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Edge Degree Sequence of Isomophic Fuzzy Graphs

Research Article

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Abstract: In this paper, edge degree sequence of isomorphic fuzzy graphs are considered and some of its properties are studied. Also a sufficient condition for a fuzzy graph and its $\mu-$complement to have an identical edge degree sequence is given.

MSC: 05C07, 05C38.

Keywords: Degree of an edge, edge degree sequence of a fuzzy graph, edge degree sequence of isomorphic fuzzy graphs.

1. Introduction

The phenomena of uncertainty in real life situation was described in a mathematical framework by Zadeh in 1965. He also introduced the concept of fuzzy relations which has a widespread application in pattern recognition. K.R. Bhutani also introduced the concepts of weak, co-weak isomorphism and isomorphism between fuzzy graphs in [2]. In [9], the $\mu-$complement was discussed by A. Nagoorgani and J. Malarvizhi. Degree of an edge in a fuzzy graph is defined by K. Radha and N. Kumaravel in [7]. In [8] K.Radha and A.Rosemine introduced degree sequence of fuzzy graph. In this paper, we discussed about the edge degree sequence of isomorphic, co-weak and weak isomorphic fuzzy graphs.

2. Preliminaries

A summary of basic definitions is given , which are represented in [1–10]. A fuzzy graph G is a pair of functions $G:(\sigma,\mu)$ where σ is a fuzzy subset of a non empty set V and μ is a symmetric fuzzy relation on σ (ie) $\mu(xy)\leq\sigma(x)\wedge\sigma(y)\ \ \forall\, x,y\in V$. The underlying crisp graph of $G:(\sigma,\mu)$ is denote by $G^*:(V,E)$ where $E\subseteq V\times V$. In a fuzzy graph $G:(\sigma,\mu)$ degree of vertex $u\in V$ is $d(u)=\sum\limits_{u\neq v}\mu(uv)$. The minimum degree of G is $\delta(G)=\wedge\{d_G(u)/u\in V\}$, the maximum degree of G is $\Delta(G)=\vee\{d_G(u)/u\in V\}$. The complement of a fuzzy graph $G:(\sigma,\mu)$ is a fuzzy graph $G^c:(\sigma^c,\mu^c)$ where $\sigma^c=\sigma$ and $\mu^c(uv)=0$ if $\mu(uv)>0$, $\mu^c(uv)=\sigma(u)\wedge\sigma(v)$ otherwise. The $\mu-$complement of a fuzzy graph $G:(\sigma,\mu)$ is a fuzzy graph $G^c:(\sigma^c,\mu^c)$ where $\sigma^c=\sigma$ and $\mu^\mu(xy)=(\sigma(x)^\sigma(y))-\mu(xy),\ \ \forall\ xy\in E$. In a fuzzy graph $G:(\sigma,\mu)$ the degree of an edge $e=uv\in V$ is $d(uv)=d(u)+d(v)-2\mu(uv)$. A sequence of real numbers $(d_1,d_2,d_3,\ldots,d_n)$ with $d_1\geq d_2\geq d_3\geq\cdots\geq d_n$, where each d_i is the degree of an edge in G, is the edge degree sequence of a fuzzy graph G.

A sequence $\xi=(d_1,d_2,d_3,\ldots,d_n)$ of real numbers is said to be edge-fuzzy graphic sequence if there exists a graph G whose edges have degree d_i and G is called realization of ξ. A homomorphism of fuzzy graphs $h:G\to G'$ is a map $h:V\to V'$

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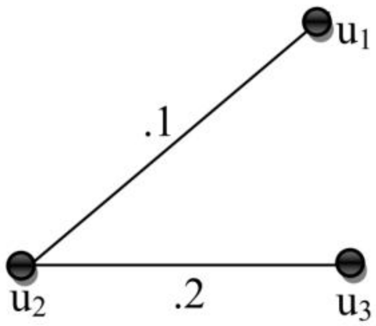

Figure 1. $G:(\sigma,\mu)$, **Edge degree sequence of G is (.2, .1)**

such that $\sigma(x) \leq \sigma'(h(x)), \forall\, x \in V, \mu(xy) \leq \mu'(h(x)h(y)), \forall\, x, y \in V$. A weak isomorphism of fuzzy graphs $h : G \to G'$ is a map $h : V \to V'$ which is a bijective homomorphism that satisfies $\sigma(x) = \sigma'(h(x)\ \ \forall\ \ x \in V, \mu(xy) \leq \mu'(h(x)h(y))\ \forall\, x, y \in V$. A co-weak isomorphism of fuzzy graphs $h : G \to G'$ is a map $h : V \to V'$ which is a bijective homomorphism that satisfies $\sigma(x) \leq \sigma'(h(x))\ \forall\, x \in V, \mu(xy) = \mu'(h(x)h(y))\ \forall\, x, y \in V$. An isomorphism $h : G \to G'$ is a bijective map $h : V \to V'$ that satisfies $\sigma(x) = \sigma'(h(x))\ \ \forall\, x \in V, \mu(xy) = \mu'(h(x)h(y))\ \forall\, x, y \in V$.

3. Edge Degree Sequence of Isomorphic Fuzzy Graphs

Theorem 3.1. *If G and G' are isomorphic fuzzy graphs, then the edge degree sequences of G and G' are same.*

Proof. Since $G : (\sigma, \mu)$ and $G' : (\sigma', \mu')$ are two isomorphic fuzzy graphs there exists a bijective map $h : V \to V'$ such that $\sigma(x) = \sigma'(h(x))\ \forall\, x \in V, \mu(xy) = \mu'(h(x), h(y))\ \forall\, x, y \in V$. Let uv be any edge of G such that $h(u) = z$ and $h(v) = x$. Then $h(u)h(v) = zx$ is an edge of G'. We have to prove that $d_G(uv) = d_{G'}(h(u)h(v))$.

Case i: $d_G(uv) = 0$.

Then no edge of G is adjacent to uv. Therefore $d_G(u) = d_G(v) = \mu(uv)$. Since G and G'are isomorphic fuzzy graphs, no edge of G' is adjacent to zx. Hence $d_{G'}(z) = d_{G'}(x) = \mu'(xz)$. Therefore $d_{G'}(h(u)h(v)) = d_{G'}(xz) = d_{G'}(x) + d_{G'}(z) - \mu'(xz) = 0 = d_G(uv)$.

Case ii: $d_G(uv) = k > 0$.

Since G and G' are isomorphic to each other and isomorphism preserves degree of the vertices, we have

$$\begin{aligned} d_G(uv) &= d_G(u) + d_G(v) - \mu(uv) \\ &= d_{G'}(h(u)) + d_{G'}(h(v)) - 2\mu'(h(u)h(v)) \\ &= d_{G'}(h(u)h(v))\ \ \forall\ \ uv \in E \end{aligned}$$

Since $uv \in E$ is arbitrarily chosen, $d_G(uv) = d_{G'}(h(u)h(v))\ \forall\ uv \in E$. Thus the edge degree sequences of G and G' are same. □

Remark 3.2. *The converse of the above Theorem 3.1 need not be true. That is, two fuzzy graphs with same edge degree sequence need not be isomorphic. In the following Figure 2, the edge degree sequence of both G and G' is (0.7, 0.7, 0.3, 0.3). But there is no bijective map from the vertex set of G to the vertex set of G' which carries v_1 with the same membership values. Hence G is not isomorphic to G'.*

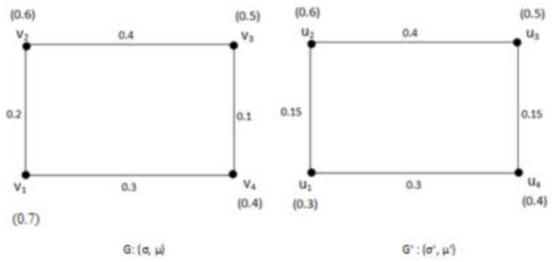

Figure 2.

Theorem 3.3. *Co-weak isomorphic fuzzy graphs preserves edge degree sequence.*

Proof. Since G is co-weak isomorphic to G', there is a bijective map $h : V \to V'$ such that $\sigma(x) \le \sigma'(h(x))\ \forall\ x \in V$, $\mu(xy) = \mu'(h(x), h(y))\ \forall\ x, y \in V$. Then proceeding as in Theorem3.1, $d_G(xy) = d_{G'}(h(x)h(y))\ \forall\ xy \in V$. Thus G and G' have identical edge degree sequences. □

Remark 3.4. *The converse of the above Theorem 3.3 need not be true. That is, two fuzzy graphs with same edge degree sequence need not be co-weak isomorphic. For example, in Figure 3, the edge degree sequence of both G and G' is (0.4, 0.3, 0.3). Under any bijective map $h : V \to V'$, at most two vertices satisfy the inequality $\sigma(x) \le \sigma'(h(x))$. Hence G is not co-weak isomorphic with G'.*

Figure 3.

Remark 3.5. *Weak isomorphic fuzzy graphs need not preserve the edge degree sequence. For example consider the following Figure 4. the bijective map $h : V_1 \to V_2$ defined by $h(v_1) = u_1, h(v_2) = u_5,\ h(v_3) = u_4, h(v_4) = u_3,\ h(v_5) = u_2$ satisfies $\sigma_1(v_i) = \sigma_2(h(v_i))\ \forall\ v_i \in V,\ \mu_1(v_iv_j) \le \mu_2(h(v_i)h(v_j))\ \forall\ v_iv_j \in E$. Hence G_1 is weak isomorphic with G_2. But the edge degree sequence (0.7, 0.7, 0.6, 0.6, 0.6) of G_1 is different from the edge degree sequence (1, 0.9, 0.8, 0.7, 0.6) of G_2.*

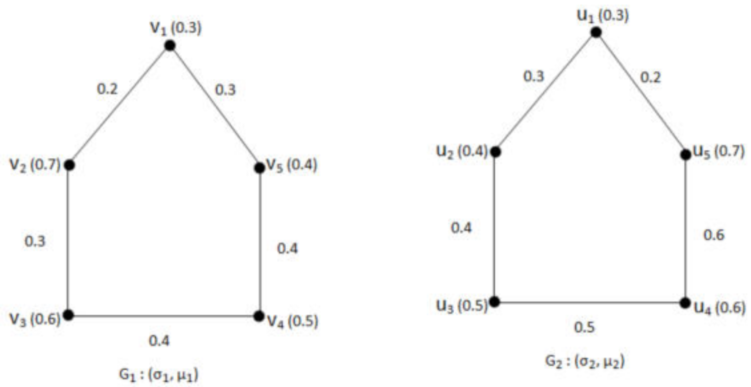

Figure 4.

Theorem 3.6. *Let $G : (\sigma, \mu)$ be a fuzzy graph. If G is self complementary fuzzy graph, then the edge degree sequences of G and G^c are identical.*

Proof. Since G is a self complementary fuzzy graph, G is isomorphic to G^c. Hence the result follows from Theorem 3.1. □

Remark 3.7. *The converse of the above Theorem 3.6 need not be true. In Figure 5 the edge degree sequence of both G and G^c is (0.4, 0.4, 0.4, 0.4, 0.4). But G is not isomorphic to G^c. Hence G is not self complementary fuzzy graph.*

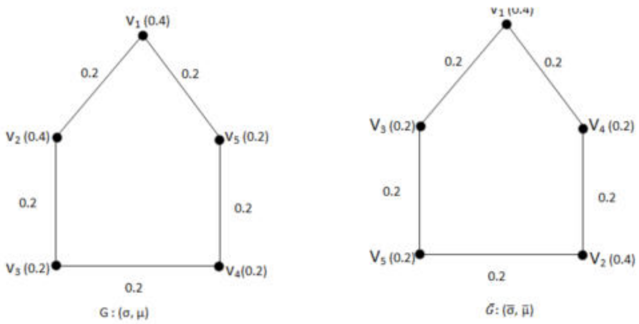

Figure 5.

Theorem 3.8. *Let $G : (\sigma, \mu)$ be a fuzzy graph such that $\mu(xy) = \frac{1}{2}(\sigma(x) \wedge \sigma(y))\ \forall\, xy \in E$. Then G and G^μ have same edge degree sequence.*

Proof. Here $\mu(xy) = \frac{1}{2}(\sigma(x) \wedge \sigma(y))\ \forall\, (xy) \in E$. By the definition of $\mu-$complement of G, we have

$$\begin{aligned}
\mu^\mu(xy) &= (\sigma(x) \wedge \sigma(y)) - \mu(xy) \\
&= (\sigma(x) \wedge \sigma(y)) - \frac{1}{2}(\sigma(x) \wedge \sigma(y)),\ \forall\, xy \in E \\
&= \frac{1}{2}((\sigma(x) \wedge \sigma(y)),\ \forall\, xy \in E \\
\mu^\mu(xy) &= \mu(xy),\ \ \forall\, xy \in E
\end{aligned}$$

Therefore $d_{G^\mu}(xy) = d_G(xy),\ \ \forall\ \ xy \in E$. Hence G and G^μ have same edge degree sequence. □

Remark 3.9. *The converse of the Theorem 3.7 need not be true. For example in Figure 6, the edge degree sequence of G and G^μ is (0.75, 0.75, 0.75, 0.75). But $\mu(uv) \neq \frac{1}{2}(\sigma(u) \wedge \sigma(v))$ for all the edges of G except zw.*

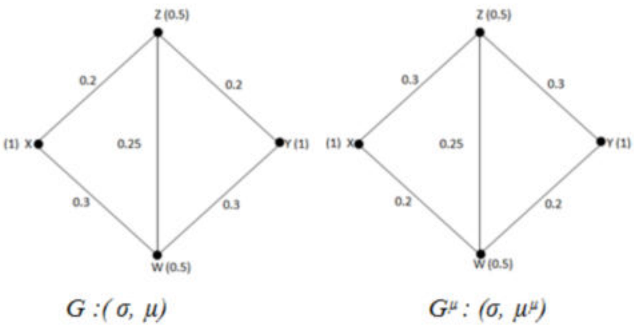

Figure 6.

Corollary 3.10. *If $G : (\sigma, \mu)$ is a fuzzy graph such that $\mu(xy) = \frac{1}{2}(\sigma(x) \wedge \sigma(y))$ $\forall$ $x, y \in V$, then G and G^{μ} have same edge degree sequence.*

Theorem 3.11. *Let $G : (\sigma, \mu)$ be a connected fuzzy graph such that such that $\sigma(v) = c$ $\forall$ $v \in V$. Then $d_G(e) = d_{G^*}(e)c$, $\forall$ $e \in E$ if and only if G is an effective fuzzy graph.*

Proof. Let $G : (\sigma, \mu)$ be a fuzzy graph such that $\sigma(v) = c$, $\forall$ $v \in V$. Assume that $d_G(e) = d_{G^*}(e).c \,\forall\, e \in E$. Suppose that G is not an effective fuzzy graph. Then there is an edge uv such that $\mu(uv) < \sigma(u) \wedge \sigma(v) = c$. Since G is connected, uv is adjacent to at least one other edge, say, uw. Then

$$\begin{aligned} d_G(uw) &= \sum_{x \neq w} \mu\,(ux) + \sum_{y \neq u} \mu\,(yw) \\ &< \sum_{x \neq w} c + \sum_{y \neq u} c < d_{G^*}(uw).c \end{aligned}$$

which is a contradiction. Hence G is effective.

Conversely assume that $G : (\sigma, \mu)$ is an effective fuzzy graph. Then $\mu(uv) = \sigma(u) \wedge \sigma(v) = c$, $\forall$ $uv \in E$ and $d_G(uv) = cd_{G^*}(uv)$, $\forall$ $uv \in E$. □

Theorem 3.12. *If $G : (\sigma, \mu)$ be a fuzzy graph such that μ is constant function with constant value c, then $d_G(e) = cd_{G^*}(e)$, $\forall$ $e \in E$.*

Proof. Since $\mu(uv) = c$ $\forall$ $uv \in E$, $d_G(uv) = \sum_{uv \in E} c = c.d_{G^*}(uv)$ □

The following theorem gives the upper and lower bounds on the sum of all the terms of the edge degree sequence.

Theorem 3.13. *If $G : (\sigma, \mu)$ be a fuzzy graph such that $r = \wedge\{\mu(e);\ e \text{ in } E\}$ and $s = \vee\{\mu(e);\ e \text{ in } E\}$, then $\sum_{e \in E} d_{G^*}(e).r \leq \sum_{e \in E} d_G(e) \leq \sum_{e \in E} d_{G^*}(e).s$.*

Proof. Here we have $r \leq \mu(uv) \leq s$ $\forall$ $uv \in E$. Therefore $d_{G^*}(uv).r \leq d_G(uv) \leq d_{G^*}(uv).s \Rightarrow \sum_{uv \in E} d_{G^*}(uv).r \leq \sum_{uv \in E} d_G(uv) \leq \sum_{uv \in V} d_{G^*}(uv).s$. □

Corollary 3.14. *If $G : (\sigma, \mu)$ is an $k-$edge regular fuzzy graph with q edges such that $r = \wedge\{\mu(e);\ e \text{ in } E\}$ and $s = \vee\{\mu(e);\ e \text{ in } E\}$, then $qkr \leq \sum_{e \in E} d_G(e) \leq qks$.*

4. Conclusion

In fuzzy graph theory degree of an edge is a parameter of a graph. In this paper we made a study about that parameter in isomorphic fuzzy graphs and in the $\mu-$complement of a fuzzy graph.

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