International Journal of Current Research in Science and Technology
Volume 3, Issue 6 (2017), 1–9.
ISSN: 2394-5745
Available Online: http://ijcrst.in/

International Journal of Current Research in Science and Technology

An Optimal Bivariate Replacement Policy for a Multistate Degenerative System with an Extreme Shock

Research Article

U.Rizwan[1], Syed Tahir Hussainy[1]* and J.Manigandan[1]

1 Department of Mathematics, Islamiah College (Autonomous), Vaniyambadi, Tamilnadu, India.

Abstract: In this paper, an optimal bivariate replacement policy for a multistate degenerative system with an extreme shock is derived.

MSC: 60K10, 90B25.

Keywords: Extreme Shocks, Degenerative System, Replacement Policy.

1. Introduction

The study of a multistate degenerative system in a maintenance model plays an important role in reliability. A multistate degenerative system is subject to shocks and the damage occurs randomly during an operating stage. Most of the maintenance models just pay attention on the internal cause of the system failure, but do not on an external cause of the system failure. A system failure may be caused by some external cause, such as a shock. The shock models have been successfully applied to different fields, such as physics, communication, electronics, engineering and medicine, etc. A very few authors have considered the deteriorating systems interrupted by random shocks. Barlow and Proschen [1988] considered an imperfect repair model, in which a repair is perfect with probability p and a minimal repair with probability $1-p$. There were many papers which consider extreme shock models. In these models, the system will fail if the amount of shock exceeds a specific threshold. Thangaraj and Rizwan [2002] have introduced and studied maintenance problems with an alternative repair model. Chen and Li [2008] have studied the extreme shock maintenance model.

In this paper, we consider an extreme shock maintenance model for a multistate stochastic degenerative system. We use a Bivariate replacement policy based either on the total repair time or the number of failures that the system encountered.

The rest of the paper is organized as follows. In section 2, we give some basic definitions and the preliminaries that are required for our discussion. We also describe the model and state the assumptions. In section 3, explicit expressions for the long-run average cost under a bivariate replacement policy (U, N) is derived. Conditions for the existence of optimality are deduced. Finally conclusion is given in section 4.

* *E-mail: rahulmorghade@gmail.com*

2. Description of the Model

In this section, we first give some definitions. Next, we describe the model of a one component multistate degenerative system. We also evaluate the conditional probabilities of the operating times and failure times, given the state of the system.

Definition 2.1. *A random variable X is said to be stochastically smaller than another random variable Y, if $P(X > \alpha) \leq P(Y > \alpha)$, for all real α. It is denoted by $X \leq_{st} Y$. A stochastic process $\{X_n, n = 1, 2, \ldots\}$ is said to be stochastically increasing, if $X_n \leq_{st} X_{n+1}$, for $n = 1, 2, \ldots$.*

Definition 2.2. *A Markov process $\{X_n, n = 1, 2, \ldots\}$ with state space $\{0, 1, 2, \ldots\}$ is said to be stochastically monotone, if $(X_{n+1}|X_n = i_1) \leq_{st} (X_{n+1}|X_n = i_2)$, for any $0 \leq i_1 \leq i_2$.*

Clearly, the stochastically monotone concept for a Markov process is defined and is based on the transition probabilities from one state to another state, conditioning on the former state. However, the stochastically monotone concept for a stochastic process defined here is for a general process and is based on the conditional distribution of two successive random variables in the process.

Definition 2.3. *A stochastic process $\{X_n, n = 1, 2, \ldots\}$ is a* `geometric process` *(GP), if there exist a constant $a > 0$ such that $\{a^{n-1}X_n, n = 1, 2, \ldots\}$ forms a renewal process. The number a is called the ratio of the geometric process.*

If $0 < a < 1$, then the GP is stochastically increasing; if $a > 1$, the GP is stochastically decreasing and if $a = 1$, the GP will reduce to a renewal process.

Definition 2.4. *An integer valued random variable N is said to be a stopping time for the sequence of independent random variables $X_1, X_2, \ldots$, if the event $\{N = n\}$ is independent of $X_{n+1}, X_{n+2}, \ldots$, for all $n = 1, 2, \ldots$.*

We shall now describe the system states. Consider a one component multistate system with $k + l$ states (k-working states and l-failure states). The system state at time t is given by

$$S(t) = \begin{cases} i & \text{if the system is in the } i\text{-th working state at time } t \\ & (i = 1, 2, \ldots, k) \\ k + j & \text{if the system is in the } j\text{-th failure state at time } t \\ & (j = 1, 2, \ldots, l) \end{cases}$$

The set of working states is $\Omega_1 = \{1, 2, \ldots, k\}$; the set of failure states is $\Omega_2 = \{k + 1, k + 2, \ldots, k + l\}$ and the state space is $\Omega = \Omega_1 \cup \Omega_2$. Initially, assume that a new system in working state 1 is installed. Whenever the system fails, it will be repaired. Let t_n be the completion time of the n-th repair, $n = 0, 1, 2, \ldots$ with $t_0 = 0$ and let s_n be the time of occurrence of the n-th failure, $n = 1, 2, 3, \ldots$. Then $t_0 < s_1 < t_1 < \cdots < s_n < t_n < \cdots$.

We next describe the probability structure of the model. Assume that the transition probability from working state i, $i = 1, 2, 3, \ldots, k$, to failure state $k + j$, $j = 1, 2, \ldots, l$, is

$$P(S(s_{n+1}) = k + j|S(t_n) = i) = q_j$$

with $\sum_{j=1}^{l} q_j = 1$. Moreover, the transition probability from failure state $k+j$, $j = 1, 2, \ldots, l$, to working state i, $i = 1, 2, \ldots, k$ is given by

$$P(S(t_n) = i|S(s_n) = k + j) = p_i$$

with $\sum_{i=1}^{k} p_i = 1$. Let X_1 be the operating time of the system after installation. In general, let $X_n,\ n = 2, 3, \ldots$ be the operating time of the system after $(n-1)$-st repair and $Y_n,\ n = 1, 2, \ldots$ be the repair time after n-th failure. Assume that there exist a life time distribution $U(t)$ and $a_i > 0, \quad i = 1, \ldots, k$ such that

$$P(X_1 \leq t) = U(t) \tag{1}$$

and

$$P(X_2 \leq t | S(t_1) = i) = U(a_i t), \quad i = 1, 2, \ldots, k \tag{2}$$

where $1 \leq a_1 \leq a_2 \leq \cdots \leq a_k$. In general, for $i_j \in \{1, 2, \ldots, k\}$,

$$P\left(X_n \leq t | S(t_1) = i_1, \ldots, S(t_{n-1}) = i_{n-1}\right) = U(a_{i_1} \cdots a_{i_{n-1}} t). \tag{3}$$

Similarly, assume that there exist a life-time distribution $V(t)$ and $b_i > 0,\ i = 1, 2, \ldots, l$ such that

$$P(Y_1 \leq t | S(s_1) = k + i) = V(b_i t), \tag{4}$$

where $1 \geq b_1 \geq b_2 \geq \cdots \geq b_l > 0$ and in general, for $i_j \in \{1, 2, \ldots, l\}$,

$$P(Y_n \leq t | S(s_1) = k + i_1, \ldots, S(s_n) = k + i_n) = V(b_{i_1} \cdots b_{i_n} t) \tag{5}$$

In particular, if $a_1 = b_1 = 1,\ a_2 = \cdots = a_k = a'$ and $b_2 = \cdots = b_l = b'$, then the $(k+l)$-state system reduces to a two-state system. In this case, equations (3) and (5) become

$$P\left(X_n \leq t\right) = U\left((a')^{n-1} t\right) \quad \text{and}$$
$$P\left(Y_n \leq t\right) = V\left((b')^{n} t\right),$$

respectively. Thus the sequence $\{X_n, n = 1, 2, \ldots\}$ forms a GP with ratio $a' > 1$, while the sequence $\{Y_n, n = 1, 2, \ldots\}$ forms a GP with ratio $0 < b' < 1$. In this case, our model reduces to the GP model for the one component two-state system introduced by Lam [1988]. For two working states $1 \leq i_1 < i_2 \leq k$, we have

$$\left(X_2 | S(t_1) = i_2\right) \leq_{st} \left(X_2 | S(t_1) = i_1\right).$$

Therefore, the working state i_1 is better than the working state i_2, in the sense that, the system in state i_1 has a stochastically large operating time than it does in state i_2. Consequently, the k working states are arranged in decreasing order, such that state 1 is the best working state and state k is the worst working state. Similarly, for two failure states $k + i_1$ and $k + i_2$ such that $k + 1 \leq k + i_1 < k + i_2 \leq k + l$, we have

$$\left(Y_1 | S(s_1) = k + i_1\right) \leq_{st} \left(Y_1 | S(s_1) = k + i_2\right).$$

Therefore, the failure state $k + i_1$ is better than the failure state $k + i_2$ in the sense that the system in state $k + i_1$ has a stochastically smaller repair time than it does in state $k + i_2$. Thus, the l failure states are also arranged in decreasing order, such that the state $k + 1$ is the best failure state and the state $k + l$ is the worst failure state. Consider a monotone process model for a multistate one component system described in this section and make the following assumptions:

A2.1 At the beginning, a new system is installed. The system has $(k+l)$ possible states, where the states $1, 2, \ldots, k$ denote, respectively, the first type working state, the second type working state, $\ldots$, k-th type working state and the states $(k+1), (k+2), \ldots, (k+l)$ denote, respectively, the first type failure state, the second type failure state $\ldots$ and the l-th type failure state of the system. The occurrences of these types of failures are stochastic and mutually exclusive.

A2.2 Whenever the system fails in any of the failure states, it will be repaired. The system will be replaced by an identical one some times later.

A2.3 Once the system is operating, the shocks from the environment arrive according to a renewal process. Let $X_{ni},\ i = 1, 2, \ldots$ be the intervals between the $(i-1)$-st and the i-th shock, after the $(n-1)$-st repair. Let $E(X_{11}) = \lambda$. We assume that $X_{ni},\ i = 1, 2, \ldots,$ are *iid* sequences, for all n.

A2.4 Let $Y_{ni},\ i = 1, 2, \ldots$ be the sequence of the amount of shock damage produced by the i-th shock, after the $(n-1)$-st repair. Let $E(Y_{11}) = \mu$. Then $\{Y_{ni},\ i = 1, 2, \ldots\}$ are *iid* sequences, for all n. If the system fails, it is closed, so that the random shocks have no effect on the system during the repair time.

A2.5 Let $Z_n,\ n = 1, 2, \ldots$ be the repair time after the n-th repair and $Z_n, n = 1, 2, \ldots$ constitute a non decreasing geometric process with $E(Z_1) = \delta$ and ratio b, such that $0 < b \le 1$. Let $N_n(t)$ be the counting process denoting the number of shocks after the $(n-1)$-st repair. The distribution of Z_n is denoted by $G_n(\cdot)$. It is clear that $E(Z_n) = \dfrac{\delta}{b^{n-1}}$.

A2.6 Let r be the reward rate per unit time of the system when it is operating and c be the repair cost rate per unit time of the system and the replacement cost is R. The replacement time is a random variable Z with $E(Z) = \tau$.

A2.7 If the system in working state i is operating, then let the reward rate be r. If the system in failure state $k+i$ is under repair, the repair cost is c. The replacement cost comprises two parts: one part is the basic replacement cost R and the other proportional to the replacement time Z at rate c_p. In other words, the replacement cost is given by $R + c_p\, Z$.

A2.8 Assume that $1 \le a_1 \le a_2 \le \cdots \le a_k$ and $1 \ge b_1 \ge b_2 \ge \cdots \ge b_l > 0$.

A2.9 Assume that $F_n(t)$ be the cumulative distribution of $L_n = \sum_{i=1}^{n} X_i$ and $G_n(t)$ be the cumulative distribution of $M_n = \sum_{i=1}^{n} Y_i$.

A2.10 The working time X_n, the repair time Y_n and the replacement time Z, [4] $(n = 1, 2, \ldots)$ are independent random variables.

3. The (U, N) Policy

In this section, we introduce and study a bivariate replacement policy (U, N) for the multistate stochastic degenerative system, under which system is replaced whenever the cumulative repair time of the system exceeds U or at the time of N-th failure, whichever occurs first. The problem is to choose an optimal replacement policy $(U, N)^*$ such that the long-run average cost per unit time is minimized. Following Lam (2005), the distribution of the survival time X_n in and the distribution of the repair time Y_n in are given by

$$P(X_n \le t) = \sum_{\sum_{i=1}^{k}\ j_i = n-1} \frac{(n-1)!}{j_1! \cdots j_k!}\ p_1^{j_1} \cdots p_k^{j_k}\ U(a_1^{j_1} \cdots a_k^{j_k} t), \tag{6}$$

where $j_1, j_2, \ldots, j_k \in \mathbb{Z}^+$ and

$$P(Y_n \leq t) = \sum_{\sum_{i=1}^{l} j_i = n} \frac{n!}{j_1! \cdots j_l!} \; q_1^{j_1} \cdots q_l^{j_l} \; V(b_1^{j_1} \cdots b_l^{j_l} t), \tag{7}$$

where $j_1, j_2, \ldots, j_l \in \mathbb{Z}^+$. If $E(X_1) = \lambda$, then the mean survival time is

$$E(X_n) = \frac{\lambda}{a^{n-1}}, \tag{8}$$

for $n > 1$, where

$$a = \left(\sum_{i=1}^{k} \frac{p_i}{a_i} \right)^{-1} \tag{9}$$

and if $E(Y_1) = \mu$, then the mean repair time is

$$E(Y_n) = \frac{\mu}{b^n} \tag{10}$$

for $n > 1$, where

$$b = \left(\sum_{j=1}^{l} \frac{q_j}{b_j} \right)^{-1}. \tag{11}$$

Further if $R_n = r_i$ where $S(s_{n-1}) = i$, $i = 1, 2, \ldots, k$ denotes the reward earned after the n-th repair, then mean reward earned after $(n-1)$-st repair is $E(R_1 X_1) = r\lambda$ and for $n \geq 2$ then expected reward after installation is given by

$$E(R_n X_n) = \frac{r\lambda}{a^{n-1}}, \tag{12}$$

where

$$r = \sum_{i=1}^{k} \frac{r_i p_i}{a_i}. \tag{13}$$

and if $C_n = c_i$ where $S(s_n) = k + i$, $i = 1, 2, \ldots, l$ denotes the repair cost after the n-th failure, then mean repair cost after n-th failure is

$$E(C_n Y_n) = \frac{c\mu}{b^{n-1}}, \tag{14}$$

where

$$c = \sum_{i=1}^{l} \frac{c_i q_i}{b_i}. \tag{15}$$

3.1. The Length of a cycle and its Mean

The length of a cycle under the bivariate replacement policy (U, N) is

$$W = \left(\sum_{i=1}^{N} X_i + \sum_{i=1}^{N-1} Y_i \right) \chi_{(M_N \leq U)} + \left(U + \sum_{i=1}^{\eta} Y_i \right) \chi_{(M_N > U)} + Z,$$

where $\eta = 0, 1, 2, \ldots, N-1$ is the number of failures before the cumulative repair time of the system exceeds U and

$$\chi_{(A)} = \begin{cases} 1 & \text{if the event } A \text{ occurs,} \\ 0 & \text{if the event } A \text{ does not occur.} \end{cases}$$

denotes the indicator function. From Leung (2006), we have

$$\begin{aligned} E\left[\chi_{(M_i \leq U < M_N)}\right] &= P(M_i \leq U < M_N) \\ &= P(M_i \leq U) - P(M_N \leq U) \\ &= G_i(U) - G_N(U). \end{aligned}$$

Lemma 3.1. *The mean length of a cycle is*

$$E(W) = \int_0^U \overline{G}_N(u)du + \sum_{i=1}^{N-1} \frac{\lambda}{a^{i-1}}\, G_{i-1}(U) + \frac{\lambda}{a^{N-1}} G_N(U). \tag{16}$$

Proof. Consider

$$\begin{aligned}
E(W) &= E\left[\left(\sum_{i=1}^{N} X_i + \sum_{i=1}^{N-1} Y_i\right)\chi_{(L_N \le U)}\right] + E\left[\left(U + \sum_{i=1}^{\eta} Y_i\right)\chi_{(L_N > U)}\right] + E(Z) \\
&= E\left\{E\left[\left(\sum_{i=1}^{N} X_i + \sum_{i=1}^{N-1} Y_i\right)\ \chi_{(L_N \le\, U)}|L_N = u\right]\right\} + E\left[U\chi_{(L_N > U)}\right] + E\left[\left(\sum_{i=1}^{\eta} Y_i\right)\ \chi_{(L_N > U)}\right] + E(Z) \\
&= \int_0^U u\mathrm{d}F_N(u) + \int_0^U \sum_{i=1}^{N-1} E(Y_i)\ \mathrm{d}\,F_N(u) + U\overline{F}_N(U) + \sum_{i=1}^{N-1} \frac{\mu}{b^{i-1}} E\left[\chi\left(L_i \le U < L_N\right)\right] + \tau \\
&= \int_0^U u\,\mathrm{d}\,F_N(u) + \sum_{i=1}^{N-1} \frac{\mu}{b^{i-1}}\ F_N(U) + U\overline{F}_N(U) + \sum_{i=1}^{N-1} \frac{\mu}{b^{i-1}} P\left(L_i \le U < L_N\right) + \tau \\
&= U\overline{F}_N(U) + \int_0^U u\,\mathrm{d}F_N(u) + \sum_{i=1}^{N-1} \frac{\mu}{b^{i-1}}\left[F_i(U) - F_N(U)\right] + \sum_{i=1}^{N-1} \frac{\mu}{b^{i-1}}\ F_N(U) + \tau \\
&= \int_0^U \overline{F}_N(u)\mathrm{d}u + \sum_{i=1}^{N-1} \frac{\mu}{b^{i-1}} F_i(U) + \tau,
\end{aligned}$$

which is (16) as desired. □

3.2. Mean Reward and Mean Repair cost

Lemma 3.2. *If* $L_N \le U$ *and* $n \ge 2$*, then the expected Reward earned is*

$$E\left[\left(\sum_{n=2}^{N} \mathrm{R}_n X_n\right)\chi(L_N \le U)\right] = \sum_{n=2}^{N} \frac{r\lambda}{a^{n-2}} \int_0^T u dF_N(u)\ . \tag{17}$$

Proof. Consider

$$\begin{aligned}
E\left[\left(\sum_{n=2}^{N} \mathrm{R}_n X_n\right)\chi(L_N \le U)\right] &= E\left\{E\left[\left(\sum_{n=2}^{N} \mathrm{R}_n X_n\right)\chi(L_N \le U)|L_N\right]\right\} \\
&= \int_0^U E\left(\sum_{n=2}^{N} \mathrm{R}_n X_n|L_N = u\right) dF_N(u) \\
&= \sum_{n=2}^{N} \frac{r\lambda}{a^{n-2}} \int_0^U u dF_N(u)\ ,
\end{aligned}$$

which is (17) and the proof is complete. □

Lemma 3.3. *If* $L_N > U$ *and* $n \ge 2$ *then the expected Reward earned is*

$$E\left[\left(\sum_{n=2}^{N} \mathrm{R}_n X_n\right)\chi(L_N > U)\right] = \sum_{n=2}^{N} \frac{r\ \lambda}{a^{n-2}}\left[F_n(U) - F_N(U)\right]\ . \tag{18}$$

Proof. Consider

$$\begin{aligned}
E\left[\left(\sum_{n=2}^{\eta} R_n X_n\right)\chi(L_N > U)\right] &= E\left[\left(\sum_{n=2}^{N} R_n X_n\right)\chi(L_n < U < L_N)\right] \\
&= \sum_{n=2}^{N} E(R_n X_n) E\left[\chi(L_n < U < L_N)\right] \\
&= \sum_{n=2}^{N} \frac{r\ \lambda}{a^{n-2}}\left[F_n(U) - F_N(U)\right]\ .
\end{aligned}$$

This completes the proof. □

Lemma 3.4. *If $L_N \leq U$, then the expected Repair Cost is*

$$E\left[\left(\sum_{n=1}^{N-1} \mathrm{C}_n Y_n\right)\chi(L_N \leq U)\right] = \sum_{n=1}^{N-1} \frac{c\mu}{b^{n-1}} F_N(U)\ . \tag{19}$$

Proof. Consider

$$\begin{aligned}
E\left[\left(\sum_{n=1}^{N-1} \mathrm{C}_n Y_n\right)\chi(L_N \leq U)\right] &= E\left[E\left(\sum_{n=1}^{N-1} \mathrm{C}_n Y_n | L_N = u\right)\chi(L_N \leq U)\right] \\
&= \int_0^U E\left(\sum_{n=1}^{N-1} \mathrm{C}_n Y_n | L_N = u\right) dF_n(u) \\
&= \int_0^U \sum_{n=1}^{N-1} E(\mathrm{C}_n Y_n) dF_N(u) \\
&= \int_0^T \sum_{i=1}^{N-1} \frac{c\mu}{b^{i-1}} dF_N(u) \\
&= \sum_{i=1}^{N-1} \frac{c\mu}{b^{i-1}} \int_0^U dF_N(u) \\
&= \sum_{n=1}^{N-1} \frac{c\mu}{b^{n-1}} F_N(U)\ .
\end{aligned}$$

This completes the proof. □

Lemma 3.5. *If $L_N > U$, then the expected Repair Cost is*

$$E\left[\left(\sum_{n=1}^{\eta-1} \mathrm{C}_n Y_n\right)\chi(L_N > U)\right] = \sum_{n=1}^{N-1} \frac{c\ \mu}{b^{n-1}} \left[F_n(U) - F_N(U)\right]\ . \tag{20}$$

Proof. Consider

$$\begin{aligned}
E\left[\left(\sum_{n=1}^{\eta-1} \mathrm{C}_n Y_n\right)\chi(L_N > U)\right] &= E\left[\left(\sum_{n=1}^{N-1} \mathrm{C}_n Y_n\right)\chi(L_n < U < L_N)\right] \\
&= \sum_{n=1}^{N-1} E(\mathrm{C}_n Y_n) E\left[\chi(L_n < U < L_N)\right] \\
&= \sum_{n=1}^{N-1} \frac{c\ \mu}{b^{n-1}} \left[F_n(U) - F_N(U)\right]\ .
\end{aligned}$$

This completes the proof. □

3.3. The Long-run Average Cost under (U, N) Policy

Let U_1 be the first replacement time and let U_n $(n \geq 2)$ be the time between $(n-1)$-st replacement and n-th replacement. Then the sequence $U_n,\ n = 1, 2, \ldots,$ forms a renewal process. The interarrival time between two consecutive replacements is a renewal cycle. By the renewal reward theorem, the long-run average cost per unit time under the bivariate replacement policy (U, N) for the multistate stochastic degenerative system is

$$\begin{aligned}
\mathcal{C}(U,N) &= \frac{\text{the expected cost incurred in a cycle}}{\text{the expected length of a cycle}} \\
&= \frac{\left[\begin{array}{l} E\left\{\left(\sum_{n=1}^{N-1} C_n Y_n - \sum_{n=1}^{N} R_n X_n\right)\chi_{(L_N \leq U)}\right\} + R \\ \\ +E\left\{\left(\sum_{n=1}^{\eta} C_n Y_n - U\sum_{n=1}^{\eta} R_n\right)\chi_{(L_N > U)}\right\} + c_p E(Z) \end{array}\right]}{E(W)}.
\end{aligned}$$

After tedious calculation and simplification, we obtain

$$\mathcal{C}(U,N) \;=\; \frac{\left[\begin{array}{c}\displaystyle\int_0^U \overline{G}_N(u)du - r\sum_{n=2}^{N}\frac{\lambda}{a^{n-1}}G_{n-1}(U) - \frac{r\lambda}{a^{N-1}}G_N(U) \\ \displaystyle +\sum_{n=2}^{N}\frac{r\lambda}{a^{n-2}}\left[G_n(U)-G_N(U)\right] + c_p\tau + R\end{array}\right]}{\displaystyle\int_0^U \overline{G}_N(u)du + \sum_{i=1}^{N-1}\frac{\lambda}{a^{i-1}}\,G_{i-1}(U) + \frac{\lambda}{a^{N-1}}G_N(U)}. \tag{21}$$

Summarizing the above results, we obtain the following.

Theorem 3.6. *The long-run average cost per unit time for an extreme shock maintenance model for a multistate stochastic degenerative system, under the the bivariate* (U,N) *replacement policy is given by the Equation* (21).

3.4. Deductions

Here $\mathcal{C}(U,N)$ is a bivariate function. Obviously, when N is fixed, $\mathcal{C}(U,N)$ is a function of U. For fixed $N=m$, it can be written as

$$\mathcal{C}(U,N) = C_m(U), \qquad m = 1,2,3,\dots\ .$$

Thus, for a fixed m, we can find U_m^* by analytical or numerical methods such that $C_m(U_m^*)$ is minimized. That is, when $N = 1,2,\dots,m,\dots,$ we can find $U_1^*,U_2^*,U_3^*,\dots,U_m^*,\dots,$ respectively, such that the corresponding $C_1(U_1^*),C_2(U_2^*),\dots,C_m(U_m^*),\dots$ are minimized.

It is logical to assume that the total repair time at any stage can never exceed the working time of the system, because in this case the repair cost will exceed the total reward earned. Further the total life time of degenerative system is limited. It follows that the total repair time of a multistate stochastic degenerative system is also limited. Therefore the long-run average cost per unit time exists. So we can determine the minimum of the long-run average cost per unit time based on $C_1(U_1^*),\ C_2(U_2^*),\dots,C_m(U_m^*),\dots$.

For example, if the minimum is denoted by $C_m(U_m^*)$, we obtain the bivariate optimal replacement policy $(U,N)^*$ such that

$$\mathcal{C}(U,N)^* = \min_m C_m(U_m^*).$$

4. Conclusion

By considering a repairable system for a monotone process model of a one component multisate stochastic degenerative system, explicit expressions for the long-run average cost per unit time under a bivariate (U,N) replacement policy is derived. Existence of optimality conditions are deduced.

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