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Control Charts Application on Hospital Length of Stay Data

Research Article

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Abstract: Statistical Process Control is an effective tool to separate the variations due to common and special causes. It was originally developed for the engineering and industrial products. As it is a powerful graphical tool it has found its application in healthcare science. The length of stay in the hospital depends on the type of disease, the type of treatment, etc. By applying XMR and EWMA charts to these data, we checked whether the length of stay variable is under control. As the companies producing goods are moving towards zero defects based on six sigma initiative, healthcare practices also should apply six sigma initiative. In this paper six sigma based individual control chart is developed for the same length of stay data from the hospital.

Keywords: XMR chart, EWMA chart, Six Sigma initiative.

1. Introduction

In 19^{th} century, around 1820 to 1840 industrial revolutions began in Great Britain and slowly the machine made products started replacing the handmade products. Many innovations took place in Great Britain and in Europe. It marks an important era in the human history. There is not much variation in the product when it is made on the machine. But the products are not exactly the same when they are inspected. Variation is present in nature and so in the product produced by a machine. There is some cause associated with each type of variation. We have to identify the cause of variation.

1.1. Causes of Variation

The variation in the data is classified into two types:

(1) **Chance cause:** The variation in the data which is natural and does not have any particular reason.

(2) **Assignable cause:** This variation in the data has some special reason behind it. This variation has to be detected and to be removed from the process.

The variation present in the data can be detected with the help of control charts. It is a simple graph of observations with three lines called as Center Line (CL), Upper Control Limit (UCL) and Lower Control Limit (LCL). When observations plotted on the graph are within these lines, usually the process is under control. There are stricter rules to judge the out-of

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control process, such as eight or more points on one side of the center line, etc. This technique has been proved to be good to detect whether the change in the process has occurred. Dr. Walter Shewhart originally developed control charts in 1924 and since then it has become one of the primary tools of quality control. Shewhart control charts are based on 3σ limits. There are charts for variables and for attributes. When the points are plotted on these graphs, if all points fall within the control limits and there is no particular pattern, then the product or the procedure is said to be in control. Then one can conclude that only natural causes are at work and there are no assignable causes present in the data.

William Woodall [1] discussed the issues related to use of control charts in health care and public surveillance. He also suggested that it is very difficult to adjust any process in health application to bring under control quickly as it can be done in industrial environment. Michael Coory et al [2] gave us insight into the way control charts can become effective tool in detecting the problems early. Control charts also provide understandable over view of the situation. Douglas Montgomery [7] gave the mathematical and statistical back ground for the control charts and the best parameter values for EWMA control chart. Six Sigma's aim is to eliminate waste and inefficiency.

This initiative sets a performance goal for everyone and accelerates the rate of improvement. The companies practicing six sigma initiatives, will produce 3.4 or less number of defectives per million opportunities. This concept was introduced by Motorola is based on many quality concepts. Dr. R. Radhakrishnan et al constructed various control charts based on six sigma initiative [3, 4].

2. Statistical Analysis

2.1. Problem Definition

The data is collected from a hospital where the patients, diagnosed with cancer and below 12 years of age, are admitted. The collected data is about the length of stay in hospital in number of days along with their sex and age. We compare the performances of XmR chart and EWMA chart for Diagnostically Related Groups (DRG).

2.2. Methodology

The data collected from the hospital is segregated into Diagnostically Related Groups (DRG). A sample size of 19 is taken for one of DGR and XMR chart and Exponentially Weighted Moving Average (EWMA) chart is drawn as these charts are used for individual observations. Then further sample of size 19 is considered to check the validity of the limits obtained earlier. Using the concepts from six sigma initiative, control chart for individual observations is constructed. For the same set of observations, six sigma based control chart is drawn. We count one day when the patient stays in hospital from morning till evening. If the patient stays for a one night and is discharged the next day, we count it as two days. So typically if a patient stays for eight days and counted as eight then it means that he has stayed there eight days and seven nights.

2.3. Analysis of the Data

XmR Chart: We considered the sample of 19 cancer patients in the same diagnostically related group and recorded their length of stay in the hospital. Since we do not have any standards (mean and variance) known, we estimate it.

Patient No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
LENGTH OF STAY	6	24	34	6	6	4	37	7	3	8	18	7	11	4	15	8	4	53	9

Table 1:

We calculate the mean of all X_i observations as well as mean of moving ranges where MR is defined as

$$\text{Moving Range (MR)} \quad MR_i = \mid X_i - X_{i-1} \mid, \quad i = 2, 3, \ldots, 19 \tag{1}$$

The values MR are tabulated below:

Patient No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
MR	–	18	10	28	0	2	33	30	4	5	10	11	4	7	11	7	4	49	44

Table 2:

$$\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n} = 13.89; \; \overline{MR} = \frac{\sum_{i=2}^{n} MR_i}{n-1} = 15.38889$$

The Moving Range chart control limits are

$$\left.\begin{aligned} CL &= \overline{MR} \\ LCL &= D_3\overline{MR} \\ UCL &= D_4\overline{MR} \end{aligned}\right\} \tag{2}$$

To set up the moving range chart, we use $D_3 = 0$ and $D_4 = 3.267$ for $n = 2$. Therefore, $CL = \overline{MR} = 15.38889$, $LCL = 0$ and $UCL = 3.267$, $\overline{MR} = 50.2755$. We plot the graph as below:

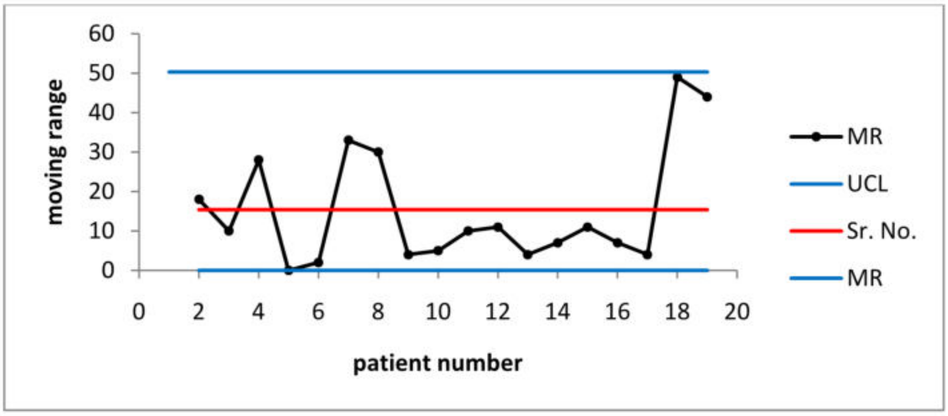


Figure 1: Moving Range Chart

The control limits for X chart are

$$\left.\begin{aligned} CL &= \overline{X} = 13.89 \\ UCL &= \overline{X} + \tfrac{3\overline{MR}}{d_2} = 54.8244474 \\ LCL &= \overline{X} - \tfrac{3\overline{MR}}{d_2} = -27.0444474 \end{aligned}\right\} \tag{3}$$

Since LCL is negative we set its value at zero. Thus the limits are $CL = 13.89$, $UCL = 54.8244474$ and $LCL = 0$. We draw the graph as below:

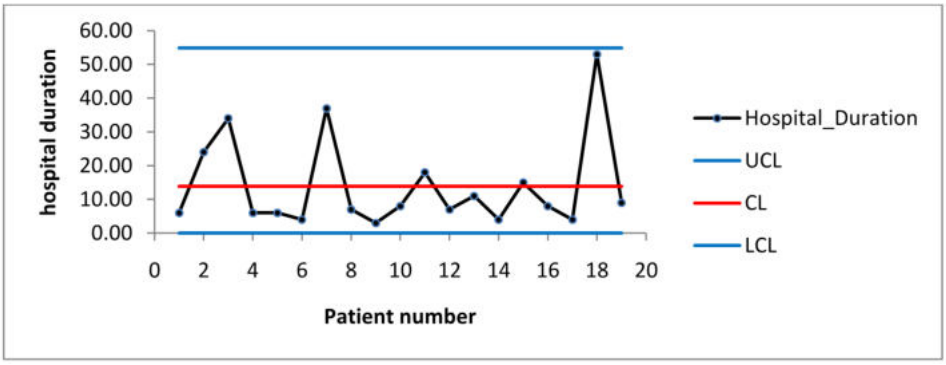


Figure 2: X chart for individual observations

From the above charts we can conclude that the process is in control. But Montgomery suggests that these charts may be good for Phase I control but not so for the Phase II, as these are unable to catch the smaller shifts.

EWMA Control Chart: We now apply Exponentially Weighted Moving Average (EWMA) chart to the same data. Let X_i be the i^{th} observation on the process (length of stay) and distributed as normal with mean μ_0 and variance σ^2. We also assume that the μ_0 as the target value for the quality characteristic x. The exponentially weighted moving average is defined as

$$Z_i = \lambda X_i + (1-\lambda) Z_{i-1} \tag{4}$$

where $0 < \lambda \leq 1$ is a constant and the starting value of z is the process target, so that $z_0 = \bar{x}$, the sample average of preliminary data. We used $\lambda = 0.05$ to calculate Z_i.

Patient No.	1	2	3	4	5	6	7	8	9	10
X_i	6	24	34	6	6	4	37	7	3	8
Z_i	13.4955	14.3955	14.8955	13.4955	13.4955	13.3955	15.0455	13.5455	13.3455	13.5955
Patient No.	11	12	13	14	15	16	17	18	19	
X_i	18	7	11	4	15	8	4	53	9	
Z_i	14.0955	13.5455	13.7455	13.3955	13.9455	13.5955	13.3955	15.8455	13.6455	

Table 3:

Since the EWMA is the weighted average of all past and current observations, it is very insensitive to the normality assumption. It is therefore the ideal control chart to use for individual observations. Let the individual observations be independent random variables with variance σ^2, then the variance of Z_i is

$$\sigma_{z_i}^2 = \sigma^2 \left(\frac{\lambda}{2-\lambda}\right)\left(1-(1-\lambda)^{2i}\right)$$

So the EWMA chart can be constructed by plotting z_i versus the sample number i (or time). The centre line and the control limits for the EWMA control chart are as follows:

$$\left.\begin{aligned} UCL &= \mu_0 + L\sigma\sqrt{\tfrac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]} \\ CL &= \mu_0 \\ LCL &= \mu_0 - L\sigma\sqrt{\tfrac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]} \end{aligned}\right\} \tag{5}$$

where L is the width of control limits. The term $\left[1-(1-\lambda)^{2i}\right]$ approaches unity as i gets larger. We considered value of $L = 2.615$ to calculate precise limits.

Patient number	Zi	UCL	CL	LCL
1	13.4955	15.6827	13.89	12.0973
2	14.3955	16.3627	13.89	11.4173
3	14.8955	16.8450	13.89	10.9350
4	13.4955	17.2208	13.89	10.5592
5	13.4955	17.5268	13.89	10.2532
6	13.3955	17.7824	13.89	9.9976
7	15.0455	17.9994	13.89	9.7806
8	13.5455	18.1859	13.89	9.5941
9	13.3455	18.3474	13.89	9.4326
10	13.5955	18.4884	13.89	9.2915
11	14.0955	18.6120	13.89	9.1680
12	13.5455	18.7209	13.89	9.0591
13	13.7455	18.8170	13.89	8.9630
14	13.3955	18.9022	13.89	8.8778
15	13.9455	18.9779	13.89	8.8021
16	13.5955	19.0453	13.89	8.7347
17	13.3955	19.1052	13.89	8.6747
18	15.8455	19.1589	13.89	8.6211
19	13.6455	19.2068	13.89	8.5732

Table 4:

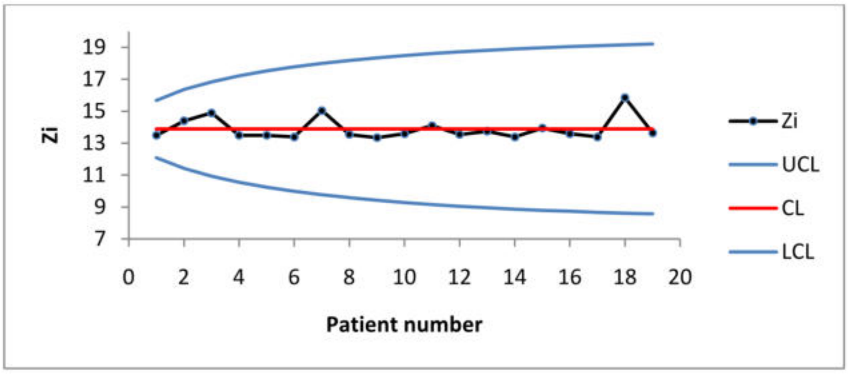


Figure 3: EWMA chart

When the second sample of size 19 is taken for the same DRG, the length of stay in the hospital is given in the table below:

Patient No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
LENGTH OF STAY	7	39	4	5	20	13	11	12	16	6	7	14	13	7	20	8	10	22	6

Table 5:

For the new data, we calculate the MR using (1) and note it down in the table below:

Patient No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
MR		32	35	1	15	7	2	1	4	10	1	7	1	6	13	12	2	12	16

Table 6:

Using the limits calculated earlier, we draw the MR chart and X chart as below:

(a) MR chart (b) X chart

Figure 4:

Both the charts are under control with no particular pattern seen in it. So we can conclude that the limits of these charts are stable. Similarly we use this new sample for EWMA chart. The limits of the earlier EWMA are used and also the parameter values of $\lambda = 0.05$ and $L = 2.615$.

Patient No.	1	2	3	4	5	6	7	8	9	10
X_i	7	39	4	5	20	13	11	12	16	6
Z_i	13.54	15.14	13.39	13.44	14.19	13.84	13.74	13.79	13.99	13.49
Patient No.	11	12	13	14	15	16	17	18	19	
Xi	7	14	13	7	20	8	10	22	6	
Z_i	13.54	13.89	13.84	13.54	14.19	13.59	13.69	14.29	13.49	

Table 7:

We calculate UCL and LCL using (5) for the EWMA chart using $CL = 13.89$ as earlier value for in-control chart.

Patient No.	Zi	UCL	CL	LCL
1	13.5455	15.6827	13.89	12.0973
2	15.1455	16.36269	13.89	11.41731
3	13.3955	16.84497	13.89	10.93503
4	13.4455	17.2208	13.89	10.5592
5	14.1955	17.5268	13.89	10.2532
6	13.8455	17.78237	13.89	9.997635
7	13.7455	17.99939	13.89	9.780609
8	13.7955	18.18585	13.89	9.594145
9	13.9955	18.34745	13.89	9.432553
10	13.4955	18.48841	13.89	9.291589
11	13.5455	18.61202	13.89	9.16798
12	13.8955	18.72086	13.89	9.059137
13	13.8455	18.81703	13.89	8.96297
14	13.5455	18.90224	13.89	8.877764
15	14.1955	18.97791	13.89	8.80209
16	13.5955	19.04525	13.89	8.734747
17	13.6955	19.10528	13.89	8.674717
18	14.2955	19.15887	13.89	8.621127
19	13.4955	19.20677	13.89	8.573226

Table 8:

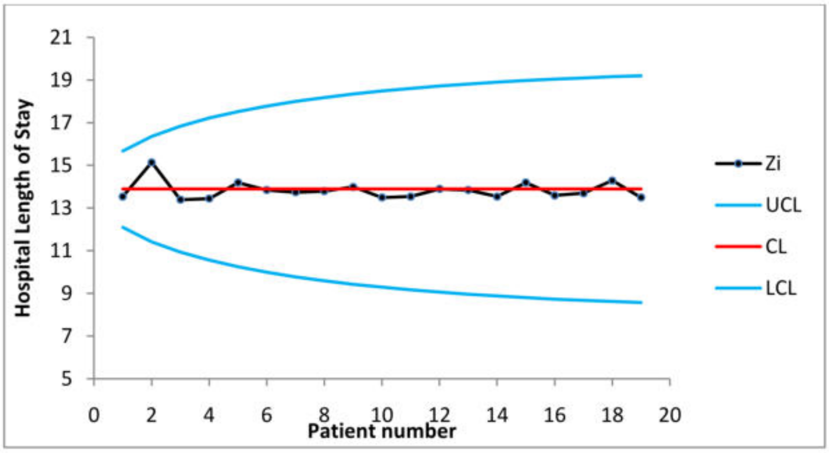


Figure 5: EWMA chart

Construction of Individual Chart Based on Six Sigma Initiative: Terminologies in Six Sigma Initiative:

1. Upper Specification Limit (USL): It is the maximum amount specified by the producer which has acceptable performance.
2. Lower Specification Limit (LSL): It is the minimum amount specified by the producer which has acceptable performance.
3. Tolerance Level (TL): It is the difference between USL and LSL. $TL = USL - LSL$
4. Process Capability (C_p): It is a ratio of tolerance level to six times standard deviation of the process.

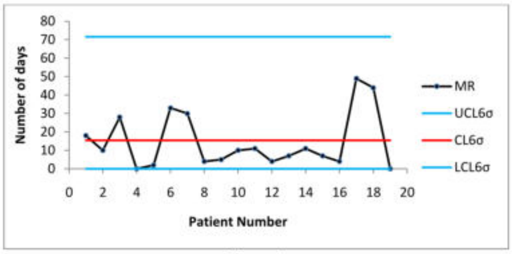
$$C_p = \frac{TL}{6\sigma} = \frac{USL - LSL}{6\sigma} \tag{6}$$

Construction of Control Chart for Individual Observations:

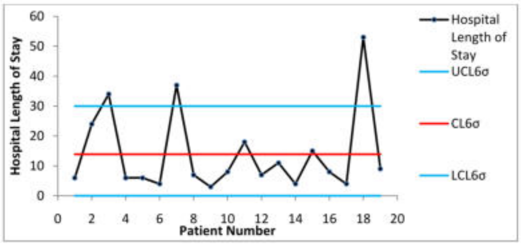
Initially we fix the tolerance level (TL) and process capability (C_p) to determine process standard deviation known as $\sigma_{6\sigma}$ using equation [6]. The 3σ control limits for individual observations control chart (X chart) are given by Center Line= $\bar{X}$ and UCL and LCL are given by $\bar{X} \pm 3\frac{\overline{MR}}{d_2}$ where $\overline{MR}$ is the mean of moving range and value of d_2 is 1.128 for sample size 2. Thus the UCL and LCL for X chart based on 3σ control limits are $\overline{X} \pm 2.66\overline{MR}$. When we apply 6σ control limits, the value of σ is estimated as $\widehat{\sigma_{6\sigma}}$.

Therefore the 6σ control limits for X chart are $CL = \bar{X}$; $UCL = \overline{X} + 4.831\widehat{\sigma_{6\sigma}}$ and $LCL = \bar{X} - 4.831\widehat{\sigma_{6\sigma}}$. The value 4.831 is obtained from $P(z \leq z_{6\sigma}) = 1 - \alpha_1$ where $\alpha_1 = 3.4 \times 10^{-6}$. The 3σ control limits for Moving Range control chart (MR chart) are given by Center Line= $\overline{MR}$ and UCL and LCL are given by $D_4\overline{MR}$ and $D_3\overline{MR}$ respectively. The values of constant $D_4 = 3.267$ and $D_3 = 0$ for a sample of size 2. The 6σ control limits for MR chart are given by $CL = \overline{MR}$; $UCL = I_{6\sigma}\overline{MR}$ and $LCL = 0$; Value of $I_{6\sigma} = 4.6506$. Thus $CL = \overline{MR}$, $UCL = 4.6505\overline{MR}$ and $LCL = 0$.

Using the data from Table 1, we get $USL = 53$, $LSL = 3$, $TL = 53 - 3 = 50$. Let $C_p = 2.5$, $\widehat{\sigma_{6\sigma}} = 3.3333$. Using this value we calculated the control limits for X chart as below: $CL_{6\sigma} = 13.89$, $UCL_{6\sigma} = 13.89 + 4.831(3.3333) = 29.9931723$, and $LCL_{6\sigma} = 13.89 - 4.831(3.3333) = -2.213 = 0$. Similarly control limits for MR chart are $CL_{6\sigma} = 15.388889$, $UCL_{6\sigma} = 4.6506(15.88889) = 71.567567$ and $LCL_{6\sigma} = 0$.



(a) MR Chart based on 6σ limits



(b) X Chart based on 6σ limits

Figure 6:

As from the above graphs we can say that the MR chart is under control but for X chart points 3, 7 and 18 are above $UCL_{6\sigma}$.

3. Conclusion

For the above data, both XMR and EWMA charts perform well. We can use these parameters for the length of stay data in hospital for cancer patients with same diagnosis in future. This shows that the treatment given to patients in this group has only natural variations and there are no special variations detected. When the XMR chart with 6σ limits are drawn, we observe that three points are out of control limits. The hospital is not fulfilling the six sigma initiative. The management needs to check for the lacuna in services rendered.

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