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Waiting Time Analysis Using Control Charts Based on Skewness and Kurtosis

Research Article

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Abstract: For (M/M/1) : (∞/FCFS) queue, the service or waiting time distribution do not conform to property of symmetry in such situation the traditional chart is improper to give satisfactory performance. In this paper, control charts based on skewness and kurtosis are used to determine the patience limits (i.e. control limits for waiting time) of customers arriving at a system for availing services. A comparative study is carried out and best control chart based on skewness and kurtosis for waiting time for queueing model is evaluated on the basis of performance measure false alarm rate. Also, simulation study is carried out to compare the performances of various control charts using lognormal distribution.

Keywords: Control limits, false alarm rate, kurtosis correction method, skewness correction method, skewness and kurtosis correction method, Shewhart method, weighted standard deviation method,(M/M/1) : (∞/FCFS) queueing system

1. Introduction

In this paper, (M/M/1) : (∞/FCFS) queue is under study, in which potential customers arrive according to a Poisson process with rate λ. There is a single server and the service times are independent and exponentially distributed with mean μ. The system uses the first come, first serve discipline and with infinite capacity. This means the customers are served in the order that they arrive to the system and they wait in a queue if the server is busy. The customers under consideration are impatient and either balk (i.e. not join the queue) or abandon the system after a random amount of time (which is referred to as their patience time) if their service has not begun by taking an overview of length of the existing queue or by sensing the amount of the time he/she has to wait. Consequence of this action is either the customer is lost or may retry. The basic performance measures for any queueing system are the average number of customers in queue and average waiting time. So, in any service system, a customer in a queue is curious in knowing how long he/she has to wait or will be delayed in queue while availing of service. If an arrival at a service system can obtain some information on the average wait or maximum waiting limit found in terms of control limits by constructing a suitable control chart for waiting time then, on that basis a customer can decide whether to stay or abandon. This information can enhance performance of the queueing system and thereby improve customer satisfaction. This will empower the organization. Thereby the customer can plan their future activities accordingly by taking into account the waiting limits.Thus controlling either balking or abandonment of customers which may otherwise result in loss of clientele, idleness of resources and money to the organization is of major concern.

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Control charts are among the most commonly used and powerful tools in statistical process control Derya and Canan [1]. In this paper, control charts are used to determine the patience limits (i.e. control limits for waiting time) of customers arriving at a system for availing services. In SPC, the objective of control charts is to determine if a process is in a statistical state or not. If not, then to bring out-of-control process into in-control and to maintain the process thereafter. Here the objective is to monitor the waiting time of a customer in a system. If the waiting time exceeds the upper limit, the customer abandons and if waiting time lies within the limits, the customer avails the service.

In practice, the service or waiting data may not conform to property of symmetry. For example, service time at a service counter like in hospitals depends on the type of ailments a patient suffers from, callers at the customer care depends on the type of query. Measurements coming from the processes often follow skewed distribution. This scenario makes the standard control chart result into a high false alarm rate.Four different methods for construction of control chart are suggested to deal with this problem, if the underlying distribution is not normal.

The remainder of the paper is organized as follows: Section 2 presents derivation of distribution of waiting time. Sections 3-8 is dedicated to the study of the control charts based on different methods for waiting time of customer in queue carried out numerically. Section 10 is for performance analysis of control charts. In Section 11 comparative study on the basis of performance measure of control charts based on different method is carried out to find the best performing control chart. The simulation study is presented in Section 12 to compare the performances of various control charts using lognormal distribution. Conclusion of the study is presented in last section.

2. Distribution of Waiting Time W_s for the M/M/1 Model Based on the FCFS Discipline

For the construction of control charts for random variable W_s, it is required to know the distribution of W_s. The probability density function of random variable W$_s$ depend on the queue discipline. Let random variable W_s denote the time spent waiting in the system (which includes both the waiting time and service time) by the customer [2]. But when considering individual waiting time, queue discipline must be specified. In this case, it is assumed to be first come, first served. Further, the waiting time random variable is part discrete and part continuous. For the most part, it is a continuous random variable, except that there is a non-zero probability that the delay will be zero, that is, a customer entering service immediately upon arrival. If there are n units in the system upon arrival with the given queue discipline, then

$$W_S \ = t_1' + t_2 + \cdots + t_{n+1}$$

where t_1' denotes the time needed by the customer who is actually in service to complete service and $t_2, \ldots, t_n$ are the service times of $n-1$ customers in the queue. The time t_{n+1} is the service time for the arriving customers. Let $f(W_S/n+1)$ be the conditional probability density function of W_s given n customers in the system ahead of the arriving customer. Since the service time distribution is exponential with parameter μ and has forgetfulness property. Thus t_1' is also exponential with parameter μ. In other words, suppose there are n units in the system upon arrival, then in order for the customer to go into service at a time between 0 and t, the distribution of the time required for n completions is independent of the time of the current arrival, this is because of memoryless property of exponential distribution and is the convolution of n exponential random variables which is an Erlang type n. Therefore, W_s is the sum of $(n+1)$ identically distributed and independent exponential random variable's, which is gamma distribution with parameters μ and $n+1$. Thus,

$$f\left(W_S\right) = \begin{cases} (\mu-\lambda)\, e^{-(\mu-\lambda)w_s}, & W_S > 0 \\ 0, & otherwise \end{cases} \tag{1}$$

2.1. Distributional Properties of Random Variable Waiting Time

The random variable W_S has exponential distribution with parameter $(\mu - \lambda)$. The distributional properties of r.v. W_S are displayed in Table 1.

$E(W_S)$	$V(W_S)$	γ_1	γ_2
$\frac{1}{\mu-\lambda}$	$\frac{1}{(\mu-\lambda)^2}$	2	6

Table 1. Distributional properties of r.v. W_S

The distribution function of random variable W_S is given by,

$$F(x) = P(W_S \leq x) = 1 - e^{-(\mu-\lambda)x}, \quad x > 0 \tag{2}$$

In the next sections, control limits for W_S are obtained by using control charts based on five methods.

3. Control Chart T_1: Shewhart Chart

The 3-σ control limits for random variable W_S are given by,

$$\left.\begin{aligned} UCL &= E[W_S] + \ 3 * \sqrt{V[\ W_S]} \\ CL &= E[W_S] \\ LCL &= E\left[W_S\right] - \ 3 * \sqrt{V[\ W_S]} \end{aligned}\right\} \tag{3}$$

where $E(W_S)$ and $V(W_S)$ are obtained in Table 1 respectively [3].

4. Control Chart T_2: Weighted Standard Deviation Method

As the waiting time distribution is positively skewed, it would be more appropriate to use the method of weighted standard deviation (WSD) [4]. This method makes no assumption about the population. In this control chart false alarm rate stays as close to the desired level as possible. It adjusts the control limits of a control chart according to the degree of skewness of the underlying population. For example, here the population is skewed to the right, so the distance of the upper control limit from the process mean will be larger than the distance of the lower limit from the process mean. This control chart is constructed by decomposing the standard deviation into two parts i.e. upper and lower deviation which is adjusted in accordance with the direction and degree of skewness. It provides asymmetric upper and lower control limits. As it is skewed to the right $P > \frac{1}{2}$ and $\sigma_U^W > \sigma_L^W$, the performance of standard control charts will be found to be poor than weighted standard deviation method. To get UCL by WSD method σ is multiplied by the factor $2P_{Ws}$ and LCL is multiplied by the factor $2(1 - P_{Ws})$, where, $P_{Ws} = P[W_S \leq E(W_S)]$. The control limits of this chart reduce to the standard charts if the underlying distribution is symmetric. Since for symmetric distribution $P_{Ws} = 0.5$.

5. Derivation and Definition of P_{W_s}

Let P_{Ws} denote the probability that random variable W_S will be less than or equal to its mean $E(W_S)$. It is known that the distribution of waiting time is exponential with parameter $\mu(1-\rho)$. Thus,

$$P_{W_S} = P[W_S \leq E(W_S)] = 0.632121 \tag{4}$$

As $P_{W_S} > 0.5$, it implies that the waiting time distribution is skewed to the right i.e. positively skewed. Hence the distance of the UCL, from the center line (CL) is larger than that of the LCL. There are more numbers of customer in lower range of waiting time in the queueing system. Also, P_{W_S} remains unaffected to variation in values of parameters λ, μ and traffic intensity. The control limits for random variable$W_S$are given by,

$$\left.\begin{aligned} UCL = E\left[W_s\right] + 3 * \sqrt{V\left(W_s\right)} * 2 * P_{W_S} \\ CL = E\left[W_s\right] \\ LCL = E\left[W_s\right] - 3 * \sqrt{V\left(W_s\right)} * 2 * \left(1 - P_{W_S}\right) \end{aligned}\right\} \tag{5}$$

where $E(W_S)$, $V\left(W_S\right)$ are as in Table 1 and P_{W_S} is as obtained by using expression (4) respectively [4].

6. Control Chart T_3: Skewness Correction(SC) Method

Construction of T_3 control chart is done by using method of skewness correction [5]. This method corrects the traditional Shewhart chart according to the skewness of random variable W_s. It gives asymmetric control limits using ± 3 standard deviations plus the same known function of the degree of skewness, $\frac{\frac{4}{3}\gamma_1}{1+0.2{\gamma_1}^2}$. This chart reduces to the Shewhart chart for symmetric distributions i.e. when $\gamma_1 = 0$. If the process distribution is closer to Weibull, lognormal ,Burr or binomial family, then simulation results shows that the SC control charts have Type I risk (i.e., probability of a false alarm) closer to 0.27% of the normal case [5]. Here, we have waiting time distribution as an exponential. For an exponential distribution with known mean, the control limits, and Type I risk, and also the Type II risk of the SC charts are found closer to those of the exact $\overline{X}$ and R charts than those of the WV and Shewhart charts.

The chart constructed by WSD method uses P_x to measure the degree of skewness, where P_x is the probability that the random variable X will be less than, or equal to, it's mean E(X). Sometimes this may not be valid. As in several distributions, P_x is close to 0.5. But, the distribution of X could be much skewed. Then the WSD method may be invalid and Type I risk can also be larger [5]. Here, SC method proposed by Chan and Cui [5] is suggested in construction of T_3 control charts, which takes into consideration the degree of skewness of the W_s distribution with no assumptions on the distribution.

The distribution of W_s is skewed to the right, γ_1 is greater than 0. Then the distance of the UCL from the CL is larger than that of the LCL from the CL. But the bandwidth of the control chart is always six. The skewness correction T_3 chart is based on the following limits when parameters are known:

$$\left.\begin{aligned} UCL = E\left(W_s\right) + \left(3 + c_4^*\right)\sqrt{V\left(W_s\right)} \\ CL = E\left(W_s\right) \\ LCL = E\left(W_s\right) + \left(-3 + c_4^*\right)\sqrt{V\left(W_s\right)} \end{aligned}\right\} \tag{6}$$

The constant c_4^* denote skewness correction and is given by $c_4^* = \frac{\frac{4}{3}\gamma_1}{1+0.2\gamma_1^2}$ and γ_1 is the skewness coefficient [5]. In this case, $c_4^* = \frac{\frac{4}{3}\gamma_1}{1+0.2{\gamma_1}^2} = 1.481481$, since $\gamma_1 = 2$. Substituting values of $E\left[Ws\right]$, $V[Ws]$ and c_4^* respectively in (6),we get UCL,CL and LCL.

Note: Here, LCL_{T_3} is set to zero, as it is found to be negative.

7. Control Chart T_4: Skewness and Kurtosis Correction Method

Construction of T_4 control chart is done by using method of skewness and kurtosis correction [6]. In this method both the degree of skewness and kurtosis is taken into account with no assumptions on the process distribution. It provides the control limits using three standard deviation with addition of the known function of skewness and kurtosis. This chart reduces to SC chart when $\gamma_2 = 0$, and further reduces to Shewhart chart when both $\gamma_1 = 0$ and $\gamma_2 = 0$. The skewness and kurtosis correction T_4 chart is based on the following limits when parameters are known are given by,

$$\left.\begin{array}{c} UCL = E\left[W_S\right] + \left(3 + \frac{\frac{4}{3}\gamma_1}{1+0.2\gamma_1^2} + \frac{\frac{3}{4}\gamma_2}{1+3|\gamma_2|}\right) * \sqrt{V[\ W_S]} \\ CL = E[W_S] \\ LCL = E\left[W_S\right] - \left(3 + \frac{\frac{4}{3}\gamma_1}{1+0.2\gamma_1^2} + \frac{\frac{3}{4}\gamma_2}{1+3|\gamma_2|}\right) * \sqrt{V[\ W_S]} \end{array}\right\} \tag{7}$$

where γ_1 denote coefficient of skewness; γ_2 denote coefficient of kurtosis.

8. Control Chart T_5: Kurtosis Correction Method

Construction of T_5 control chart is done by using method of kurtosis correction [7]. In this method control chart is constructed when the process distribution is symmetrical, but has a kurtosis greater than zero. It makes no assumptions on the functional form of underlying distribution. This method shifts the control limits to both sides by the same amount which is function of Kurtosis. When kurtosis is zero, the KC method control chart reduces to Shewhart control chart. The control limits of T_5 chart are [7],

$$\left.\begin{array}{c} UCL = E\left[W_S\right] + \left(3 + \frac{\gamma_2}{1+0.33\gamma_2}\right) * \sqrt{V[\ W_S]} \\ CL = E\left[W_S\right] \\ LCL = E\left[W_S\right] - \left(3 + \frac{\gamma_2}{1+0.33\gamma_2}\right) * \sqrt{V[\ W_S]} \end{array}\right\} \tag{8}$$

where γ_2 is coefficient of kurtosis.

9. Performance Measures

In this section performance measures of control chart are studied.

9.1. False Alarm Rate for Control Chart

Let α denote the probability of type-I, which is referred as false alarm rate (FAR). Then, $\alpha = \alpha_l + \alpha_u$ where α_l and α_u are the risk probabilities generated in the lower and upper tail, respectively i.e.

$$\alpha_u = P[W_S > UCL] = e^{-\mu(1-\rho)UCL}, \ \ UCL > 0 \ \text{ and}$$

$$\alpha_l = P\left[W_S < LCL\right] = 1 - e^{-\mu(1-\rho)LCL}, \ \ LCL > 0$$

Note: As LCL is negative, hence we take $LCL = 0$. Therefore $\alpha_{T_i} = (\alpha_u)_{T_i}, \ \ i = 1, ..., 5$ denote the FAR of T_i chart.

9.2. Average Run Length for Control Chart

Let $ARL_{T_i}, i = 1, ..., 5$ denote the average run length of T_i chart and is given by $ARL_{T_i} = \frac{1}{\alpha_{T_i}}, i = 1, ..., 5$ where α_{T_i} is the false alarm rate of T_i chart

10. Performance Analysis of Control Charts

To study the effect of traffic intensity on control limits, the same set of values of λ and μ were taken and ρ, CL, UCL, FAR and ARL were computed using control chart based on various methods and are displayed in Tables 2,3 and 4. These tables exhibit following results:

- As traffic intensity increases, control limits also increases.
- $(UCL)_{KC} > (UCL)_{SKC} > (UCL)_{SC} > (UCL)_{WSD} > (UCL)_{SHRT}$
- $(\alpha)_{KC} < (\alpha)_{SKC} < (\alpha)_{SC} < (\alpha)_{WSD} < (\alpha)_{SHRT}$. Because underlying distribution is positively skewed and highly leptokurtic. Therefore, UCL obtained from Shewhart method, weighted standard deviation, skewness correction and skewness and kurtosis correction are low, which results in high value for $(\alpha)_{SHRT}$, $(\alpha)_{WSD}$, $(\alpha)_{SC}$ and $(\alpha)_{SKC}$.
- Irrelevant of values of λ and μ, a constant value are observed for α_u and ARL. For T_5: Observe for $L = 3$, $\alpha_l = 0$ and $\alpha_u = 0.002446$. The performance measure, $ARL_{T_5} \approx 409$. This indicates that waiting time of customer will be plot outside the control limits every 409 customers, on average. For a process in control, we prefer the ARL to be large because an observation plotting outside the control limits represents a false alarm.
- $ARL_{T_5} > ARL_{T_4} > ARL_{T_3} > ARL_{T_2} > ARL_{T_1}$. It implies that performance of control chart T_5 is found to be better than performance of control charts T_4, T_3, T_2 and control chart T_1.

11. Comparative Study

In this section, the comparative study is carried out, through numerical analysis of the upper control limits and performance measures of the various methods considered in the study with Shewhart control chart and are displayed in various tables. From Table 2, it is concluded that kurtosis correction method is best in providing upper control limits of all the methods as it controls the effect of kurtosis which the underlying distribution has. It is observed that Shewhart chart has the lowest valued upper control limit of all, this is because it is based on the assumption of normality. In Table 3, false alarm rates are displayed for control chart obtained by various methods.

Thus, the control chart based on kurtosis correction method is superior to all other methods in providing upper control limit for random variable waiting time of customer in M/M/1 queueing system as the waiting time distribution is highly leptokurtic.

Hence, it can be concluded that for leptokurtic distributions, it will be appropriate to use control chart based on kurtosis correction method for achieving accuracy in control limits. Since, KC method in its construction of control limits corrects it for leptokurtic.

λ	μ	ρ	UCL_{SHRT}	UCL_{WSD}	UCL_{SC}	UCL_{SKC}	UCL_{KC}
11	38	0.289474	0.148148	0.177508	0.203018	0.21179	0.222719
8	22	0.363636	0.285714	0.342338	0.391534	0.408452	0.42953
19	43	0.441861	0.166667	0.199697	0.228395	0.238264	0.250559
24	48	0.5	0.166667	0.199697	0.228395	0.238264	0.250559
19	35	0.542857	0.25	0.299545	0.342593	0.357395	0.375839
21	37	0.567568	0.25	0.299545	0.342593	0.357395	0.375839
18	31	0.580645	0.307692	0.368671	0.421652	0.439871	0.462571
16	25	0.64	0.444444	0.532525	0.609053	0.635369	0.668158
8	12	0.666667	1	1.198182	1.37037	1.429581	1.503356
13	19	0.684211	0.666667	0.798788	0.91358	0.953054	1.002237
7	10	0.7	1.333333	1.597575	1.82716	1.906108	2.004474
15	20	0.75	0.8	0.958545	1.096296	1.143665	1.202685
14	18	0.777778	1	1.198182	1.37037	1.429581	1.503356
16	20	0.8	1	1.198182	1.37037	1.429581	1.503356
14	17	0.823529	1.333333	1.597575	1.82716	1.906108	2.004474

λ	μ	ρ	UCL_{SHRT}	UCL_{WSD}	UCL_{SC}	UCL_{SKC}	UCL_{KC}
21	25	0.84	1	1.198182	1.37037	1.429581	1.503356
17	19	0.894737	2	2.396363	2.740741	2.859162	3.006712
12	13	0.923077	4	4.792726	5.481481	5.718323	6.013423
15	16	0.9375	4	4.792726	5.481481	5.718323	6.013423
22	23	0.956522	4	4.792726	5.481481	5.718323	6.013423

Table 2. **Upper control limits of r.v. W_s obtained by using various methods**

Note: Control limits are expressed in minutes

$(\alpha_u)_{SHRT}$	$(\alpha_u)_{WSD}$	$(\alpha_u)_{SC}$	$(\alpha_u)_{SKC}$	$(\alpha_u)_{KC}$
0.018316	0.00829	0.004163	0.003285	0.002446

Table 3. **Comparison of FAR's of control charts obtained by various methods**

ARL_{SHRT}	ARL_{WSD}	ARL_{SC}	ARL_{SKC}	ARL_{KC}
54.59815	120.6298	240.2022	304.394	408.8805

Table 4. **Comparison of ARL's of control charts obtained by various methods**

12. Simulation and Comparison

A simulation study is conducted to compare the upper control limit (UCL) for waiting time of customer for (M/M/1): (∞/FCFS) system, obtained from control charts, based on different methods, classical Shewhart, weighted standard deviation(WSD), skewness correction(SC), skewness kurtosis correction(SKC) and kurtosis correction(KC). The simulated data is generated from lognormal distribution.

Note, the skewness γ_1 and kurtosis γ_2 of lognormal distribution are independent of the location parameter μ. Hence we take $\mu = 0$ and denote this distribution by lognormal (σ). In the application, assume the waiting time in system has lognormal distribution with location parameter equal to zero and scale parameter σ, where $\sigma = 0.75$. The steps of the simulation are,

Step 1. Construction of control chart.

(1.1) For sample size $n = 5$, generate a random subgroup from lognormal $(0, \sigma)$ distribution, where $\sigma = 0.75$ Using R software.

(1.2) Repeat Step 1.1, $k = 30$ times.

(1.3) Compute the sample mean $\overline{\overline{X}}$, $\overline{R}$, γ_1 and γ_2.

(1.4) Compute the control limits and false alarm rate of Shewhart (T_1), $WSD(T_2)$, $SC(T_3)$, $SKC(T_4)$ and $KC(T_5)$ charts.

$P(X \leq E(X))$	Mean	Range	Variance	Skewness	Kurtosis
0.613333	1.275644	1.925564	0.793784	1.663632	3.485207

Table 5. **Distributional properties of lognormal distribution(0,0.75)**

From Table 5, the underlying distribution is positively skewed and leptokurtic

Sample Number	Mean	Range
1	0.903746	1.736835
2	2.241626	4.511319
3	1.192032	1.157486
4	1.954647	1.597729
5	1.192876	1.418676
6	1.224972	1.313121
7	0.859493	1.842674
8	1.409145	3.615503
9	1.58536	2.221986
10	0.840678	0.704479
11	1.83004	2.120632
12	1.386482	2.33127
13	1.257965	3.166707
14	1.038803	1.232959
15	0.887862	1.499697
16	1.154682	1.079473
17	0.918269	1.322851
18	1.018968	0.475907
19	1.368213	1.118607
20	1.42549	2.982077
21	0.701097	1.541844
22	1.562116	1.622381
23	1.702025	2.795658
24	0.972562	1.422687
25	0.930072	1.754398
26	0.740289	1.374197
27	2.378788	3.581713
28	0.852353	1.066398
29	1.616178	2.685819
30	1.122501	2.471844

Table 6. **Sample mean and sample range**

Control chart	UCL	LCL	FAR
Shewhart	2.386695	0.164594	0.192175
WSD	2.753718	0.343815	0.155542
SC	2.820717	0.40914	0.14987
SKC	2.868086	0.366778	0.146023
KC	2.633167	-0.08188	0.166475

Table 7. **Control limits using different methods**

Note: Control limits are expressed in minutes

From Table 7, it is observed that upper control limit obtained from control chart based on skewness and kurtosis correction method is maximum of all the limits.As it takes into consideration correction for both skewness and kurtosis in its control limits.

13. Conclusion

The paper proposes performance wise for highly leptokurtic distribution use of control chart based on kurtosis correction (KC) method for obtaining UCL for waiting time of customer in M/M/1 queueing system (W_s). Since, it out performs charts based on all other methods as it considers in its construction of control limits kurtosis of underlying distribution of r.v. W_s . Whereas, on the basis of simulation, it can be concluded that, if the distribution is positively skewed and not highly leptokurtic, then it will be appropriate to use control chart based on skewness and kurtosis correction method for achieving accuracy in control limits. These control charts can be suggested for intimating system management for taking precautionary measure. R software was used in computation.

References

[1] K.Derya and H.Canan, *Control charts for skewed distributions: Weibull, gamma, and lognormal*, Metodoloskizvezki, 9(2)(2012), 95-106.

[2] D.Gross and C.M.Harris, *Fundamentals of queueing theory*, Third edition. A Wiley-Interscience Publication, (1998).

[3] D.C.Montgomery, *Statistical Quality Control: A Modern Introduction*, Sixth edition, Wiley-India Edition, India, (2010).

[4] Y.S.Chang and D.S.Bai, *Control charts for positively-skewed populations with weighted standard deviations*, Quality and Reliability Engineering International, 17(2001), 397-406.

[5] L.K.Chan and H.J.Cui, *Skewness correction* $\overline{X}$ *and R charts for skewed distributions*, Naval Research Logistics 50(6)(2003), 555-573.

[6] S.B.Wang, *Skewness and kurtosis correction for* $\overline{X}$ *and R control charts*, Institute of Statistics, National University of Kaohsiung, Kaohsiung, Taiwan 811 R.O.C,(2009).

[7] P.R.Tadikamalla and D.G.Popescu, *Kurtosis correction method for* $\overline{X}$ *and R control chartsfor long-tailed symmetrical distributions*, Naval Research Logistics, 54(2007), 371-383.