma_1(u_1)] + d_{\bar{G}_1^*}(u_1)[td_{G_2}(u_2) - \sigma_2(u_2)] + \sigma_1(u_1) \wedge \sigma_2(u_2)$.*

Proof. Since $d_{G_i}(u_i) = d_{G_i}(u_i) + \sigma_i(u_i) - \sigma_i(u_i) = td_{G_i}(u_i) - \sigma_i(u_i)$, replacing $d_{G_i}(u_i)$ by $td_{G_i}(u_i) - \sigma_i(u_i)$, $i = 1, 2$, in Theorem 3.1. gives the result. □

Corollary 3.3. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs on complete crisp graphs G_1^* and G_2^* respectively. If $\sigma_1 \geq \mu_2$, $\sigma_2 \geq \mu_1$ and $\mu_1 \leq \mu_2$, then $td_{G_1 \times_\alpha G_2}(u_1, u_2) = d_{G_1}(u_1) d_{G_2^*}(u_2) + \sigma_1(u_1) \wedge \sigma_2(u_2)$.*

Proof. Since both G_1^* and G_2^* are complete graphs, $d_{\bar{G_1^*}}(u_1) = 0$ and $d_{G_2^*}(u_2) = [p_2 - 1]$. Therefore Theorem 3.1 becomes, $td_{G_1 \times_\beta G_2}(u_1, u_2) = d_{G_1}(u_1) d_{G_2^*}(u_2) + \sigma_1(u_1) \wedge \sigma_2(u_2)$. □

Corollary 3.4. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs on complete crisp graphs G_1^* and G_2^* respectively. If $\sigma_1 \geq \mu_2$, $\sigma_2 \geq \mu_1$ and $\mu_1 \leq \mu_2$, then $td_{G_1 \times_\alpha G_2}(u_1, u_2) = [td_{G_1}(u_1) - \sigma_1(u_1)] d_{G_2^*}(u_2) + \sigma_1(u_1) \wedge \sigma_2(u_2)$.*

Theorem 3.5. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs. If $\sigma_1 \geq \mu_2$, $\sigma_2 \geq \mu_1$ and $\mu_2 \leq \mu_1$, then $td_{G_1 \times_\beta G_2}(u_1, u_2) = [p_1 - 1] d_{G_2}(u_2) + d_{\bar{G_2^*}}(u_2) d_{G_1}(u_1) + \sigma_1(u_1) \wedge \sigma_2(u_2)$.*

Proof. Suppose that $\sigma_1 \geq \mu_2$, $\sigma_2 \geq \mu_1$ and $\mu_2 \leq \mu_1$. Then from (1) for any vertex $(u_1, u_2) \in V_1 \times V_2$

$$\begin{aligned}
td_{G_1 \times_\beta G_2}(u_1, u_2) &= \sum_{\substack{u_1v_1 \in E_1, \\ u_2v_2 \notin E_2}} \mu_2(u_2v_2) + \sum_{\substack{u_1v_1 \in E_1, \\ u_2v_2 \notin E_2}} \mu_1(u_1v_1) + \sum_{\substack{u_1v_1 \notin E_1, \\ u_2v_2 \in E_2}} \mu_2(u_2v_2) + \sigma_1(u_1) \wedge \sigma_2(u_2) \\
&= d_{G_2}(u_2) d_{G_1^*}(u_1) + d_{\bar{G_2^*}}(u_2) d_{G_1}(u_1) + d_{\bar{G_1^*}}(u_1) d_{G_2}(u_2) + \sigma_1(u_1) \wedge \sigma_2(u_2) \\
&= [d_{G_1^*}(u_1) + d_{\bar{G_1^*}}(u_1)] d_{G_2}(u_2) + d_{\bar{G_2^*}}(u_2) d_{G_1}(u_1) + \sigma_1(u_1) \wedge \sigma_2(u_2) \\
&= [p_1 - 1] d_{G_2}(u_2) + d_{\bar{G_2^*}}(u_2) d_{G_1}(u_1) + \sigma_1(u_1) \wedge \sigma_2(u_2)
\end{aligned}$$

□

Corollary 3.6. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs. If $\sigma_1 \geq \mu_2$, $\sigma_2 \geq \mu_1$ and $\mu_2 \leq \mu_1$, then $td_{G_1 \times_\beta G_2}(u_1, u_2) = [p_1 - 1][td_{G_2}(u_2) - \sigma_2(u_2)] + d_{\bar{G_2^*}}(u_2)[td_{G_1}(u_1) - \sigma_1(u_1)] + \sigma_1(u_1) \wedge \sigma_2(u_2)$.*

Corollary 3.7. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs on complete crisp graphs G_1^* and G_2^* respectively. If $\sigma_1 \geq \mu_2$, $\sigma_2 \geq \mu_1$ and $\mu_2 \leq \mu_1$, then $td_{G_1 \times_\beta G_2}(u_1, u_2) = d_{G_2}(u_2) d_{G_1^*}(u_1) + \sigma_1(u_1) \wedge \sigma_2(u_2)$.*

Proof. Since both G_1^* and G_2^* are complete graphs, $d_{\bar{G_2^*}}(u_2) = 0$ and $d_{G_1^*}(u_1) = p_1 - 1$. Therefore Theorem 3.5 becomes, $td_{G_1 \times_\beta G_2}(u_1, u_2) = d_{G_2}(u_2) d_{G_1^*}(u_1) + \sigma_1(u_1) \wedge \sigma_2(u_2)$. □

Corollary 3.8. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs on complete crisp graphs G_1^* and G_2^* respectively. If $\sigma_1 \geq \mu_2$, $\sigma_2 \geq \mu_1$ and $\mu_2 \leq \mu_1$, then $td_{G_1 \times_\beta G_2}(u_1, u_2) = d_{G_1^*}(u_1)[td_{G_2}(u_2) - \sigma_2(u_2)] + \sigma_1(u_1) \wedge \sigma_2(u_2)$.*

Theorem 3.9. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs such that $\sigma_1 \leq \mu_2$. Then $td_{G_1 \times_\beta G_2}(u_1, u_2) = d_{G_1}(u_1)[p_2 - 1] + \sigma_1(u_1)[d_{\bar{G_1^*}}(u_1) d_{G_2^*}(u_2) + 1]$.*

Proof. We have $\sigma_1 \leq \mu_2$. Hence $\sigma_2 \geq \mu_1$ and $\sigma_1 \leq \sigma_2$. From (1),

$$\begin{aligned}
td_{G_1 \times_\beta G_2}(u_1, u_2) &= \sum_{\substack{u_1v_1 \in E_1, \\ u_2v_2 \in E_2}} \mu_1(u_1v_1) + \sum_{\substack{u_1v_1 \in E_1, \\ u_2v_2 \notin E_2}} \mu_1(u_1v_1) + \sum_{\substack{u_1v_1 \notin E_1, \\ u_2v_2 \in E_2}} \sigma_1(u_1) \wedge \sigma_1(v_1) + \sigma_1(u_1) \\
&= d_{G_1}(u_1) d_{G_2^*}(u_2) + d_{\bar{G_2^*}}(u_2) d_{G_1}(u_1) + \sigma_1(u_1) d_{\bar{G_1^*}}(u_1) d_{G_2^*}(u_2) + \sigma_1(u_1) \\
&= d_{G_1}(u_1)[d_{G_2^*}(u_2) + d_{\bar{G_2^*}}(u_2)] + \sigma_1(u_1)[d_{\bar{G_1^*}}(u_1) d_{G_2^*}(u_2) + 1] \\
&= d_{G_1}(u_1)[p_2 - 1] + \sigma_1(u_1)[d_{\bar{G_1^*}}(u_1) d_{G_2^*}(u_2) + 1].
\end{aligned}$$

□

Corollary 3.10. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs such that $\sigma_1 \leq \mu_2$. Then $td_{G_1 \times_\beta G_2}(u_1, u_2) = td_{G_1}(u_1)[p_2 - 1] + \sigma_1(u_1)[d_{\bar{G_1^*}}(u_1) d_{G_2^*}(u_2) - p_2 + 2]$.*

Proof. The proof follows by replacing $d_{G_1}(u_1)$ by $td_{G_1}(u_1) - \sigma_1(u_1)$, $i = 1, 2$, in Theorem 3.9. □

Corollary 3.11. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs on complete crisp graphs G_1^* and G_2^* respectively such that $\sigma_1 \leq \mu_2$. Then $td_{G_1 \times_\beta G_2}(u_1, u_2) = d_{G_1}(u_1) d_{G_2^*}(u_2) + \sigma_1(u_1)$.*

Proof. Since both G_1^* and G_2^* are complete graphs, $d_{\bar{G_1^*}}(u_1) = 0$ and $d_{G_2^*}(u_2) = [p_2 - 1]$. Therefore Theorem 3.9 becomes, $td_{G_1 \times_\beta G_2}(u_1, u_2) = d_{G_1}(u_1) d_{G_2^*}(u_2) + \sigma_1(u_1)$. □

Corollary 3.12. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs on complete crisp graphs G_1^* and G_2^* respectively such that $\sigma_1 \leq \mu_2$. Then $td_{G_1 \times_\beta G_2}(u_1, u_2) = td_{G_1}(u_1) d_{G_2^*}(u_2) + \sigma_1(u_1)[2 - p_2]$.*

Proof. Since both G_1^* and G_2^* are complete graphs, $d_{\bar{G_1^*}}(u_1) = 0$ and $d_{G_2^*}(u_2) = [p_2 - 1]$. Therefore Corollary 3.10 becomes, $td_{G_1 \times_\beta G_2}(u_1, u_2) = td_{G_1}(u_1) d_{G_2^*}(u_2) + \sigma_1(u_1)[2 - p_2]$. □

Theorem 3.13. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs such that $\sigma_2 \leq \mu_1$. Then $td_{G_1 \times_\beta G_2}(u_1, u_2) = d_{G_2}(u_2)[p_1 - 1] + \sigma_2(u_2)[d_{\bar{G_2^*}}(u_2) d_{G_1^*}(u_1) + 1]$.*

Corollary 3.14. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs such that $\sigma_2 \leq \mu_1$. Then $td_{G_1 \times_\beta G_2}(u_1, u_2) = td_{G_2}(u_2)[p_1 - 1] + \sigma_2(u_2)[d_{\bar{G_2^*}}(u_2) d_{G_1^*}(u_1) - p_1 + 2]$.*

Corollary 3.15. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs on complete crisp graphs G_1^* and G_2^* respectively such that $\sigma_2 \leq \mu_1$. Then $td_{G_1 \times_\beta G_2}(u_1, u_2) = d_{G_2}(u_2) d_{G_1^*}(u_1) + \sigma_2(u_2)$.*

Corollary 3.16. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs on complete crisp graphs G_1^* and G_2^* respectively such that $\sigma_2 \leq \mu_1$. Then $td_{G_1 \times_\beta G_2}(u_1, u_2) = td_{G_2}(u_2) d_{G_1^*}(u_1) + \sigma_2(u_2)[2 - p_1]$.*

4. Totally Regular Property of Beta Product of Two Fuzzy Graphs

Theorem 4.1. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs such that $\sigma_1 \geq \mu_2$, $\sigma_2 \geq \mu_1$, $\mu_1 \leq \mu_2$ and $\sigma_1 \wedge \sigma_2$ is a constant function and let G_1^* be a regular graph. Then $G_1 \times_\beta G_2$ is a totally regular fuzzy graph if and only if G_1 and G_2 are regular fuzzy graphs.*

Proof. Let $\sigma_1(u) \wedge \sigma_2(v) = c$, a constant for all $u \in V_1$ and $v \in V_2$. Let G_1^* be r_1-regular graph. Assume that $G_1 \times_\beta G_2$ is a totally regular fuzzy graph. Then for any two points (u_1, u_2) and (v_1, v_2) in $V_1 \times V_2$.

$$td_{G_1 \times_\beta G_2}(u_1, u_2) = td_{G_1 \times_\beta G_2}(v_1, v_2)$$

From Theorem 3.1,

$$[p_2 - 1] d_{G_1}(u_1) + d_{\bar{G_1^*}}(u_1) d_{G_2}(u_2) + c = [p_2 - 1] d_{G_1}(v_1) + d_{\bar{G_1^*}}(v_1) d_{G_2}(v_2) + c$$
$$\Rightarrow [p_2 - 1] d_{G_1}(u_1) + [p_1 - 1 - d_{G_1^*}(u_1)] d_{G_2}(u_2) = [p_2 - 1] d_{G_1}(v_1) + [p_1 - 1 - d_{G_1^*}(v_1)] d_{G_2}(v_2)$$
$$\Rightarrow [p_2 - 1] d_{G_1}(u_1) + [p_1 - 1 - r_1] d_{G_2}(u_2) = [p_2 - 1] d_{G_1}(v_1) + [p_1 - 1 - r_1] d_{G_2}(v_2) \quad (2)$$

Fix $u \in V_1$ and consider (u, u_2) and (u, v_2) in $V_1 \times V_2$, where $u_2, v_2 \in V_2$ are arbitrary. From(2),

$$[p_2 - 1]d_{G_1}(u) + [p_1 - 1 - r_1]d_{G_2}(u_2) = [p_2 - 1]d_{G_1}(u) + [p_1 - 1 - r_1]d_{G_2}(v_2)$$
$$\Rightarrow d_{G_2}(u_2) = d_{G_2}(v_2)$$

This is true for all $u_2, v_2 \in V_2$. Thus G_2 is a regular fuzzy graph. Fix $v \in V_2$ and consider (u_1, v) and (v_1, v) in $V_1 \in V_2$, where $u_1, v_1 \in V_1$ are arbitrary. From (2),

$$[p_2 - 1]d_{G_1}(u_1) + [p_1 - 1 - r_1]d_{G_2}(v) = [p_2 - 1]d_{G_1}(v_1) + [p_1 - 1 - r_1]d_{G_2}(v)$$
$$\Rightarrow d_{G_1}(u_1) = d_{G_1}(v_1)$$

This is true for all $u_1, v_1 \in V_1$. Thus G_1 is a regular fuzzy graph.
Conversely, Let G_1 be a k_1-regular fuzzy graph and G_2 be a k_2-regular fuzzy graph. Then for any vertex $(u_1, u_2) \in V_1 \times V_2$, from Theorem 3.1, $td_{G_1 \times_\beta G_2}(u_1, u_2) = [p_2 - 1]k_1 + [p_1 - 1 - r_1]k_2 + c$. Hence $G_1 \times_\beta G_2$ is a totally regular fuzzy graph. □

Theorem 4.2. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs such that $\sigma_1 \geq \mu_2$, $\sigma_2 \geq \mu_1$, $\mu_1 \leq \mu_2$ and σ_1 and σ_2 are constant functions and let G_1^* be a regular graph. Then $G_1 \times_\beta G_2$ is a totally regular fuzzy graph if and only if G_1 and G_2 are totally regular fuzzy graphs.*

Proof. Let c_1 and c_2 be the constant values of σ_1 and σ_2 respectively. Without loss of generality assume that $c_1 c_2$. Then $\sigma_1(u) \wedge \sigma_2(v) = \min\{c_1, c_2\} = c_1$. Let G_1^* be r_1-regular graph. Assume that $G_1 \times_\beta G_2$ is a totally regular fuzzy graph. Then for any two points (u_1, u_2) and (v_1, v_2) in $V_1 \times V_2$.

$$td_{G_1 \times_\beta G_2}(u_1, u_2) = td_{G_1 \times_\beta G_2}(v_1, v_2)$$

From Corollary 3.2,

$$[p_2 - 1][td_{G_1}(u_1) - \sigma_1(u_1)] + d_{\bar{G_1^*}}(u_1)[td_{G_2}(u_2) - \sigma_2(u_2)] + c_1 = [p_2 - 1][td_{G_1}(v_1) - \sigma_1(v_1)] + d_{\bar{G_1^*}}(v_1)[td_{G_2}(v_2) - \sigma_2(v_2)] + c_1$$
$$\Rightarrow [p_2 - 1][td_{G_1}(u_1) - c_1] + d_{\bar{G_1^*}}(u_1)[td_{G_2}(u_2) - c_2] + c_1 = [p_2 - 1][td_{G_1}(v_1) - c_1] + d_{\bar{G_1^*}}(v_1)[td_{G_2}(v_2) - \sigma_2(v_2)] + c_1$$
$$\Rightarrow [p_2 - 1]td_{G_1}(u_1) + [p_1 - 1 - r_1]td_{G_2}(u_2) = [p_2 - 1]d_{G_1}(v_1) + [p_1 - 1 - r_1]td_{G_2}(v_2) \qquad (3)$$

Fix $u \in V_1$ and consider (u, u_2) and (u, v_2) in $V_1 \times V_2$ where $u_2, v_2 \times V_2$ are arbitrary. From (3),

$$[p_2 - 1]td_{G_1}(u) + [p_1 - 1 - r_1]td_{G_2}(u_2) = [p_2 - 1]d_{G_1}(u) + [p_1 - 1 - r_1]td_{G_2}(v_2)$$
$$\Rightarrow [p_1 - 1 - r_1]td_{G_2}(u_2) = [p_1 - 1 - r_1]td_{G_2}(v_2)$$
$$\Rightarrow d_{G_2}(u_2) = d_{G_2}(v_2)$$

This is true for all $u_2, v_2 \in V_2$. Thus G_2 is a regular fuzzy graph. Fix $v \in V_2$ and consider (u_1, v) and (v_1, v) in $V_1 \times V_2$ where $u_1, v_1 \in V_1$ are arbitrary. From (3),

$$[p_2 - 1]td_{G_1}(u_1) + [p_1 - 1 - r_1]td_{G_2}(v) = [p_2 - 1]d_{G_1}(v_1) + [p_1 - 1 - r_1]td_{G_2}(v)$$
$$\Rightarrow [p_2 - 1]td_{G_1}(u_1) = [p_2 - 1]td_{G_1}(v_1)$$
$$\Rightarrow td_{G_1}(u_1) = td_{G_1}(v_1)$$

This is true for all $u_1, v_1 \in V_1$. Thus G_1 is a totally regular fuzzy graph.

Conversely, Let G_1 be a k_1-regular fuzzy graph and G_2 be a k_2-regular fuzzy graph. Then for any vertex $(u_1, u_2) \in V_1 \times V_2$, from Corollary 3.2, $td_{G_1 \times_\beta G_2}(u_1, u_2) = [p_2 - 1]k_1 + [p_1 - 1 - r_1]k_2 + c$. Hence $G_1 \times_\beta G_2$ is a totally regular fuzzy graph. □

Theorem 4.3. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_{2,2})$ be two fuzzy graphs on complete crisp graphs G_1^* and G_2^* respectively such that $\sigma_1 \geq \mu_2$, $\sigma_2 \geq \mu_1$, $\mu_1 \leq \mu_2$ and $\sigma_1 \wedge \sigma_2$ is a constant function. Then $G_1 \times_\beta G_2$ is a totally regular fuzzy graph if and only if G_1 is a regular fuzzy graph.*

Theorem 4.4. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs on complete crisp graphs G_1^* and G_2^* respectively such that $\sigma_1 \geq \mu_2$, $\sigma_2 \geq \mu_1$, $\mu_1 \leq \mu_2$ and σ_1 and $\sigma_1 \wedge \sigma_2$ are constant functions. Then $G_1 \times_\beta G_2$ is a totally regular fuzzy graph if and only if G_1 is a totally regular fuzzy graph.*

Theorem 4.5. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs such that $\sigma_1 \geq \mu_2$, $\sigma_2 \geq \mu_1$, $\mu_2 \leq \mu_1$ and $\sigma_1 \wedge \sigma_2$ is a constant function and let G_2^* be a regular graph. Then $G_1 \times_\beta G_2$ is a totally regular fuzzy graph if and only if G_1 and G_2 are regular fuzzy graphs.*

Proof. Let $\sigma_1(u) \wedge \sigma_2(v) = c$, a constant for all $u \in V_1$ and $v \in V_2$. Let G_2^* be r_2-regular graph. Assume that $G_1 \times_\beta G_2$ is a totally regular fuzzy graph. Then for any two points (u_1, u_2) and (v_1, v_2) in $V_1 \times V_2$.

$$td_{G_1 \times_\beta G_2}(u_1, u_2) = td_{G_1 \times_\beta G_2}(v_1, v_2)$$

From Theorem 3.5,

$$[p_1 - 1]d_{G_2}(u_2) + d_{\bar{G_2^*}}(u_2)d_{G_1}(u_1) + c = [p_1 - 1]d_{G_2}(v_2) + d_{\bar{G_2^*}}(v_2)d_{G_1}(v_1) + c$$
$$\Rightarrow [p_1 - 1]d_{G_2}(u_2) + [p_2 - 1 - d_{G_2^*}(u_2)]d_{G_1}(u_1) = [p_1 - 1]d_{G_2}(v_2) + [p_2 - 1 - d_{G_2^*}(v_2)]d_{G_1}(v_1)$$
$$\Rightarrow [p_1 - 1]d_{G_2}(u_2) + [p_2 - 1 - r_2]d_{G_1}(u_1) = [p_1 - 1]d_{G_2}(v_2) + [p_2 - 1 - r_2]d_{G_1}(v_1) \quad (4)$$

Fix $u \in V_1$ and consider (u, u_2) and (u, v_2) in $V_1 \times V_2$ where $u_2, v_2 \in V_2$ are arbitrary. From (4),

$$[p_1 - 1]d_{G_2}(u_2) + [p_2 - 1 - r_2]d_{G_1}(u) = [p_1 - 1]d_{G_2}(v_2) + [p_2 - 1 - r_2]d_{G_1}(u)$$
$$\Rightarrow d_{G_2}(u_2) = d_{G_2}(v_2)$$

This is true for all $u_2, v_2 \in V_2$. Thus G_2 is a regular fuzzy graph. Fix $v \in V_2$ and consider (u_1, v) and (v_1, v) in $V_1 \times V_2$ where $u_1, v_1 \in V_1$ are arbitrary. From (4),

$$[p_1 - 1]d_{G_2}(v) + [p_2 - 1 - r_2]d_{G_1}(u_1) = [p_1 - 1]d_{G_2}(v) + [p_2 - 1 - r_2]d_{G_1}(v_1)$$
$$\Rightarrow d_{G_1}(u_1) = d_{G_1}(v_1)$$

This is true for all $u_1, v_1 \in V_1$. Thus G_1 is a regular fuzzy graph.

Conversely, Let G_1 be a k_1-regular fuzzy graph and G_2 be a k_2-regular fuzzy graph. Then for any vertex $(u_1, u_2) \in V_1 \times V_2$, from Theorem 3.4,

$$td_{G_1 \times_\beta G_2}(u_1, u_2) = [p_1 - 1]k_2 + [p_2 - 1 - r_2]k_1$$

Hence $G_1 \times_\beta G_2$ is a totally regular fuzzy graph. □

Theorem 4.6. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs such that $\sigma_1 \geq \mu_2, \sigma_2 \geq \mu_1, \mu_2 \leq \mu_1$ and $\sigma_1 \wedge \sigma_2$ is a constant function and let G_2^* be a regular graph. Then $G_1 \times_\beta G_2$ is a totally regular fuzzy graph if and only if G_1 and G_2 are totally regular fuzzy graphs.*

Theorem 4.7. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs on complete crisp graphs G_1^* and G_2^* respectively such that $\sigma_1 \geq \mu_2$, $\sigma_2 \geq \mu_1$, $\mu_2 \leq \mu_1$ and $\sigma_1 \wedge \sigma_2$ is a constant function. Then $G_1 \times_\beta G_2$ is a totally regular fuzzy graph if and only if G_2 is a regular fuzzy graph.*

Theorem 4.8. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs on complete crisp graphs G_1^* and G_2^* respectively such that $\sigma_1 \geq \mu_2$, $\sigma_2 \geq \mu_1$, $\mu_2 \leq \mu_1$ and σ_1 and $\sigma_1 \wedge \sigma_2$ are constant functions. Then $G_1 \times_\beta G_2$ is a totally regular fuzzy graph if and only if G_2 is a totally regular fuzzy graph.*

Theorem 4.9. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs such that σ_1 is a constant function with $\sigma_1 \leq \mu_2$ and let $G_1^*(/G_2^*)$ be a regular graph. Then $G_1 \times_\beta G_2$ is a totally regular fuzzy graph if and only if G_1 is a regular fuzzy graph and $G_2^*(/G_1^*)$ is a regular graph.*

Proof. Let $\sigma_1(u_1) = c_1$ for all $u \in V_1$, where c_1 is the constant value of σ_1. Let G_1^* be r_1-regular graph. Assume that $G_1 \times_\beta G_2$ is a totally regular fuzzy graph. Then for any two points (u_1, u_2) and (v_1, v_2) in $V_1 \times V_2$.

$$td_{G_1 \times_\beta G_2}(u_1, u_2) = td_{G_1 \times_\beta G_2}(v_1, v_2)$$

From Theorem 3.9,

$$\begin{aligned} d_{G_1}(u_1)[p_2 - 1] + c_1[(p_1 - 1 - r_1)d_{G_2^*}(u_2) + 1] &= d_{G_1}(v_1)[p_2 - 1] + c_1[(p_1 - 1 - r_1)d_{G_2^*}(v_2) + 1] \\ \Rightarrow d_{G_1}(u_1)[p_2 - 1] + c_1(p_1 - 1 - r_1)d_{G_2^*}(u_2) &= d_{G_1}(v_1)[p_2 - 1] + c_1(p_1 - 1 - r_1)d_{G_2^*}(v_2) \quad (5) \end{aligned}$$

Fix $u \in V_1$ and consider (u, u_2) and (u, v_2) in $V_1 \times V_2$ where $u_2, v_2 \in V_2$ are arbitrary. From (5),

$$d_{G_1}(u)[p_2 - 1] + c_1(p_1 - 1 - r_1)d_{G_2^*}(u_2) = d_{G_1}(u)[p_2 - 1] + c_1(p_1 - 1 - r_1)d_{G_2^*}(v_2)$$
$$\Rightarrow d_{G_2}^*(u_2) = d_{G_2}^*(v_2)$$

This is true for every u_2, v_2 in V_2. Thus G_2^* is a regular graph. Fix $v \in V_2$ and consider (u_1, v) and (v_1, v) in $V_1 \times V_2$ where $u_1, v_1 \in V_1$ are arbitrary. From (5),

$$d_{G_1}(u_1)[p_2 - 1] + c_1(p_1 - 1 - r_1)d_{G_2^*}(v) = d_{G_1}(v_1)[p_2 - 1] + c_1(p_1 - 1 - r_1)d_{G_2^*}(v)$$
$$\Rightarrow d_{G_1}(u_1) = d_{G_1}(v_1)$$

This is true for all $u_1, v_1 \in V_1$. Thus G_1 is a regular fuzzy graph.

Conversely, Let G_1 be a k_1-regular fuzzy graph and G_2^* is a r_2-regular graph. Then for any vertex $(u_1, u_2) \in V_1 \times V_2$, from Theorem 3.7,

$$td_{G_1 \times_\beta G_2}(u_1, u_2) = k_1[p_2 - 1] + c_1[p_1 - 1 - r_1]r_2 + c_1$$

Hence $G_1 \times_\beta G_2$ is a totally regular fuzzy graph. □

Theorem 4.10. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs such that σ_1 is a constant function with $\sigma_1 \leq \mu_2$ and let $G_1^*(/G_2^*)$ be a regular graph. Then $G_1 \times_\beta G_2$ is a totally regular fuzzy graph if and only if G_1 is a totally regular fuzzy graph and $G_2^*(/G_1^*)$ is a regular graph.*

Theorem 4.11. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs on complete crisp graphs G_1^* and G_2^* respectively such that σ_1 is a constant function with $\sigma_1 \leq \mu_2$ and let $G_1^*(/G_2^*)$ be a regular graph. Then $G_1 \times_\beta G_2$ is a totally regular fuzzy graph if and only if G_1 is a regular fuzzy graph and $G_2^*(/G_1^*)$ is a regular graph.*

Theorem 4.12. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs on complete crisp graphs G_1^* and G_2^* respectively such that σ_1 is a constant function with $\sigma_1 \leq \mu_2$ and let $G_1^*(/G_2^*)$ be a regular graph. Then $G_1 \times_\beta G_2$ is a totally regular fuzzy graph if and only if G_1 is a regular fuzzy graph and $G_2^*(/G_1^*)$ is a regular graph.*

Theorem 4.13. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs such that σ_2 is a constant function with $\sigma_2 \leq \mu_1$ and let $G_2^*(/G_1^*)$ be a regular graph. Then $G_1 \times_\beta G_2$ is a totally regular fuzzy graph if and only if G_2 is a regular fuzzy graph and $G_1^*(/G_2^*)$ is a regular graph.*

Theorem 4.14. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs such that σ_2 is a constant function with $\sigma_2 \leq \mu_1$ and let $G_2^*(/G_1^*)$ be a regular graph. Then $G_1 \times_\beta G_2$ is a totally regular fuzzy graph if and only if G_2 is a totally regular fuzzy graph and $G_1^*(/G_2^*)$ is a regular graph.*

Theorem 4.15. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs on complete crisp graphs G_1^* and G_2^* respectively such that σ_2 is a constant function with $\sigma_2 \leq \mu_1$. Then $G_1 \times_\beta G_2$ is a totally regular fuzzy graph if and only if G_2 is a regular fuzzy graph.*

Theorem 4.16. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs on complete crisp graphs G_1^* and G_2^* respectively such that σ_2 is a constant function with $\sigma_2 \leq \mu_1$. Then $G_1 \times_\beta G_2$ is a totally regular fuzzy graph if and only if G_2 is a totally regular fuzzy graph.*

5. Total Degree of a Vertex in Gamma Product on Fuzzy Graphs

For any vertex $(u_1, u_2) \in V_1 \times V_2$

$$td_{G_1 \times_\gamma G_2}(u_1, u_2) = \sum_{(u_1,u_2)(v_1,v_2) \in E} ((\mu_1 \times_\gamma \mu_2)(u_1, u_2)(v_1, v_2)) + (\sigma_1 \times_\gamma \sigma_2)(u_1, u_2)$$

$$\begin{aligned} td_{G_1 \times_\gamma G_2}(u_1, u_2) = &\sum_{u_1 = v_1, u_2v_2 \in E_2} \sigma_1(u_1) \wedge \mu_2(u_2v_2) + \sum_{u_2 = v_2, u_1v_1 \in E_1} \sigma_2(u_2) \wedge \mu_1(u_1v_1) \\ &+ \sum_{u_1v_1 \in E_1, u_2v_2 \in E_2} \sigma_2(u_2) \wedge \sigma_2(v_2) \wedge \mu_1(u_1v_1) + \sum_{u_1v_1 \in E_1, u_2v_2 \in E_2} \sigma_1(u_1) \wedge \sigma_1(v_1) \wedge \mu_2(u_2v_2) \\ &+ \sum_{u_1v_1 \in E_1, u_2v_2 \in E_2} \mu_1(u_1v_1) \wedge \mu_2(u_2v_2) + \sigma_1(u_1) \wedge \sigma_2(u_2) \end{aligned} \tag{6}$$

In the following theorems, we obtain the total degree of a vertex in the gamma product of two fuzzy graphs in terms of the degrees of vertices in the given fuzzy graphs in some particular cases.

Theorem 5.1. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs. If $\sigma_1 \geq \mu_2, \sigma_2 \geq \mu_1$ and $\mu_1 \leq \mu_2$, then $td_{G_1 \times_\gamma G_2}(u_1, u_2) = d_{G_2}(u_2)[p_1 - d_{G_1}(u_1)] + p_2 d_{G_1}(u_1) + \sigma_1(u_1) \wedge \sigma_2(u_2)$.*

Proof. Given $\sigma_1 \geq \mu_2, \sigma_2 \geq \mu_1$ and $\mu_1 \leq \mu_2$. From (3.1)

$$\begin{aligned} td_{G_1 \times_\gamma G_2}(u_1,u_2) &= \sum_{u_1=v_1, u_2v_2 \in E_2} \mu_2(u_2v_2) + \sum_{u_2=v_2, u_1v_1 \in E_1} \mu_1(u_1v_1) \\ &+ \sum_{u_1v_1 \in E_1, u_2v_2 \in E_2} \mu_2(u_2v_2) + \sum_{u_1v_1 \in E_1, u_2v_2 \in E_2} \mu_1(u_1v_1) + \sum_{u_1v_1 \in E_1, u_2v_2 \in E_2} \mu_1(u_1v_1) + \sigma_1(u_1) \wedge \sigma_2(u_2) \\ &= d_{G_2}(u_2) + d_{G_1}(u_1) + d_{\bar{G_1^*}}(u_1)d_{G_2}(u_2) + d_{\bar{G_2^*}}(u_2)d_{G_1}(u_1) + d_{G_1}(u_1)d_{G_2^*}(u_2) + \sigma_1(u_1) \wedge \sigma_2(u_2) \\ &= d_{G_2}(u_2)[1 + d_{\bar{G_1^*}}(u_1)] + d_{G_1}(u_1)[1 + d_{\bar{G_2^*}}(u_2) + d_{G_2^*}(u_2)] + \sigma_1(u_1) \wedge \sigma_2(u_2) \\ &= d_{G_2}(u_2)[1 + p_1 - 1 - d_{G_1^*}(u_1)] + d_{G_1}(u_1)[1 + p_2 - 1] + \sigma_1(u_1) \wedge \sigma_2(u_2) \\ &= d_{G_2}(u_2)[p_1 - d_{G_1^*}(u_1)] + p_2 d_{G_1}(u_1) + \sigma_1(u_1) \wedge \sigma_2(u_2) \end{aligned}$$

□

Corollary 5.2. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs. If $\sigma_1 \geq \mu_2, \sigma_2 \geq \mu_1$ and $\mu_1 \leq \mu_2$, then $td_{G_1 \times_\gamma G_2}(u_1,u_2) = td_{G_2}(u_2)[p_1 - d_{G_1}(u_1)] - [p_1 - d_{G_1}(u_1)]\sigma_2(u_2) + p_2 td_{G_1}(u_1) - p_2\sigma_1(u_1) + \sigma_1(u_1)?\sigma_2(u_2)$.*

Theorem 5.3. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs. If $\sigma_1 \geq \mu_2, \sigma_2 \geq \mu_1$ and $\mu_2 \leq \mu_1$, then $td_{G_1 \times_\gamma G_2}(u_1,u_2) = d_{G_1}(u_1)[p_2 - d_{G_2^*}(u_2)] + p_1 d_{G_2}(u_2) + \sigma_1(u_1) \wedge \sigma_2(u_2)$.*

Corollary 5.4. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs. If $\sigma_1 \geq \mu_2, \sigma_2 \geq \mu_1$ and $\mu_2 \leq \mu_1$, then $td_{G_1 \times_\gamma G_2}(u_1,u_2) = td_{G_1}(u_1)[p_2 - d_{G_2^*}(u_2)] - [p_2 - d_{G_2^*}(u_2)]\sigma_1(u_1) + p_1 td_{G_2}(u_2) - p_1\sigma_2(u_2) + \sigma_1(u_1) \wedge \sigma_2(u_2)$.*

Theorem 5.5. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs. If $\sigma_1 \leq \mu_2$ and σ_1 is a constant function with $\sigma_1(u_1) = c_1$ for all $u \in V_1$, then $td_{G_1 \times_\gamma G_2}(u_1,u_2) = c_1[(p_1 - d_{G_1}(u_1))d_{G_2}(u_2) + 1] + p_2 d_{G_1}(u_1)$, where c_1 is the constant value of σ_1.*

Proof. Given $\sigma_1 \leq \mu_2$ and σ_1 is a constant function with $\sigma_1(u_1) = c_1$ for all $u \in V_1$. If $\sigma_1 \leq \mu_2$, then $\sigma_2 \geq \mu_1$ and $\mu_1 \leq \mu_2$. Also $\sigma_1 \leq \sigma_2$. From (6),

$$\begin{aligned} td_{G_1 \times_\gamma G_2}(u_1,u_2) &= \sigma_1(u_1)d_{G_2^*}(u_2) + d_{G_1}(u_1) + \sigma_1(u_1)d_{\bar{G_1^*}}(u_1)d_{G_2^*}(u_2) + d_{\bar{G_2^*}}(u_2)d_{G_1}(u_1) + d_{G_2^*}(u_2)d_{G_1}(u_1) + \sigma_1(u_1) \\ &= c_1 d_{G_2^*}(u_2) + d_{G_1}(u_1) + c_1 d_{\bar{G_1^*}}(u_1)d_{G_2^*}(u_2) + d_{\bar{G_2^*}}(u_2)d_{G_1}(u_1) + d_{G_2^*}(u_2)d_{G_1}(u_1) + c_1 \\ &= c_1 d_{G_2^*}(u_2) + d_{G_1}(u_1) + c_1[p_1 - 1 - d_{G_1^*}(u_1)]d_{G_2^*}(u_2) + [p_2 - 1 - d_{G_2^*}(u_2)]d_{G_1}(u_1) + d_{G_2^*}(u_2)d_{G_1}(u_1) + c_1 \\ &= c_1 d_{G_2^*}(u_2) + d_{G_1}(u_1) + c_1 p_1 d_{G_2^*}(u_2) - c_1 d_{G_2^*}(u_2) - c_1 d_{G_1^*}(u_1)d_{G_2^*}(u_2) + p_2 d_{G_1}(u_1) - d_{G_1}(u_1) \\ &\quad - d_{G_2^*}(u_2)d_{G_1}(u_1) + d_{G_2^*}(u_2)d_{G_1}(u_1) + c_1 \\ &= c_1 p_1 d_{G_2^*}(u_2) - c_1 d_{G_1^*}(u_1)d_{G_2^*}(u_2) + p_2 d_{G_1}(u_1) + c_1 \\ &= c_1[(p_1 - d_{G_1^*}(u_1))d_{G_2^*}(u_2) + 1] + p_2 d_{G_1}(u_1) \end{aligned}$$

□

Corollary 5.6. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs. If$\sigma_1 \leq \mu_2$ and σ_1 is a constant function with $\sigma_1(u_1) = c_1$ for all $u \in V_1$, then $td_{G_1 \times_\gamma G_2}(u_1,u_2) = c_1[(p_1 - d_{G_1^*}(u_1))d_{G_2^*}(u_2) - p_2 + 1] + p_2 td_{G_1}(u_1)$, where c_1 is the constant value of σ_1.*

Theorem 5.7. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs. If $\sigma_2 \leq \mu_1$ and σ_2 is a constant function with $\sigma_2(u_2) = c_2$ for all $u \in V_2$, then $td_{G_1 \times_\gamma G_2}(u_1,u_2) = p_1 d_{G_2}(u_2) + \sigma_2(u_2)[p_2 d_{G_1^*}(u_1) + 1] - d_{G_2^*}(u_2)d_{G_1^*}(u_1)$, where c_2 is the constant value of σ_2.*

Proof. Given $\sigma_2 \leq \mu_1$ and σ_2 is a constant function with $\sigma_2(u_2) = c_2$ for all $v \in V_2$. If $\sigma_2 \leq \mu_1$, then $\sigma_1 \geq \mu_2$ and $\mu_2 \leq \mu_1$. Also $\sigma_2 \leq \sigma_1$.

$$\begin{aligned} td_{G_1 \times_\gamma G_2}(u_1, u_2) &= d_{G_2}(u_2) + \sigma_2(u_2) d_{G_1^*}(u_1) + d_{\bar{G_1^*}}(u_1) d_{G_2}(u_2) + d_{\bar{G_2^*}}(u_2)\sigma_2(u_2) d_{G_1^*}(u_1) + d_{G_2}(u_2) d_{G_1^*}(u_1) + \sigma_2(u_2) \\ &= d_{G_2}(u_2) + \sigma_2(u_2) d_{G_1^*}(u_1) + [p_1 - 1 - d_{G_1^*}(u_1)] d_{G_2}(u_2) + [p_2 - 1 - d_{G_2^*}(u_2)]\sigma_2(u_2) d_{G_1^*}(u_1) \\ &\quad + d_{G_2}(u_2) d_{G_1^*}(u_1) + \sigma_2(u_2) \\ &= p_1 d_{G_2}(u_2) + c_2[p_2 d_{G_1^*}(u_1) + 1] - d_{G_2^*}(u_2) d_{G_1^*}(u_1) \end{aligned}$$

□

Corollary 5.8. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs. If $\sigma_2 \leq \mu_1$ and σ_2 is a constant function with $\sigma_2(u_2) = c_2$ for all $u \in V_2$, then $td_{G_1 \times_\gamma G_2}(u_1, u_2) = p_1 td_{G_2}(u_2) + c_2[p_2 d_{G_1^*}(u_1) - p_1 + 1] - d_{G_2^*}(u_2) d_{G_1^*}(u_1)$, where c_2 is the constant value of σ_2.*

6. Totally Regular Property in Gamma Product of Two Fuzzy Graphs

In general, there does not exist any relationship between the totally regular property of G_1 and G_2 and the totally regular property of $G_1 \wedge G_2$.

1. If G_1 or G_2 is a totally regular fuzzy graphs, then $G_1 \wedge G_2$ need not be a totally regular fuzzy graph in Fig. 1.

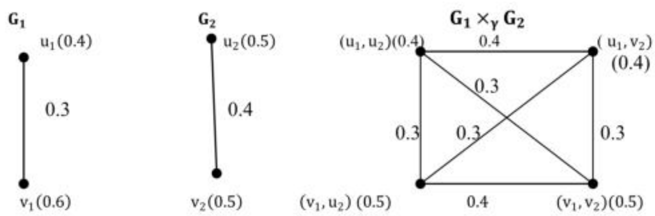

Figure 1.

2. If $G_1 \wedge G_2$ is a totally regular fuzzy graph, then G_1 or G_2 need not be a totally regular in Fig 2.

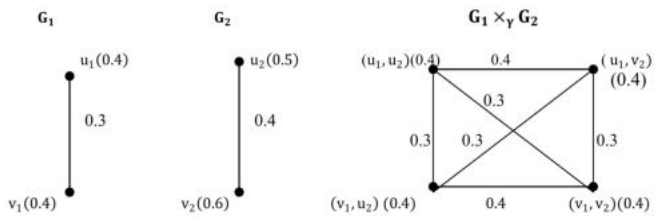

Figure 2.

Here $\sigma_1 \geq \mu_2, \sigma_2 \geq \mu_1$ and $\mu_1 \leq \mu_2$.

$$\begin{aligned} td_{G_1 \times_\gamma G_2}(u_1, u_2) &= d_{G_2}(u_2)[p_1 - d_{G_1^*}(u_1)] + p_2 d_{G_1}(u_1) + \sigma_1(u_1) \wedge \sigma_2(u_2) \\ &= 0.4[2 - 1] + 2(0.3) + 0.4 = 0.4 + 0.6 + 0.4 = 1.4 \end{aligned}$$

Theorem 6.1. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs such that $\sigma_1 \geq \mu_2$, $\sigma_2 \geq \mu_1$, $\mu_1 \leq \mu_2$ and $\sigma_1 \wedge \sigma_2$ is a constant function and let G_1^* be a regular graph. Then $G_1 \times_\gamma G_2$ is a totally regular fuzzy graph if and only if G_1 and G_2 are regular fuzzy graphs.*

Proof. Let $\sigma_1(u) \wedge \sigma_2(v) = c$, a constant for all $u \in V_1$ and $v \in V_2$. Let G_1^* be r_1-regular graph. Assume that $G_1 \times G_2$ is a totally regular fuzzy graph. Then for any two points (u_1, u_2) and (v_1, v_2) in $V_1 \times V_2$.

$$td_{G_1 \times G_2}(u_1, u_2) = td_{G_1 \times G_2}(v_1, v_2)$$

From Theorem 5.1,

$$d_{G_2}(u_2)[p_1 - r_1] + p_2 d_{G_1}(u_1) + c = d_{G_2}(v_2)[p_1 - r_1] + p_2 d_{G_1}(v_1) + c$$
$$\Rightarrow d_{G_2}(u_2)[p_1 - r_1] + p_2 d_{G_1}(u_1) = d_{G_2}(v_2)[p_1 - r_1] + p_2 d_{G_1}(v_1) \quad (7)$$

Fix $u \in V_1$ and consider (u, u_2) and (u, v_2) in $V_1 \times V_2$ where $u_2, v_2 \in V_2$ are arbitrary. From (7),

$$d_{G_2}(u_2)[p_1 - r_1] + p_2 d_{G_1}(u) = d_{G_2}(u_2)[p_1 - r_1] + p_2 d_{G_1}(u)$$
$$\Rightarrow d_{G_2}(u_2) = d_{G_2}(v_2)$$

This is true for all $u_2, v_2 \in V_2$. Thus G_2 is a regular fuzzy graph. Fix $v \in V_2$ and consider (u_1, v) and (v_1, v) in $V_1 \times V_2$ where $u_1, v_1 \in V_1$ are arbitrary. From (7),

$$d_{G_2}(v)[p_1 - r_1] + p_2 d_{G_1}(u_1) = d_{G_2}(v)[p_1 - r_1] + p_2 d_{G_1}(u_1)$$
$$\Rightarrow d_{G_1}(u_1) = d_{G_1}(v_1)$$

This is true for all $u_1, v_1 \in V_1$. Thus G_1 is a regular fuzzy graph.
Conversely, Let G_1 be k_1-regular fuzzy graph and G_2 be k_2-regular fuzzy graph. From Theorem 5.1, $td_{G_1 \times_\gamma G_2}(u_1, u_2) = k_2[p_1 - r_1] + p_2 k_1 + c$. Hence $G_1 \times_\gamma G_2$ is a totally regular fuzzy graph. □

Theorem 6.2. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs such that $\sigma_1 \geq \mu_2$, $\sigma_2 \geq \mu_1$, $\mu_1 \leq \mu_2$ and $\sigma_1 \wedge \sigma_2$ is a constant function and let G_1^* be a regular graph. Then $G_1 \times_\gamma G_2$ is a totally regular fuzzy graph if and only if G_1 and G_2 are totally regular fuzzy graphs.*

Proof. Let $\sigma_1(u) \wedge \sigma_2(v) = c$, a constant for all $u \in V_1$ and $v \in V_2$. Let $\sigma_i(u_i) = c_i$ for all $u_i \in V_i$, where c_i is a constant, $i = 1, 2$. Let G_1^* be r_1-regular graph. Assume that $G_1 \times G_2$ is a totally regular fuzzy graph. Then for any two points (u_1, u_2) and (v_1, v_2) in $V_1 \times V_2$.

$$td_{G_1 \times G_2}(u_1, u_2) = td_{G_1 \times G_2}(v_1, v_2)$$

From Corollary 5.2,

$$td_{G_2}(u_2)[p_1 - r_1] - [p_1 - r_1]c_2 + p_2 td_{G_1}(u_1) - p_2 c_1 + c = td_{G_2}(v_2)[p_1 - r_1] - [p_1 - r_1]c_2 + p_2 td_{G_1}(v_1) - p_2 c_1 + c$$
$$\Rightarrow td_{G_2}(u_2)[p_1 - r_1] + p_2 td_{G_1}(u_1) = td_{G_2}(v_2)[p_1 - r_1] + p_2 td_{G_1}(v_1) \quad (8)$$

Fix $u \in V_1$ and consider (u, u_2) and (u, v_2) in $V_1 \times V_2$ where $u_2, v_2 \in V_2$ are arbitrary. From (8),

$$td_{G_2}(u_2)[p_1 - r_1] + p_2 td_{G_1}(u) = td_{G_2}(v_2)[p_1 - r_1] + p_2 td_{G_1}(u)$$
$$\Rightarrow td_{G_2}(u_2) = td_{G_2}(v_2)$$

This is true for all $u_2, v_2 \in V_2$. Thus G_2 is a totally regular fuzzy graph. Fix $v \in V_2$ and consider (u_1, v) and (v_1, v) in $V_1 \times V_2$ where $u_1, v_1 \in V_1$ are arbitrary. From (8),

$$td_{G_2}(v)[p_1 - r_1] + p_2 td_{G_1}(u_1) = td_{G_2}(v)[p_1 - r_1] + p_2 td_{G_1}(v_1)$$
$$\Rightarrow td_{G_1}(u_1) = td_{G_1}(v_1)$$

This is true for all $u_1, v_1 \in V_1$. Thus G_1 is a totally regular fuzzy graph.

Conversely, Let G_1 be k_1-totally regular fuzzy graph and G_2 be k_2-totally regular fuzzy graph. From corollary 5.2, $td_{G_1 \times_\gamma G_2}(u_1, u_2) = k_2[p_1 - r_1] - [p_1 - r_1]c_2 + p_2 k_2 - p_2 c_1 + c$. Hence $G_1 \times_\gamma G_2$ is a totally regular fuzzy graph. □

Theorem 6.3. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs such that $\sigma_1 \geq \mu_2$, $\sigma_2 \geq \mu_1$, $\mu_2 \leq \mu_1$ and $\sigma_1 \wedge \sigma_2$ is a constant function and let G_2^* be a regular graph. Then $G_1 \times_\gamma G_2$ is a totally regular fuzzy graph if and only if G_1 and G_2 are regular fuzzy graphs.*

Theorem 6.4. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs such that $\sigma_1 \geq \mu_2$, $\sigma_2 \geq \mu_1$, $\mu_2 \leq \mu_1$ and $\sigma_1 \wedge \sigma_2$ is a constant function and let G_2^* be a regular graph. Then $G_1 \times_\gamma G_2$ is a totally regular fuzzy graph if and only if G_1 and G_2 are totally regular fuzzy graphs.*

Theorem 6.5. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs such that $\sigma_1 \leq \mu_2$ and σ_1 is a constant function with $\sigma_1(u_1) = c_1$ for all $u \in V_1$ and let $G_1^*(/G_2^*)$ be a regular graph. Then $G_1 \times_\gamma G_2$ is a totally regular fuzzy graph if and only if G_1 is a regular fuzzy graph and $G_2^*(/G_1^*)$ is a regular graph.*

Proof. Let $\sigma_1(u_1) = c_1$ for all $u \in V_1$, where c_1 is a constant. Let G_1^* be r_1-regular graph. We have $\sigma_1 \leq \mu_2$. Hence $\sigma_2 \geq \mu_1$ and $\sigma_1 \leq \sigma_2$. Assume that $G_1 \times G_2$ is a totally regular fuzzy graph. Then for any two points (u_1, u_2) and (v_1, v_2) in $V_1 \times V_2$.

$$td_{G_1 \times G_2}(u_1, u_2) = td_{G_1 \times G_2}(v_1, v_2)$$

From Theorem 5.5,

$$c_1[(p_1 - r_1)d_{G_2}(u_2) + 1] + p_2 d_{G_1}(u_1) = c_1[(p_1 - r_1)d_{G_2}(v_2) + 1] + p_2 d_{G_1}(v_1) \tag{9}$$

Fix $u \in V_1$ and consider (u, u_2) and (u, v_2) in $V_1 \times V_2$ where $u_2, v_2 \in V_2$ are arbitrary. From (9),

$$c_1[(p_1 - r_1)d_{G_2}(u_2) + 1] + p_2 d_{G_1}(u) = c_1[(p_1 - r_1)d_{G_2}(v_2) + 1] + p_2 d_{G_1}(u)$$
$$\Rightarrow d_{G_2^*}(u_2) = d_{G_2^*}(v_2)$$

This is true for all $u_2, v_2 \in V_2$. Thus G_2^* is a regular graph. Fix $v \in V_2$ and consider (u_1, v) and (v_1, v) in $V_1 \times V_2$ where $u_1, v_1 \in V_1$ are arbitrary. From (9),

$$c_1[(p_1 - r_1)d_{G_2^*}(v) + 1] + p_2 d_{G_1}(u_1) = c_1[(p_1 - r_1)d_{G_2^*}(v) + 1] + p_2 d_{G_1}(v_1)$$
$$\Rightarrow d_{G_1}(u_1) = d_{G_1}(v_1)$$

This is true for all $u_1, v_1 \in V_1$. Thus G_1 is a regular fuzzy graph.

Conversely, Let G_1 be k_1-regular fuzzy graph and G_2^* be r_2-regular graph. From Theorem 5.5, $td_{G_1 \times_\gamma G_2}(u_1, u_2) = c_1[(p_1 - r_1)r_2 + 1] + p_2 k_1$. Hence $G_1 \times_\gamma G_2$ is a totally regular fuzzy graph. □

Theorem 6.6. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs such that $\sigma_1 \leq \mu_2$ and σ_1 is a constant function and let $G_1^*(/G_2^*)$ be a regular graph. Then $G_1 \times_\gamma G_2$ is a totally regular fuzzy graph if and only if G_1 is a totally regular fuzzy graph and $G_2^*(/G_1^*)$ is a regular graph.*

Theorem 6.7. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs such that $\sigma_2 \leq \mu_1$ and σ_2 is a constant function with $\sigma_2(u_2) = c_2$ for all $u \in V_2$ and let $G_2^*(/G_1^*)$ be a regular graph. Then $G_1 \times_\gamma G_2$ is a totally regular fuzzy graph if and only if G_2 is a regular fuzzy graph and $G_1^*(/G_2^*)$ is a regular graph.*

Proof. Let $\sigma_2(u_2) \wedge \sigma_2(v_2) = c_2$ for all $v \in V_2$, where c_2 is a constant. Let G_2^* be r_2-regular graph. We have $\sigma_2 \leq \mu_1$. Hence $\sigma_1 \geq \mu_2$ and $\sigma_2 \leq \sigma_1$. Assume that $G_1 \times G_2$ is a totally regular fuzzy graph. Then for any two points (u_1, u_2) and (v_1, v_2) in $V_1 \times V_2$.

$$td_{G_1 \times G_2}(u_1, u_2) = td_{G_1 \times G_2}(v_{1,2})$$

From Theorem 5.7,

$$p_1 d_{G_2}(u_2) + c_2[p_2 d_{G_1^*}(u_1) + 1] - r_2 d_{G_1^*}(u_1) = p_1 d_{G_2}(v_2) + c_2[p_2 d_{G_1^*}(v_1) + 1] - r_2 d_{G_1^*}(v_1) \tag{10}$$

Fix $u \in V_1$ and consider (u, u_2) and (u, v_2) in $V_1 \times V_2$ where $u_2, v_2 \in V_2$ are arbitrary. From (10),

$$p_1 d_{G_2}(u_2) + c_2[p_2 d_{G_1^*}(u) + 1] - r_2 d_{G_1^*}(u) = p_1 d_{G_2}(v_2) + c_2[p_2 d_{G_1^*}(u) + 1] - r_2 d_{G_1^*}(u)$$
$$\Rightarrow d_{G_2}(u_2) = d_{G_2}(v_2)$$

This is true for all $u_2, v_2 \in V_2$. Thus G_2 is a regular fuzzy graph. Fix $v \in V_2$ and consider (u_1, v) and (v_1, v) in $V_1 \times V_2$ where $u_1, v_1 \in V_1$ are arbitrary. From (10),

$$p_1 d_{G_2}(v) + c_2[p_2 d_{G_1^*}(u_1) + 1] - r_2 d_{G_1^*}(u_1) = p_1 d_{G_2}(v) + c_2[p_2 d_{G_1^*}(v_1) + 1] - r_2 d_{G_1^*}(v_1)$$
$$c_2[p_2 d_{G_1^*}(u_1) + 1] - r_2 d_{G_1^*}(u_1) = c_2[p_2 d_{G_1^*}(v_1) + 1] - r_2 d_{G_1^*}(v_1)$$
$$\Rightarrow c_2 p_2 d_{G_1^*}(u_1) + c_2 - r_2 d_{G_1^*}(u_1) = c_2 p_2 d_{G_1^*}(v_1) + c_2 - r_2 d_{G_1^*}(v_1)$$
$$\Rightarrow (c_2 p_2 - r_2) d_{G_1^*}(u_1) = (c_2 p_2 - r_2) d_{G_1^*}(v_1)$$
$$\Rightarrow d_{G_1^*}(u_1) = d_{G_1^*}(v_1)$$

This is true for all $u_1, v_1 \in V_1$. Thus G_1^* is a regular graph.

Conversely, Let G_2 be k_2-regular fuzzy graph and G_1^* be r_1-regular graph. From Theorem 5.7,

$$td_{G_1 \times_\gamma G_2}(u_1, u_2) = p_1 k_2 + c_2[p_2 r_1 + 1] - r_2 r_1$$

Hence $G_1 \times_\gamma G_2$ is a totally regular fuzzy graph. □

Theorem 6.8. *Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs such that $\sigma_2 \leq \mu_1$ and σ_2 is a constant function with $\sigma_2(u_2) = c_2$ for all $u \in V_2$ and let $G_2^*(/G_1^*)$ be a regular graph. Then $G_1 \times_\gamma G_2$ is a totally regular fuzzy graph if and only if G_2 is a totally regular fuzzy graph and $G_1^*(/G_2^*)$ is a regular graph.*

7. Conclusion

In this paper, we have obtained the total degree of a vertex in $G_1 \times_\beta G_2$ in terms of degree and total degree of vertices in G_1 and G_2 in some particular cases. The total degree of vertices in Gamma product in terms of the total degree of vertices in G_1 and G_2 under some conditions are obtained. It will be helpful especially when the graphs are very large and useful in studying various properties of Beta product and Gamma product of two fuzzy graphs. Also we have shown that the Beta product and Gamma product of two totally regular fuzzy graphs need not be a totally regular fuzzy graph. We have obtained necessary and sufficient condition for the Beta product and Gamma product of two fuzzy graphs to be totally regular in some particular cases.

References

[1] P.Bhattacharya, *Some Remarks on Fuzzy Graphs*, Pattern Recognition Letter, 6(1987), 297-302.

[2] John N.Modeson, Premch and S.Nair, *Fuzzy Graphs and Fuzzy Hypergraphs*, Physica-verlag, Heidelberg, (2000).

[3] L.A.Zadeh, *Fuzzy Sets*, Inform. Control., 8(1965), 338-53.

[4] J.N.Mordeson and C.S.Peng, *Operations on Fuzzy Graphs*, Inform. Sci., 79(1994), 159-170.

[5] A.Nagoorgani and M.BasheerAhamed, *Order and Size in Fuzzy Graph*, Bulletin of Pure and Applied Sciences, 22E(1)(2003), 145-148.

[6] A.Nagoorgani and K.Radha, *Some Sequences in Fuzzy Graphs*, Far East Journal of Applied Mathematics, 31(3)(2008), 321-335.

[7] A.Nagoorgani and K.Radha, *On Regular Fuzzy Graphs*, Journal of Physical Sciences, 12(2008), 33-40.

[8] A.Nagoorgani and K.Radha, *The Degree of a vertex in some fuzzy graphs*, International Journal of Algorithms, Computing and Mathematics, 3(2009).

[9] A.Rosenfeld, *Fuzzy Graphs*, In: L. A. Zadeh, K.S. Fu, M. Shimura, Eds., Fuzzy sets and Their Applications, Academic Press, (1975), 77-95.

[10] K.Radha and M.Vijaya, *The Total Degree of a vertex in some fuzzy graphs*, Jamal Academic Research Journal : An Interdisciplinary, special issue, (2014), 160-168.

[11] A.Nagoorgani and B.Fathima Kani, *Degree of a vertex in Alpha, Beta, Gamma Product of Fuzzy Graphs*, Jamal Academic Research Journal : An Interdisciplinary, special issue, (2014), 104-114.

[12] K.Radha and M.Vijaya, *Totally Regular Property of Cartesian Product of two fuzzy graphs*, Jamal Academic Research Journal : An Interdisciplinary, special issue, (2015), 647-652.

[13] K.Radha and M.Vijaya, *Totally Regular Property of Composition of two fuzzy graphs*", International journal of Pure and Applied Mathematical Sciences, 8(1)(2015), 87-100.

[14] K.Radha and M.Vijaya, *Totally Regular Property of the join of two fuzzy graphs*, International journal of Fuzzy Mathematical Archieve, 8(1)(2015), 9-17.

[15] K.Radha and M.Vijaya, *Totally Regular Property of Alpha Product of two fuzzy graphs*, International Journal of Multidisciplinary Research and Development, 3(4)(2016), 125-130.

[16] K.Radha and M.Vijaya, *Totally Regular Property of Conjunction of two fuzzy graphs*, Jamal Academic Research journal : An Interdisciplinary, special issue, (2016), 157-163.

[17] K.Radha and M.Vijaya, *Regular and totally regular property of disjunction of two fuzzy graphs*, International Journal of Multidisciplinary Research and Development, 4(4)(2017), 63-71.