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# Bivariate Maintenance Models for Multistate Degenerative Systems Under Quasi Renewal Process

**Research Article**

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**Abstract:** In this paper, optimal bivariate replacement models for a multistate degenerative system under quasi renewal process are derived. Numerical examples are included to strengthen the theoretical results.

**MSC:** 60K10, 90B25.

**Keywords:** Extreme Shocks, Degenerative System, Replacement Policy.



## 1. Introduction

The maintenance problem of a multistate degenerative system with $k$-working states and $l$-failure states under quasi renewal process is studied. The long-run average cost for a multistate stochastic degenerative system under the following bivariate replacement policies.

(T, N) - policy,
(U, N) - policy,
$(T^+,\ N)$ - policy, and
$(U^-,\ N)$ - policy,

where $T$ is the working age of the system, $N$ is the number of failures of the system, $U$ is the cumulative repair time of the system, $T^+$ is the system replaced at the first failure point after the cumulative operating time exceeds $T$ and $U^-$ is the failure point just before the total repair time exceed $U$, under quasi renewal process are derived. Existence of optimality under the aforesaid bivariate replacement polices are deduced. Numerical examples are given to illustrate the results developed in this paper.

The rest of the paper is structured as follows: In Section 2, we give a general description of the model. We also present the monotone process model of a one component multistate stochastic degenerative system and the relevant results regarding their probability structure. In Section 3, we derive explicit expressions for the long-run average cost per unit time for this model under the bivariate replacement policies $(T, N)$, $(U, N)$, $(T^+, N)$ and $(U^-, N)$. Numerical examples are given in Section 4 to illustrate the results developed in this paper. Finally, conclusion is given Section 5.

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## 2. Preliminaries

In this section, we first give the definitions required for further discussion. If the sequence of non-negative random variable $\{X_1, X_2, \ldots\}$ is independent and $X_i = \alpha X_{i-1}$ for $i > 2$ where $\alpha > 0$ is a constant, then the counting process $N(t), t \geq 0$ is said to be a quasi renewal process with parameter $\alpha$ and the first inter arrival time $X_1$. When $\alpha = 1$ this process becomes the classical renewal process. The quasi renewal process can be used to model a maintenance process when $\alpha \leq 1$ and a reliability growth process in product developing and burn-in for $\alpha > 1$. Assuming that the probability density function, cumulative distribution function, survival function and failure rates of random variable $X_1$ are $f_1(x)$, $F_1(x)$, $s_1(x)$ and $r_1(x)$, respectively, then the probability density function, cumulative distribution function, survival function, failure rate, mean and variance of $X_n$, for $n = 1, 2, \ldots$ are

$$f_n(x) = \frac{1}{\alpha^{n-1}} f_1\left(\frac{1}{\alpha^{n-1}} x\right) \qquad s_n(x) = s_1\left(\frac{x}{\alpha^{n-1}}\right)$$

$$E(X_n) = \alpha^{n-1} E(X_1) \qquad F_n(x) = F_1\left(\frac{x}{\alpha^{n-1}}\right)$$

$$r_n(x) = \left(\frac{r_1}{\alpha^{n-1}}\right)\left(\frac{x}{\alpha^{n-1}}\right) \qquad Var(X_n) = \alpha^{2n-2} Var(X_1)$$

Because of the non-negativity of $X_1$ and the fact that $X_1$ is not identically 0, we conclude that $E(X_1) = \mu_1 \neq 0$.

## 3. Description of the Model

We consider the model of a one-component multistate system. We also evaluate the conditional probabilities of the operating times and failure times given the state of the system. Consider a one-component multistate system with $(k + l)$ states $k$-working states and $l$-failure states. The system state at time $t$ is given by

$$S(t) = \begin{cases} i & if\ thesystem\ is\ in\ the\ i-th\ working\ state\ at\ time\ t \\ & (i = 1, 2, \ldots, k) \\ k+j & if\ the\ system\ is\ in\ the\ j-thfailure\ state\ at\ time\ t \\ & (j = 1, 2, \ldots, l) \end{cases}$$

The set of working states is $\Omega_1 = \{1, 2, \ldots, k\}$; the set of failure states is $\Omega_2 = \{k + 1, k + 2, \ldots, k + l\}$ and the state space is $\Omega = \Omega_1 \cup \Omega_2$. Initially, assume that a new system in working state 1 is installed. Whenever the system fails, it will be repaired. Let $t_n$ be the completion time of the $n$-th repair, $n = 0, 1, \ldots$ with $t_0 = 0$ and let $s_n$ be the time of occurrence of the $n$-th failure, $n = 1, 2, \ldots$ . Then $t_0 < s_1 < t_1 < \cdots < s_n < t_n < \cdots$. Consider a monotone process model for a multistate one-component system described in this section and make the assumptions.

**A1** At the beginning, a new system is installed. The system has $(k+l)$ possible states, where the states $1, 2, \ldots, k$ denote, respectively, the first-type working state, the second-type working state $, \ldots,$ $k$-th-type working state and the states $(k + 1)$, $(k + 2), \ldots, (k + l)$ denote, respectively, the first-type failure state, the second-type failure state, $\ldots$ and the $l$-th type failure state of the system. The occurrences of these types of failures are stochastic and mutually exclusive.

**A2** Whenever the system fails in any of the working states, it will be repaired. The system will be replaced by an identical new one some times later.

**A3** Let $X_n$ be the survival time of the system after $(n - 1)$-st repair. Then $\{X_n,\ n = 1, 2, \ldots\}$ forms a non-decreasing quasi renewal process with parameter $0 < \alpha < 1$ and $E(X_1) = \lambda > 0$.

**A4** Let $Y_n$ be the repair time after $n$-th failure. Then $\{Y_n,\ n = 1, 2, \ldots\}$ forms a non-increasing quasi renewal process with parameter $\beta, \beta > 1$ and $E(Y_1) = \mu \geq 0$. Here $\mu = 0$ means that repair time is negligible.

**A5** If the system in working state $i$ is operating, then let the reward rate be $r_i$. If the system in failure state $(k+i)$ is under repair, the repair cost is $C_i$. The replacement cost comprises two parts: one part is the basic replacement cost $B$ and the other proportional to the replacement time $Z$ at rate $c_p$. In other words, the replacement cost is given by $B + c_p Z$.

**A6** Assume that $F_n(t)$ is the cumulative distribution of $L_n = \sum\limits_{i=1}^{n} X_i$ and $G_n(t)$ be the cumulative distribution of $M_n = \sum\limits_{i=1}^{n} Y_i$.

**A7** The random variables $X_n,\ n = 1, 2, \ldots,\ \ Y_n,\ n = 1, 2, \ldots,$ and the replacement time $Z$ are independent.

**A8** During the repair time, the system is closed so that any further repair during repair time is ineffective.

We now describe the probability structure of the model. Assume that the transition probability from working state $i,\ i = 1, 2, \ldots, k$, to failure state $k+j,\ \ j = 1, 2, \ldots, l$, is

$$P(S(s_{n+1}) = k + j | S(t_n) = i) = q_j,$$

with $\sum\limits_{j=1}^{l} q_j = 1$. Moreover, the transition probability from failure state $k+j,\ j = 1, 2, \ldots, l$, to working state $i,\ i = 1, 2, \ldots, k$ is given by

$$P(S(t_n) = i | S(s_n) = k + j) = p_i,$$

with $\sum\limits_{i=1}^{k} p_i = 1$. Assume that there exist a life-time distribution $U(t)$ and $a_i > 0,\ i = 1, \ldots, k$ such that

$$P(X_1 \leq t) = U(t)$$

and

$$P(X_2 \leq t | S(t_1) = i) = U(a_i t),\quad i = 1, 2, \ldots, k$$

where $1 \leq a_1 \leq a_2 \leq \cdots \leq a_k$. In general, for $i_j \in \{1, 2, \ldots, k\}$,

$$P\left(X_n \leq t | S(t_1) = i_1, \ldots, S(t_{n-1}) = i_{n-1}\right) = U(a_{i_1} \cdots a_{i_{n-1}} t).$$

Similarly, assume that there exist a life-time distribution $V(t)$ and $b_i > 0,\ i = 1, 2, \ldots, l$ such that

$$P(Y_1 \leq t | S(s_1) = k + i) = V(b_i t),$$

where $1 \geq b_1 \geq b_2 \geq \cdots \geq b_l > 0$ and in general, for $i_j \in \{1, 2, \ldots, l\}$,

$$P(Y_n \leq t | S(s_1) = k + i_1, \ldots, S(s_n) = k + i_n) = V(b_{i_1} \cdots b_{i_n} t)$$

A life distribution $F(\cdot)$ is said to be new better than used in expectation (NBUE), if

$$\int_0^\infty \overline{F}(t + x)dx \leq \overline{F}(t) \int_0^\infty \overline{F}(x)dx, \qquad \forall\, t \geq 0.$$

To say that the life distribution of an item is $NBUE$ is equivalent to saying that the mean life length of a new item is greater than the mean residual life length of a non-failed item of age $t > 0$. At every failure point, a decision is taken whether the failed system can be sent for repair. If the cumulative repair time after this repair is expected to exceed a threshold value $U$, the repair need not be initiated at that failure time. Such a fictitious repair time is called a virtual repair time. For two working states $1 \leq i_1 < i_2 \leq k$, we have

$$(X_2|S(t_1) = i_2) \leq_{st} (X_2|S(t_1) = i_1).$$

Therefore, the working state $i_1$ is better than the working state $i_2$, in the sense that, the system in state $i_1$ has a stochastically large operating time than it does in state $i_2$. Consequently, the $k$ working states are arranged in decreasing order, such that state 1 is the best working state and state $k$ is the worst working state. Similarly, for two failure states $k + i_1$ and $k + i_2$ such that $k + 1 \leq k + i_1 < k + i_2 \leq k + l$, we have

$$(Y_1|S(s_1) = k + i_1) \leq_{st} (Y_1|S(s_1) = k + i_2).$$

Therefore, the failure state $k + i_1$ is better than the failure state $k + i_2$ in the sense that the system in state $k + i_1$ has a stochastically smaller repair time than it does in state $k + i_2$. Thus, the $l$ failure states are also arranged in decreasing order, such that the state $k + 1$ is the best failure state and the state $k + l$ is the worst failure state.

# 4. Bivariate Replacement Policies

## 4.1. The (T, N) Policy

In this section, we introduce and study a bivariate replacement policy $(T, N)$ for the multistate stochastic degenerative system, under which system is replaced at the working age $T$ or at the time of $N$-th failure, whichever occurs first. The problem is to choose an optimal replacement policy $(T, N)^*$ such that the long-run average cost per unit time is minimized. The working age $T$ of the system at time $t$ is the cumulative life-time given by

$$T = \begin{cases} t - M_n, & 3cmL_n + M_n \leq t < L_{n+1} + M_n \\ L_{n+1}, & 3cmL_{n+1} + M_n \leq t < L_{n+1} + M_{n+1} \end{cases}$$

where $L_n = \sum\limits_{i=1}^{n} X_i$ and $M_n = \sum\limits_{i=1}^{n} Y_i$ and $L_0 = M_0 = 0$. Following Lam (2005), the distribution of the survival time $X_n$ in assumption **A3** and the distribution of the repair time $Y_n$ in assumption **A4** are given by

$$P(X_n \leq t) = \sum_{\sum\limits_{i=1}^{k} j_i = n-1} \frac{(n-1)!}{j_1! \cdots j_k!}\ p_1^{j_1} \cdots p_k^{j_k}\ U_1(a_1^{j_1} \cdots a_k^{j_k} t),$$

where $j_1, j_2, \ldots, j_k \in \mathrm{Z}^+$ and

$$P(Y_n \leq t) = \sum_{\sum\limits_{i=1}^{l} j_i = n} \frac{n!}{j_1! \cdots j_l!}\ q_1^{j_1} \cdots q_l^{j_l}\ V_1(b_1^{j_1} \cdots b_l^{j_l} t), \tag{1}$$

where $j_1, j_2, \ldots, j_l \in Z^+$. Further, if $E(X_1) = \lambda$, then the mean survival time is

$$E(X_n) = \lambda \alpha^{n-1}, \tag{2}$$

for $n > 1$, where

$$\alpha = \left( \sum_{i=1}^{k} \frac{p_i}{a_i} \right) \tag{3}$$

and if $E(Y_1) = \mu$, then the mean repair time is

$$E(Y_n) = \beta^n \ \mu \tag{4}$$

for $n > 1$, where

$$\beta = \left( \sum_{j=1}^{l} \frac{q_j}{b_j} \right). \tag{5}$$

## 4.2. The Length of a Cycle and its Mean

The length of a cycle under the bivariate replacement policy $(T, N)$ is

$$W = \left( \sum_{i=1}^{N} X_i + \sum_{i=1}^{N-1} Y_i \right) \chi_{(L_N \leq T)} + \left( T + \sum_{i=1}^{\eta} Y_i \right) \chi_{(L_N > T)} + Z,$$

where $\eta = 0, 1, 2, \ldots, N-1$ is the number of failures before the working age of the system exceeds $T$ and

$$\chi_{(A)} = \begin{cases} 1 & if\ the\ event\ \ A\ occurs, \\ 0 & if\ the\ event\ A\ does\ not\ occur \end{cases}$$

denotes the indicator function and $E[\chi_{(A)}] = P(A)$. From Leung (2006), we have

$$\begin{aligned} E\left[\chi_{(L_i \leq T < L_N)}\right] &= P(L_i \leq T < L_N) \\ &= P(L_i \leq T) - P(L_N \leq T) \\ &= F_i(T) - F_N(T). \end{aligned}$$

**Lemma 4.1.** *The mean length of a cycle under the policy $(T, N)$ is*

$$E(W) = \int_0^T \overline{F}_N(u) du + \sum_{i=1}^{N-1} \beta^{i-1} \ \mu \ F_i(T) + \tau. \tag{6}$$

*Proof.* Consider

$$\begin{aligned} E(W) &= E\left[ \left( \sum_{i=1}^{N} X_i + \sum_{i=1}^{N-1} Y_i \right) \ \chi_{(L_N \leq T)} \right] + E\left[ \left( T + \sum_{i=1}^{\eta} Y_i \right) \ \chi_{(L_N > T)} \right] + E(Z) \\ &= E\left\{ E\left[ \left( \sum_{i=1}^{N} X_i + \sum_{i=1}^{N-1} Y_i \right) \chi_{(L_N \leq \ T)} | L_N \right] \right\} + E\left[T \chi_{(L_N > T)}\right] + E\left[ \left( \sum_{i=1}^{\eta} Y_i \right) \ \chi_{(L_N > T)} \right] + E(Z) \\ &= \int_0^T u dF_N(u) + \int_0^T \sum_{i=1}^{N-1} E(Y_i) \ dF_N(u) + T\overline{F}_N(T) + \sum_{i=1}^{N-1} \beta^{i-1} \ \mu E\left[\chi_{(L_i \leq T < L_N)}\right] + \tau \\ &= \quad \int_0^T u \, dF_N(u) + \sum_{i=1}^{N-1} \beta^{i-1} \ \mu \ F_N(T) + T\overline{F}_N(T) + \sum_{i=1}^{N-1} \beta^{i-1} \ \mu P\left(L_i \leq T < L_N\right) + \tau \\ &= T\overline{F}_N(T) + \int_0^T u \, dF_N(u) + \sum_{i=1}^{N-1} \beta^{i-1} \ \mu \left[F_i(T) - F_N(T)\right] + \sum_{i=1}^{N-1} \beta^{i-1} \ \mu \ F_N(T) + \tau \\ &= \int_0^T \overline{F}_N(u) du + \sum_{i=1}^{N-1} \beta^{i-1} \ \mu F_i(T) + \tau, \end{aligned}$$

as desired and this completes the proof. □

## 4.3. The Long-run Average Cost under (T, N) Policy

Let $T_1$ be the first replacement time and let $T_n$ $(n \geq 2)$ be the time between $(n-1)$-st replacement and $n$-th replacement. Then the sequence $T_n,\ n = 1, 2, \ldots,$ forms a renewal process. The inter arrival time between two consecutive replacements is a renewal cycle. By the renewal reward theorem, the long-run average cost per unit time under the multistate bivariate replacement policy $(T, N)$ is given by

$$\mathcal{C}(T,N) = \frac{\textit{the expected costincurred in a cycle}}{\textit{the expected length of acycle}}$$
$$= \frac{\left[\begin{array}{l} E\left\{\left(C\sum_{i=1}^{N-1} Y_i - R\sum_{i=1}^{N} X_i\right)\chi_{(L_N \leq T)}\right\} + B \\ +E\left\{\left(C\sum_{i=1}^{\eta} Y_i - R\,T\right)\chi_{(L_N > T)}\right\} + c_p E(Z) \end{array}\right]}{E(W)}.$$

Using Lemma 4.1 and simplifying, we have the following result.

**Theorem 4.2.** *For the model described in section 3, under the assumptions **A1** to **A8**, the long-run average cost per unit time under the bivariate replacement policy $(T, N)$ for a multistate stochastic degenerative system under quasi renewal process is given by*

$$\mathcal{C}(T,N) = \frac{C\sum\limits_{i=1}^{N-1}\beta^{i-1}\,\mu\,F_i(T) - R\int_0^T \overline{F}_N(u)du + c_p\tau + B}{\int_0^T \overline{F}_N(u)du + \sum\limits_{i=1}^{N-1}\beta^{i-1}\,\mu\,F_i(T) + \tau}, \tag{7}$$

**Deductions:** Here $\mathcal{C}(T, N)$ is a bivariate function. Obviously, when $N$ is fixed, $\mathcal{C}(T, N)$ is a function of $T$. For fixed $N = m$, it can be written as

$$\mathcal{C}(T,N) = C_m(T), \quad m = 1, 2, \ldots\ .$$

Thus, for a fixed $m$, we can find $T_m^*$ by analytical or numerical methods such that $C_m(T_m^*)$ is minimized. That is, when $N = 1, 2, \ldots, m, \ldots,$ we can find $T_1^*, T_2^*, T_3^*, \ldots, T_m^*, \ldots,$ respectively, such that the corresponding $C_1(T_1^*), C_2(T_2^*), \ldots,$ $C_m(T_m^*), \ldots$ are minimized. Because the total lifetime of a multistate degenerative system is limited, the minimum of the long-run average cost per unit time exists. So we can determine the minimum of the long-run average cost per unit time based on $C_1(T_1^*), C_2(T_2^*), \ldots,\ C_m(T_m^*), \ldots$. For example, if the minimum is denoted by $C_n(T_n^*)$, we obtain the bivariate optimal replacement policy $(T, N)^*$ such that

$$\mathcal{C}((T,N)^*) = \min_n C_n(T_n^*).$$

**The (U, N) Policy:** In this section, we introduce and study a bivariate replacement policy $(U, N)$ for the multistate stochastic degenerative system, under which the system will be replaced at the time of $N$-th failure or the total repair time exceeds $U$, whichever occurs earlier. The problem is to choose an optimal replacement policy $(U, N)^*$ such that the long-run average cost per unit time is minimized.

**The length of a Cycle and its Mean:** The length of a cycle $W$ under the bivariate replacement policy $(U, N)$ is

$$W = \left(\sum_{i=1}^{N} X_i + \sum_{i=1}^{N-1} Y_i\right)\chi_{(M_N \leq U)} + \left(\sum_{i=1}^{\eta} X_i + U\right)\chi_{(M_N > U)} + Z,$$

where $\eta = 0, 1, 2, \ldots, N-1$ is the number of failures before the total repair time exceeds $U$ and $\chi_{(\cdot)}$ denotes the indicator function.

**Lemma 4.3.** *The mean length of the cycle under the policy $(U, N)$ is*

$$E(W) = \sum_{i=1}^{N-1} \alpha^{i-1}\, \lambda\, G_{i-1}(U) + \alpha^{N-1}\, \lambda G_N(U) + \int_0^U \overline{G}_N(u)du + \tau. \tag{8}$$

*Proof.* Consider

$$\begin{aligned}
E(W) &= E\left[\left(\sum_{i=1}^{N} X_i + \sum_{i=1}^{N-1} Y_i\right)\ \chi_{(M_N \le U)}\right] + \left[\left(\sum_{i=1}^{\eta} X_i + U\right)\ \chi_{(M_N > U)}\right] + E(Z) \\
&= E\left\{E\left[\left(\sum_{i=1}^{N} X_i + \sum_{i=1}^{N-1} Y_i\right)\ \chi_{(M_N \le\, U)} | M_N = u\right]\right\} + E\left[\sum_{i=1}^{\eta} X_i \chi_{(M_N > U)}\right] + E\left[U\ \chi_{(M_N > U)}\right] + E(Z) \\
&= E\left(\sum_{i=1}^{N} X_i\right) E\left(\chi_{(M_N \le U)}\right) + \int_0^U u dG_N(u) + \sum_{i=1}^{N-1} E\left(X_i\right) E[\chi_{(M_{i-1} \le U < M_N)}] + UE\left[\ \chi_{(M_N > U)}\right] + \tau \\
&= \sum_{i=1}^{N} \alpha^{i-1}\, \lambda + \int_0^U u\, dG_N(u) + \sum_{i=1}^{N-1} E\left(X_i\right) P(M_{i-1} \le U < M_N) + U\overline{G}_N(U) + \tau \\
&= \sum_{i=1}^{N} \alpha^{i-1}\, \lambda G_N(U) + \int_0^U u\, dG_N(u) + \sum_{i=1}^{N-1} \alpha^{i-1}\, \lambda\, [G_{i-1}(U) - G_N(U)] + U\overline{G}_N(U) + \tau,
\end{aligned}$$

which on simplification yields (8). □

**The Long-run Average Cost under (U, N) Policy:** Let $U_1$ be the first replacement time and let $U_n$ $(n \ge 2)$ be the time between the $(n-1)$-st replacement and the $n$-th replacement. Then the sequence $U_n$, $n = 1, 2, \ldots$, forms a renewal process. The interarrival time between two consecutive replacement is a renewal cycle. By the renewal reward theorem, the long-run average cost per unit time under the bivariate replacement policy-$(U, N)$ for a multistate stochastic degenerative system is given by

$$\begin{aligned}
\mathcal{C}(U, N) &= \frac{\textit{the expected costincurred in a cycle}}{\textit{the expected length of acycle}} \\
&= \frac{\begin{bmatrix} E\left\{\left(c \sum_{i=1}^{N-1} Y_i - r \sum_{i=1}^{N} X_i\right) \chi_{(M_N \le U)}\right\} + R \\ +E\left\{\left(cU - r \sum_{i=1}^{\eta} X_i\right) \chi_{(M_N > U)}\right\} + c_p E(Z) \end{bmatrix}}{E(W)}.
\end{aligned}$$

Using Lemma 4.2 and simplifying, we have the following result.

**Theorem 4.4.** *For the model described in section 3, under the assumptions **A1** to **A8**, the long-run average cost per unit time under the bivariate replacement policy $(U, N)$ for a multistate stochastic degenerative system under quasi renewal process is given by*

$$\mathcal{C}(U, N)\ = \frac{C \int_0^U \overline{G}_N(u)du - R \sum_{i=1}^{N-1} \alpha^{i-1}\ \lambda G_{i-1}(U) + \alpha^{N-1}\ \lambda G_N(U) + c_p \tau + B}{\sum_{i=1}^{N-1} \alpha^{i-1}\ \lambda G_{i-1}(U) + \alpha^{N-1}\ \lambda G_N(U) + \int_0^U \overline{G}_N(u)du + \tau}. \tag{9}$$

**Deductions:** Here $\mathcal{C}(U, N)$ is a bivariate function. Obviously, when $N$ is fixed, $\mathcal{C}(U, N)$ is a function of $U$. For fixed $N = m$, it can be written as

$$\mathcal{C}(U, N) = C_m(U), \quad m = 1, 2, 3, \ldots .$$

Thus, for a fixed $m$, we can find $U_m^*$ by analytical or numerical methods such that $C_m(U_m^*)$ is minimized. That is, when $N = 1, 2, \ldots, m, \ldots$, we can find $U_1^*, U_2^*, U_3^*, \ldots, U_m^*, \ldots$, respectively, such that the corresponding $C_1(U_1^*), C_2(U_2^*), \ldots, C_m(U_m^*), \ldots$ are minimized.

It is logical to assume that the total repair time at any stage can never exceed the working time of the system, because in this case the total repair cost will exceed the total reward earned. Further the total life time of a multistate stochastic degenerative system is limited. It follows that the total repair time of a multistate stochastic degenerative system is also limited. Therefore the long-run average cost per unit time exist. So we can determine the minimum of the long-run average cost per unit time based on $C_1(U_1^*), C_2(U_2^*), \ldots, C_m(U_m^*), \ldots$. For example, if the minimum is denoted by $C_m(U_m^*)$, we obtain the bivariate optimal replacement policy $(U, N)^*$ such that

$$\mathcal{C}((U, N)^*) = \min_m C_m(U_m^*).$$

**The $(T^+, N)$ Policy:** It is a policy for which the multistate stochastic degenerative system, under which system will be replaced at the first failure point after the cumulative operating time exceeds $T$ or at the occurrence of the $N$-th failure, whichever occurs earlier. The technique of replacing at the first failure point after the cumulative operating time exceeds a predetermined value is used in Muth (1977).

**The Length of a Cycle and its Mean:** The length of the cycle under the bivariate replacement policy $(T^+, N)$ is

$$W = \left(\sum_{i=1}^{N} X_i + \sum_{i=1}^{N-1} Y_i\right) \chi_{(L_N \le T)} + \left(\sum_{i=1}^{\eta} X_i + \sum_{i=1}^{\eta} Y_{i-1}\right) \chi_{(L_N > T)} + Z,$$

where $\eta = 1, 2, \ldots, N-1$ is the number of failures before the total operating time exceeds $T$. The random variable $\eta$ has a geometric distribution given by

$$\begin{aligned} P(\eta = j) &= P(X_1 \le T, X_2 \le T, \ldots, X_{\eta-1} \le T, X_\eta > T); \;\; j = 1, 2, \ldots \\ &= F^{j-1}(T)\overline{F}(T). \end{aligned}$$

Since $\eta$ is a random variable,

$$\begin{aligned} E(\eta - 1) &= \sum_{j=1}^{\infty} (j-1) P(\eta = j) \\ &= \overline{F}(T) \sum_{j=1}^{\infty} (j-1) F^{j-1}(T) \\ &= \frac{F(T)}{\overline{F}(T)}. \end{aligned}$$

**Lemma 4.5.** *The mean length of the cycle under the policy $(T^+, N)$ is*

$$E(W) = \int_0^T u\, dF_N(u) + \frac{F(T)}{\overline{F}(T)} \sum_{i=1}^{N-1} \alpha^{i-1}\, \lambda \left[F_i(T) - F_N(T)\right] + \sum_{i=1}^{N-1} \beta^{i-1}\, \mu \left[(1-b)F_N(T) + bF_i(T)\right] + \tau. \tag{10}$$

*Proof.* Consider

$$\begin{aligned} E(W) &= E\left[\left(\sum_{i=1}^{N} X_i + \sum_{i=1}^{N-1} Y_i\right) \chi_{(L_N \le T)}\right] + E\left[\left(\sum_{i=1}^{\eta} X_i + \sum_{i=1}^{\eta} Y_{i-1}\right) \chi_{(L_N > T)}\right] + E(Z) \\ &= E\left\{E\left[\left(\sum_{i=1}^{N} X_i + \sum_{i=1}^{N-1} Y_i\right) \chi_{(L_N \le\, T)} | L_N\right]\right\} + E\left[\sum_{i=1}^{\eta} X_i \chi_{(L_N > T)}\right] + E\left[\left(\sum_{i=1}^{\eta} Y_{i-1}\right) \chi_{(L_N > T)}\right] + E(Z) \\ &= \int_0^T u dF_N(u) + \int_0^T \sum_{i=1}^{N-1} E(Y_i)\, dF_N(u) + \sum_{i=1}^{N-1} E(X_i | \eta = N-1)\, P(L_i \le T < L_N) + \sum_{i=1}^{N-1} E(Y_{i-1}) E\left[\chi\left(L_i \le T < L_N\right)\right] + \tau \\ &= \int_0^T u\, dF_N(u) + \sum_{i=1}^{N-1} \beta^{i-1}\, \mu F_N(T) + \frac{F(T)}{\overline{F}(T)} \sum_{i=1}^{N-1} \alpha^{i-1}\, \lambda \left[F_i(T) - F_N(T)\right] + \sum_{i=1}^{N-1} \beta^{i-2}\, \mu \left[F_i(T) - F_N(T)\right] + \tau, \end{aligned}$$

which on simplification yields (10) and the proof is complete. □

**The Long-run Average Cost under $(T^+,N)$ Policy:** By the renewal reward theorem, the long-run average cost per unit time under the bivariate replacement policy-$(T^+,N)$ for a multistate stochastic degenerative system is given by

$$\mathcal{C}(T^+,N)=\frac{\textit{the expected costincurred in a cycle}}{\textit{the expected length of acycle}}$$
$$=\frac{\left[\begin{array}{l} E\left\{\left(C\sum_{i=1}^{N-1}Y_i-r\sum_{i=1}^{N}X_i\right)\chi_{(L_N\le T)}\right\}+B\\ +E\left\{\left(C\sum_{i=1}^{\eta}Y_{i-1}-R\sum_{i=1}^{\eta}X_i\right)\chi_{(L_N>T)}\right\}+c_pE(Z)\end{array}\right]}{E(W)},$$

Using Lemma 4.3 and simplifying, we have the following result.

**Theorem 4.6.** *For the model described in section 3, under the assumptions **A1** to **A8**, the long-run average cost per unit time using the bivariate replacement policy $(T^+,N)$ for a multistate stochastic degenerative system under quasi renewal process is given by*

$$\mathcal{C}(T^+,N)=\frac{\left[\begin{array}{c} C\sum\limits_{i=1}^{N-1}\alpha^{i-1}\,\lambda\left[(1-b)F_N(T)+bF_i(T)\right]-R\int_0^T u\,dF_N(u)\\ +\frac{F(T)}{\overline{F}(T)}\sum\limits_{i=1}^{N-1}\alpha^{i-1}\,\lambda\left[F_i(T)-F_N(T)\right]+c_p\tau+B\end{array}\right]}{\left[\begin{array}{c}\int_0^T u\,dF_N(u)+\frac{F(T)}{\overline{F}(T)}\sum\limits_{i=1}^{N-1}\alpha^{i-1}\lambda\left[F_i(T)-F_N(T)\right]\\ +\sum\limits_{i=1}^{N-1}\alpha^{i-1}\,\lambda\left[(1-b)F_N(T)+bF_i(T)\right]+\tau\end{array}\right]}, \tag{11}$$

The bivariate optimal replacement policy $(T^+,N)^*$ is obtained in a manner similar to that used for finding optimal policy $(T,N)^*$.

**The $(U^-,N)$ Policy:** Under the policy $(U^-,N)$, we will replace the multistate stochastic degenerative system at the failure point just before the total repair time exceed $U$ or at the occurrence of the $N$-th failure, whichever occurs earlier. In the policy $(U,N)$, an optimal policy may be such that we will have to replace the system, in the middle of a repair, after (say) $s$ units of repair time. Then the question naturally arises is whether it would not have been more profitable to replace the system at the failure point itself, as the repair cost could be saved. Since no additional costs are involved for replacing at failure in our policies, it is profitable not to replace the system in the middle of an operating interval.

**The Length of a Cycle and its Mean:** The length of a cycle $W$ under the bivariate replacement policy $(U^-,N)$ is

$$W=\left(\sum_{i=1}^{N}X_i+\sum_{i=1}^{N-1}Y_i\right)\chi_{(M_N\le U)}+\left(\sum_{i=1}^{\eta}X_i+\sum_{i=0}^{\nu}Y_i\right)\chi_{(M_N>U)}+Z,$$

where $\eta=1,2,\ldots,N-1$ is the number of failures before the total repair time exceeds $U$ and $\nu=0,1,\ldots,N-1$ is the number of repairs before the total repair time is expected to exceed $U$. If $M_i\le U<M_{i+1}$ for $i=1,2,\ldots,N-1$, then $U-M_i$ will be the virtual repair time.

**Lemma 4.7.** *The mean length of the cycle under policy $(U^-,N)$ is*

$$E(W)=\sum_{i=1}^{N-1}\alpha^{i-1}\,\lambda G_{i-1}(U)+\frac{\lambda}{a^{N-1}}G_N(U)+\int_0^U u dG_N(u)+\frac{G(U)}{\overline{G}(U)}\sum_{i=0}^{N-1}\beta^{i-1}\,\mu\left[G_i(U)-G_N(U)\right]+\tau. \tag{12}$$

*Proof.* Consider

$$\begin{aligned}E(W)&=E\left[\left(\sum_{i=1}^{N}X_i+\sum_{i=1}^{N-1}Y_i\right)\chi_{(M_N\le U)}\right]+\left[\left(\sum_{i=1}^{\eta}X_i+\sum_{i=0}^{\nu}Y_i\right)\chi_{(M_N>U)}\right]+E(Z)\\ &=\sum_{i=1}^{N}\alpha^{i-1}\,\lambda G_N(U)+\int_0^U u dG_N(u)+E\left[\sum_{i=1}^{\eta}X_i\chi_{(M_N>U)}\right]+E\left[\sum_{i=0}^{\nu}Y_i\,\chi_{(M_N>U)}\right]+E(Z)\\ &=\sum_{i=1}^{N}\alpha^{i-1}\,\lambda G_N(U)+\int_0^U u dG_N(u)+\sum_{i=1}^{N-1}E(X_i)P\left[M_{i-1}\le U<M_N\right]+\sum_{i=0}^{N-1}E(Y_i|\nu)P(M_i\le U<M_n)+E(Z)\end{aligned}$$

$$= \sum_{i=1}^{N} \alpha^{i-1}\, \lambda G_N(U) + \int_0^U u\, dG_N(u) + \sum_{i=1}^{N-1} \alpha^{i-1}\, \lambda \left[G_{i-1}(U) - G_N(U)\right] + \sum_{i=0}^{N-1} E(Y_i)E(\nu - 1)\left[G_i(U) - G_N(U)\right] + \tau$$
$$= \sum_{i=1}^{N} \alpha^{i-1}\, \lambda G_N(U) + \int_0^U u\, dG_N(u) + \sum_{i=1}^{N-1} \alpha^{i-1}\, \lambda \left[G_{i-1}(U) - G_N(U)\right] + \frac{G(U)}{\overline{G}(U)} \sum_{i=0}^{N-1} \beta^{i-1}\, \mu \left[G_i(U) - G_N(U)\right] + \tau,$$

which on simplification yields (12) . □

**The Long-run Average Cost under the Policy** $(U^-, N)$**:** By the renewal reward theorem, the long-run average cost per unit time under the bivariate replacement policy-$(U^-, N)$ for a multistate stochastic degenerative system is given by

$$\mathcal{C}(U^-, N) = \frac{\textit{the expected costincurred in a cycle}}{\textit{the expected length of acycle}}$$
$$= \frac{\begin{bmatrix} E\left\{\left(C \sum\limits_{i=1}^{N-1} Y_i - R \sum\limits_{i=1}^{N} X_i\right) \chi_{(M_N \le U)}\right\} + B \\ +E\left\{\left(C \sum\limits_{i=0}^{\nu} Y_i - R \sum\limits_{i=1}^{\eta} X_i\right) \chi_{(M_N > U)}\right\} + c_p E(Z) \end{bmatrix}}{E(W)}.$$

Using Lemma 4.4. and simplifying, we have the following result.

**Theorem 4.8.** *For the model described in section 3, under the assumptions* ***A1*** *to* ***A8****, the long-run average cost per unit time using the bivariate replacement policy* $(U^-, N)$ *for a multistate stochastic degenerative system under quasi renewal process is given by*

$$\mathcal{C}(U^-, N) = \frac{\begin{bmatrix} C \int_0^U u\, dG_N(u) + \frac{G(U)}{\overline{G}(U)} \sum\limits_{i=0}^{N-1} \beta^{i-1}\, \mu \left[G_i(U) - G_N(U)\right] \\ -R \sum\limits_{i=1}^{N-1} \alpha^{i-1}\, \lambda G_{i-1}(u) + \alpha^{N-1}\, \lambda G_N(U) + c_p \tau + B \end{bmatrix}}{\begin{bmatrix} \sum\limits_{i=1}^{N-1} \alpha^{i-1}\, \lambda G_{i-1}(u) + \alpha^{N-1}\, \lambda G_N(U) + \int_0^U u\, dG_N(u) \\ +\frac{G(U)}{\overline{G}(U)} \sum\limits_{i=0}^{N-1} \beta^{i-1}\, \mu \left[G_i(U) - G_N(U)\right] + \tau \end{bmatrix}}. \tag{13}$$

## 5. Numerical Examples

Consider a degenerative simple repairable system with five states including three working states and two failure states, that is $k = 3$ and $l = 2$. Assume that Then $p_1 + p_2 + p_3 = 1$; $1 < a_1 < a_2 < a_3$ and $\alpha = \left(\frac{p_1}{a_1} + \frac{p_2}{a_2} + \frac{p_3}{a_3}\right) = 0.9792$. Assume

| | |
|---|---|
| $p_1 = 0.312$ | $a_1 = 1.016$ |
| $p_2 = 0.329$ | $a_2 = 1.021$ |
| $p_3 = 0.359$ | $a_3 = 1.026$ |

further that Then $q_1 + q_2 = 1$; $1 > b_1 > b_2$ and $\beta = \left(\frac{q_1}{b_1} + \frac{q_2}{b_2}\right) = 1.0821$. Let

| | |
|---|---|
| $q_1 = 0.48$ | $b_1 = 0.94$ |
| $q_2 = 0.52$ | $b_2 = 0.91$ |

| | | | |
|---|---|---|---|
| $R = 6$ | $C = 50$ | $\lambda = 90$ | $\tau = 13$ |
| $B = 6000$ | $c_p = 4$ | $\mu = 32$ | |

**The (T, N) Policy:** Assume that the distribution function of $X_n$ is exponential, that is

$$F_n(T) = F(a^{n-1}T)$$
$$= 1 - \exp\left(-\frac{a^{n-1}T}{\lambda}\right),$$

where $T \geq 0$; $\frac{1}{\lambda} > 0$; $a \geq 1$ *and* $n = 1, 2, \ldots$, and $F(T) = 1 - \exp\left(-\frac{T}{\lambda}\right)$. If $X_1, X_2, \ldots, X_n$ are independent and each $X_i$ has exponential distribution with mean $\lambda_i$ $(i = 1, 2, \ldots, n)$, then probability density function of $\sum_{i=1}^{n} X_i$ is

$$f_n(t) = (-1)^{n-1}\,\lambda_1\lambda_2\cdots\lambda_n \sum_{i=1}^{n} \frac{\exp(-\lambda_i t)}{\prod_{j=1;i\neq j}^{n}(\lambda_{i-1} - \lambda_{j-1})}, \qquad for\ t \geq 0.$$

The distribution function of $\sum_{i=1}^{n} X_i$ in this paper is given by

$$F_n(T) = \begin{cases} 1 - \sum_{i=1}^{n}\left[\prod_{j=1;i\neq j}^{n} \frac{a^{j-1}}{a^{j-1} - a^{i-1}}\right]\exp\left(-\frac{a^{i-1}T}{\lambda}\right), & for\ \ T \geq 0 \\ 0, & otherwise \end{cases}$$

and $F(T) = 1 - \exp\left(-\frac{T}{\lambda}\right)$.

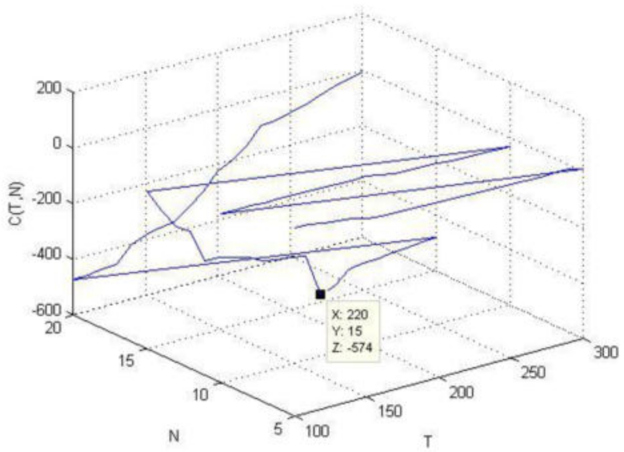


**Figure 1.** **The Graph of** $C(T, N)$

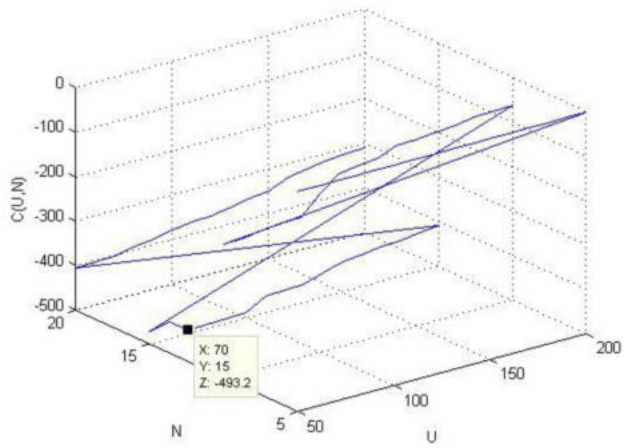


**Figure 2.** **The Graph of** $C(U, N)$

The equation (7) then becomes

$$\mathcal{C}(T,N)=\frac{\left[\begin{array}{c} C\sum\limits_{n=1}^{N-1}\frac{\mu}{b^{n-1}}\left[1-\sum\limits_{i=1}^{n}\left(\prod\limits_{j=1;i\neq j}^{n}\frac{a^{j-1}}{a^{j-1}-a^{i-1}}\right)\exp\left(-\frac{a^{i-1}T}{\lambda}\right)\right] \\ -R\sum\limits_{i=1}^{N}\left[\prod\limits_{j=1;j\neq i}^{N}\frac{a^{j-1}}{a^{j-1}-a^{i-1}}\right]\alpha^{i-1}\lambda\left[1-\exp\left(-\frac{a^{i-1}T}{\lambda}\right)\right]+c_p\tau+B\end{array}\right]}{\left[\begin{array}{c}\sum\limits_{i=1}^{N}\left[\prod\limits_{j=1;j\neq i}^{N}\frac{a^{j-1}}{a^{j-1}-a^{i-1}}\right]\alpha^{i-1}\lambda\left[1-\exp\left(-\frac{a^{i-1}T}{\lambda}\right)\right] \\ +\sum\limits_{n=1}^{N-1}\frac{\mu}{b^{n-1}}\left[1-\sum\limits_{i=1}^{n}\left(\prod\limits_{j=1;i\neq j}^{n}\frac{a^{j-1}}{a^{j-1}-a^{i-1}}\right)\exp\left(-\frac{a^{i-1}T}{\lambda}\right)\right]+\tau\end{array}\right]}, \tag{14}$$

In this case, using equation (14) and overpassing numerical calculations, we arrive at $(T,N)^*=(220,15)$, such that $\mathcal{C}(T,N)$ is minimum at $(T,N)^*$ and the long-run average cost is $\mathcal{C}(T,N)=\mathcal{C}(220,15)=-573.9723$ monetary units. The values of $\mathcal{C}(T,N)$ for $T$ ranging from 100 to 300 time units and $N$ ranging from 5 to 20, are evaluated. Some of the values of $\mathcal{C}(T,N)$ are given in Table 1. These values are plotted in the Figure 1.

**The $(U,N)$ Policy:** The equation (9), then becomes

$$\mathcal{C}(U,N)=\frac{\left[\begin{array}{c} C\sum\limits_{n=1}^{N-1}\left[\prod\limits_{j=1;j\neq i}^{N-1}\frac{b^{j-1}}{b^{j-1}-b^{i-1}}\right]\left(\frac{\mu}{b^{i-1}}\right)\left[1-\exp\left(-\frac{b^{i-1}U}{\mu}\right)\right] \\ -R\sum\limits_{n=1}^{N-1}\alpha^{i-1}\lambda\left[1-\sum\limits_{i=1}^{n}\left(\prod\limits_{j=1;j\neq i}^{n}\frac{b^{j-1}}{b^{j-1}-b^{i-1}}\right)\exp\left(-\frac{b^{i-1}U}{\mu}\right)\right] \\ +\alpha^{N-1}\lambda\left[1-\sum\limits_{i=1}^{N}\left(\prod\limits_{j=1;j\neq i}^{N}\frac{b^{j-1}}{b^{j-1}-b^{i-1}}\right)\exp\left(-\frac{b^{i-1}U}{\mu}\right)\right]+c_p\tau+B\end{array}\right]}{\left[\begin{array}{c}\sum\limits_{n=1}^{N-1}\alpha^{i-1}\lambda\left[1-\sum\limits_{i=1}^{n}\left(\prod\limits_{j=1;j\neq i}^{n}\frac{b^{j-1}}{b^{j-1}-b^{i-1}}\right)\exp\left(-\frac{b^{i-1}U}{\mu}\right)\right] \\ +\alpha^{N-1}\lambda\left[1-\sum\limits_{i=1}^{N}\left(\prod\limits_{j=1;j\neq i}^{N}\frac{b^{j-1}}{b^{j-1}-b^{i-1}}\right)\exp\left(-\frac{b^{i-1}U}{\mu}\right)\right] \\ +\sum\limits_{n=1}^{N-1}\left[\prod\limits_{j=1;j\neq i}^{N-1}\frac{b^{j-1}}{b^{j-1}-b^{i-1}}\right]\left(\frac{\mu}{b^{i-1}}\right)\left[1-\exp\left(-\frac{b^{i-1}U}{\mu}\right)\right]+\tau\end{array}\right]}, \tag{15}$$

| $(T,N)$ | $\mathcal{C}(T,N)$ | $(T,N)$ | $\mathcal{C}(T,N)$ | $(T,N)$ | $\mathcal{C}(T,N)$ | $(T,N)$ | $\mathcal{C}(T,N)$ |
|---|---|---|---|---|---|---|---|
| (100,5) | 81.2465 | (100,10) | 8.9498 | (100,15) | −33.8084 | (100,20) | −472.7234 |
| (110,5) | 79.7345 | (110,10) | 8.0123 | (110,15) | −121.0084 | (110,20) | −470.9998 |
| (120,5) | 69.7345 | (120,10) | 7.8984 | (120,15) | −189.7643 | (120,20) | −465.1267 |
| (130,5) | 61.4993 | (130,10) | 5.9998 | (130,15) | −221.0438 | (130,20) | −461.2244 |
| (140,5) | 55.7213 | (140,10) | 5.9183 | (140,15) | −339.9407 | (140,20) | −402.2344 |
| (150,5) | 41.4482 | (150,10) | 5.1182 | (150,15) | −341.0492 | (150,20) | −384.8194 |
| (160,5) | 35.3817 | (160,10) | 4.3341 | (160,15) | −353.0821 | (160,20) | −372.9128 |
| (170,5) | 34.9992 | (170,10) | 2.9838 | (170,15) | −367.2108 | (170,20) | −362.8219 |
| (180,5) | 33.1983 | (180,10) | 2.1584 | (180,15) | −395.0812 | (180,20) | −323.9121 |
| (190,5) | 33.0083 | (190,10) | 1.2973 | (190,15) | −405.2483 | (190,20) | −283.4456 |
| (200,5) | 32.9945 | (200,10) | 1.1924 | (200,15) | −411.3849 | (200,20) | −224.3927 |
| (210,5) | 31.4599 | (210,10) | −7.1948 | (210,15) | −423.1234 | (210,20) | −198.2573 |
| (220,5) | 27.4563 | (220,10) | −18.4538 | **(220,15)** | **−573.9723** | (220,20) | −158.4338 |
| (230,5) | 26.8381 | (230,10) | −19.9992 | (230,15) | −555.2974 | (230,20) | −103.5581 |
| (240,5) | 25.9493 | (240,10) | −20.1998 | (240,15) | −510.1432 | (240,20) | −97.9127 |
| (250,5) | 25.0021 | (250,10) | −21.4585 | (250,15) | −502.3218 | (250,20) | −81.2792 |
| (260,5) | 24.9391 | (260,10) | −25.8131 | (260,15) | −499.9992 | (260,20) | −72.4318 |
| (270,5) | 24.2521 | (270,10) | −26.3418 | (270,15) | −493.2998 | (270,20) | −53.3841 |
| (280,5) | 23.9995 | (280,10) | −28.4531 | (280,15) | −491.1273 | (280,20) | −32.4882 |
| (290,5) | 23.8417 | (290,10) | −29.3845 | (290,15) | −482.7324 | (290,20) | −21.4331 |
| (300,5) | 11.2414 | (300,10) | −31.8184 | (300,15) | −480.1998 | (300,20) | −9.1946 |

**Table 1.** **The values of $\mathcal{C}(T,N)$**

| $(U,N)$ | $\mathcal{C}(U,N)$ | $(U,N)$ | $\mathcal{C}(U,N)$ | $(U,N)$ | $\mathcal{C}(U,N)$ |
|---|---|---|---|---|---|
| (50,10) | −198.7865 | (50,15) | −471.2356 | (50,20) | −405.5679 |
| (60,10) | −197.8432 | (60,15) | −463.3301 | (60,20) | −403.9912 |
| (70,10) | −191.4331 | **(70,15)** | − **493.1928** | (70,20) | −400.8137 |
| (80,10) | −187.9924 | (80,15) | −491.2893 | (80,20) | −391.4383 |
| (90,10) | −184.2311 | (90,15) | −489.1214 | (90,20) | −387.8843 |
| (100,10) | −145.3213 | (100,15) | −487.4123 | (100,20) | −378.1812 |
| (110,10) | −111.4528 | (110,15) | −463.1122 | (110,20) | −373.1199 |
| (120,10) | −105.3521 | (120,15) | −461.6472 | (120,20) | −369.9911 |
| (130,10) | −99.9921 | (130,15) | −459.4382 | (130,20) | −363.0011 |
| (140,10) | −81.9824 | (140,15) | −444.4482 | (140,20) | −353.5236 |
| (150,10) | −80.4531 | (150,15) | −431.8231 | (150,20) | −349.1015 |
| (160,10) | −75.6312 | (160,15) | −429.3481 | (160,20) | −330.2927 |
| (170,10) | −71.4531 | (170,15) | −421.1789 | (170,20) | −324.1082 |
| (180,10) | −65.6341 | (180,15) | −419.9943 | (180,20) | −316.9857 |
| (190,10) | −63.9928 | (190,15) | −413.4382 | (190,20) | −312.8923 |
| (200,10) | −61.7529 | (200,15) | −410.1284 | (200,20) | −310.9932 |

**Table 2.** **The values of $\mathcal{C}(U,N)$**

In this case, using equation (15) and overpassing numerical calculations, we arrive at $(U,N)^* = (70,15)$, such that $\mathcal{C}(U,N)$ is minimum at $(U,N)^*$ and the long-run average cost is $\mathcal{C}(U,N) = \mathcal{C}(70,15) = -493.1928$ monetary units. The values of $\mathcal{C}(U,N)$ for $U$ ranging from 50 to 200 time units and $N$ ranging from 10 to 20, are evaluated. Some of the values of $\mathcal{C}(U,N)$ are given in Table 2.

## 6. Conclusion

By considering a repairable system for a monotone process model of a one component multisate degenerative system, explicit expressions for the long-run average cost per unit time under the bivariate replacement policies $(T,N)$, $(U,N)$, $(T^+,N)$ and $(U^-,N)$ have been derived. Numerical examples for some of the bivariate replacement policies are given to illustrate the models and methodology developed in this paper.

### References


[1] M.S.A-Hameed and F.Proschan, *Shock Models with underlying Birth Process*, Journal of Applied Probability, 12(1975), 18-28.

[2] R.Barlow and F.Proschan, *Mathematical Theory of Reliability*, (1965).

[3] R.Barlow and F.Proschan, *Statistical Theory of Reliability and life testing*, John Wiley, New York, (1975).

[4] W.Feller, *An introduction to Probability Theory and its Applications*, John Wiley, New York, (1965).

[5] P.Govindaraju, U.Rizwan and V.Thagaraj, *Optimal Replacement Policy for a Multistate Degenrative System*, Int. J. of Computational and Applied Mathematics, 4(3)(2009), 175-186.

[6] P.Govindaraju, U.Rizwan and V.Thagaraj, *An extreme shock maintenance model under a Bivariate Reaplacement Policy*, Research Methods in Mathematical Sciences, (2011), 1-10.

[7] Y.Lam, *Geometric Processes and Replacement Problem*, Acta. Math. Sinica, 4(1988), 366-377.

[8] Y.Lam, *An Optimal Repairable Replacement Model for Deteriorating System*, J. App. Prob., 28(1991), 843-851.

[9] Y.Lam, *A Monotone Process Maintenance Model for a Multistate System*, J. App. Prob., 42(2005), 1-14.

[10] Y.Lam and Yeekit, *Optimal Maintenance Model for a Multistate Deteriorating System*, Int. J. Sys. Sci., 34(5)(2003), 303-308.

[11] Y.Lam and Y.L.Zhang, *A Geometric Process Maintenance Model and Deteriorating System under a Random Environment*, IEEE Transactions on Reliability, 52(2003), 83-89.

[12] Y.Lam and Y.L.Zhang, *A Shock Model for the Maintenance Problem of a Repairable System*, Computers and Operations Research, 31(2004), 1807-1820.

[13] K.N.F.Leung, *A Note on A Bivariate Optimal Replacement Policy for a Repairable System*, Engineering Optimization, 38(5)(2006), 621-625.

[14] E.J.Muth, *An Optimal Decision Rule for Repair Vs. Replacement*, IEEE Trans. Reliability., 3(1977), 179-181.

[15] F.Pellerey, *Shock Models with underlying Counting Process*, J. Appl. Prob., 31(1994), 156-166.

[16] Philip J.F.Boland and Frank Proschan, *Optimum Replacement of a system subject to shocks*, FSU Statistics Report, (1981), 81-131.

[17] S.M.Ross, *Stochastic Processes*, (2nd ed), John Wiley and Sons, New York, (1996).

[18] S.M.Ross, *Introduction to Probability Modelling*, Academic Press, New York, (1997).

[19] A.Rangan and A.Tansu, *A New Shock Model for Systems Subject to Random Threshold Failure*, PWASET, 30(2008).

[20] U.Rizwan, *On Stochastic Life Time Models*, J. Madras University - Section B : Sciences, 52(2000), 121-143.

[21] J.G.Shantikumar and U.Sumita, *General Shock Models Associated with Correlated Renewal Sequences*, J. Appl. Prob., 20(1983), 600-614.

[22] J.G.Shantikumar and U.Sumita, *Distribution Properties of the System Failure Time in a General Shock Model*, Advances in Applied Probability, 16(1984), 363-377.

[23] A.D.J.Stanley, *On geometric processes and repair replacement problems*, Micro elce. Reliab., 33(1993), 489-491.

[24] H.M.Taylor and S.Karlin, *An Introduction to Stochastic Modelling*, Academic Press, New York, (1994).

[25] V.Thangaraj and U.Rizwan, *Optimal Replacement Policies in Burn-in Process for an Alternative Repair Model*, International Journal of Information and Management Sciences, 12(3)(2001), 43-56.

[26] Y.L.Zhang, *A Bivariate Optimal Replacement Policy for a Repairable System*, J. Appl. Prob., 31(1994), 1123-1127.

[27] Y.L.Zhang, R.C.M.Yam and M.J.Zuo, *A Bivariate Optimal Replacement Policy for a Multistate Repairable System*, Reliability Engineering and System Safety., 92(2007), 535-542.

[28] D.Zuckerman, *Replacement Models under Additive Damage*, Naval Research Logistics Quarterly, 24(4)(1977), 549-558.