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# Weakly $\mathcal{I}_g$-$\omega$-closed Sets

**Research Article**

**K.Banumathi[1], I.Rajasekaran[2] and O.Ravi[2]*

1 Department of Mathematics, St. Aloysius College of Education, Royappanpatti, Theni, Tamil Nadu, India.
2 Department of Mathematics, P.M.Thevar College, Usilampatti, Madurai, Tamil Nadu, India.

**Abstract:** In this paper, another generalized class of $\tau^\star$ called weakly $\mathcal{I}_g$-$\omega$-closed sets is studied and the notion of weakly $\mathcal{I}_g$-$\omega$-open sets in ideal topological spaces is also studied. The relationships of weakly $\mathcal{I}_g$-$\omega$-closed sets with various other sets are investigated.

**MSC:** 54A05, 54A10.

**Keywords:** $\tau^\star$, generalized class, weakly $\mathcal{I}_g$-$\omega$-closed set, ideal topological space, generalized closed set, $\mathcal{I}_g$-$\omega$-closed set, $pre^\star_{\mathcal{I}}$-closed set, $pre^\star_{\mathcal{I}}$-open set.



## 1. Introduction

The first step of generalizing closed sets (briefly, $g$-closed sets) was done by Levine in 1970 [13]. He defined a subset $S$ of a topological space $(X, \tau)$ to be $g$-closed if its closure is contained in every open superset of $S$. As the weak form of $g$-closed sets, the notion of weakly $g$-closed sets was introduced and studied by Sundaram and Nagaveni [18]. Sundaram and Pushpalatha [19] introduced and studied the notion of strongly $g$-closed sets, which are weaker than closed sets and stronger than $g$-closed sets. Park and Park [17] introduced and studied the notion of mildly $g$-closed sets, which is properly placed between the class of strongly $g$-closed sets and the class of weakly $g$-closed sets. Moreover, the relations with other notions directly or indirectly connected with $g$-closed sets were investigated by them. In 1999, Dontchev et al. [6] studied the notion of generalized closed sets in ideal topological spaces called $\mathcal{I}_g$-closed sets. In 2008, Navaneethakrishnan and Paulraj Joseph studied some characterizations of normal spaces via $\mathcal{I}_g$-open sets [14]. In 2013, Ekici and Ozen [8] introduced a generalized class of $\tau^\star$ in ideal topological spaces. The notion of $\omega$-open sets in topological spaces introduced by Hdeib [9] has been studied in recent years by a good number of researchers like Noiri et al [16], Al-Omari and Noorani [3, 4] and Khalid Y. Al-Zoubi [11]. The main aim of this paper is to study another generalized class of $\tau^\star$ called weakly $\mathcal{I}_g$-$\omega$-open sets in ideal topological spaces. Moreover, this generalized class of $\tau^\star$ generalize $\mathcal{I}_g$-$\omega$-open sets and weakly $\mathcal{I}_g$-$\omega$-open sets. The relationships of weakly $\mathcal{I}_g$-$\omega$-closed sets with various other sets are discussed.

## 2. Preliminaries

Throughout this paper, $\mathbb{R}$ (resp. $\mathbb{Q}$, $(\mathbb{R}-\mathbb{Q})$, $(\mathbb{R}-\mathbb{Q})_-$ and $(\mathbb{R}-\mathbb{Q})_+$) denotes the set of real numbers (resp. the set of rational numbers, the set of irrational numbers, the set of negative irrational numbers and the set of positive irrational

* *E-mail: siingam@yahoo.com*

numbers). In this paper, $(X, \tau)$ represents a topological space on which no separation axioms are assumed unless explicitly stated. The closure and interior of a subset $G$ of a topological space $(X, \tau)$ will be denoted by $cl(G)$ and $int(G)$, respectively.

**Definition 2.1.** *A subset $G$ of a topological space $(X, \tau)$ is said to be*

*(1). g-closed [13] if $cl(G) \subseteq H$ whenever $G \subseteq H$ and $H$ is open in $X$;*

*(2). g-open [13] if $X \backslash G$ is g-closed;*

*(3). weakly g-closed [18] if $cl(int(G)) \subseteq H$ whenever $G \subseteq H$ and $H$ is open in $X$;*

*(4). strongly g-closed [19] if $cl(G) \subseteq H$ whenever $G \subseteq H$ and $H$ is g-open in $X$.*

**Definition 2.2** ([21]). *In a topological space $(X, \tau)$, a point $p$ in $X$ is called a condensation point of a subset $H$ if for each open set $U$ containing $p$, $U \cap H$ is uncountable.*

**Definition 2.3** ([9]). *A subset $H$ of a topological space $(X, \tau)$ is called $\omega$-closed if it contains all its condensation points. The complement of an $\omega$-closed set is called $\omega$-open.*

It is well known that a subset $W$ of a topological space $(X, \tau)$ is $\omega$-open if and only if for each $x \in W$, there exists $U \in \tau$ such that $x \in U$ and $U - W$ is countable. The family of all $\omega$-open sets, denoted by $\tau_\omega$, is a topology on $X$, which is finer than $\tau$. The interior and closure operator in $(X, \tau_\omega)$ are denoted by $int_\omega$ and $cl_\omega$ respectively.

**Lemma 2.4** ([9]). *Let $H$ be a subset of a topological space $(X, \tau)$. Then*

*(1). $H$ is $\omega$-closed in $X$ if and only if $H = cl_\omega(H)$.*

*(2). $cl_\omega(X \backslash H) = X \backslash int_\omega(H)$.*

*(3). $cl_\omega(H)$ is $\omega$-closed in $X$.*

*(4). $x \in cl_\omega(H)$ if and only if $H \cap G \neq \phi$ for each $\omega$-open set $G$ containing $x$.*

*(5). $cl_\omega(H) \subseteq cl(H)$.*

*(6). $int(H) \subseteq int_\omega(H)$.*

**Lemma 2.5** ([11]). *If $A$ is an $\omega$-open subset of a space $(X, \tau)$, then $A - C$ is $\omega$-open for every countable subset $C$ of $X$.*

An ideal $\mathcal{I}$ on a topological space $(X, \tau)$ is a nonempty collection of subsets of $X$ which satisfies

(1). $A \in \mathcal{I}$ and $B \subseteq A$ imply $B \in \mathcal{I}$ and

(2). $A \in \mathcal{I}$ and $B \in \mathcal{I}$ imply $A \cup B \in \mathcal{I}$ [12].

Given a topological space $(X, \tau)$ with an ideal $\mathcal{I}$ on $X$, if $\mathbb{P}$(X) is the set of all subsets of $X$, a set operator $(.)^\star : \mathbb{P}(X) \to \mathbb{P}(X)$, called a local function [12] of A with respect to $\tau$ and $\mathcal{I}$ is defined as follows: for $A \subseteq X$, $A^\star(\mathcal{I}, \tau) = \{x \in X | U \cap A \notin \mathcal{I}$ for every $U \in \tau(x)\}$ where $\tau(x) = \{U \in \tau | x \in U\}$. A Kuratowski closure operator $cl^\star(.)$ for a topology $\tau^\star(\mathcal{I}, \tau)$, called the $\star$-topology and finer than $\tau$, is defined by $cl^\star(A) = A \cup A^\star(\mathcal{I}, \tau)$ [20]. We will simply write $A^\star$ for $A^\star(\mathcal{I}, \tau)$ and $\tau^\star$ for $\tau^\star(\mathcal{I}, \tau)$. If $\mathcal{I}$ is an ideal on $X$, then $(X, \tau, \mathcal{I})$ is called an ideal topological space. On the other hand, $(A, \tau_A, \mathcal{I}_A)$ where $\tau_A$ is the relative topology on $A$ and $\mathcal{I}_A = \{A \cap J : J \in \mathcal{I}\}$ is an ideal topological subspace for the ideal topological space $(X, \tau, \mathcal{I})$ and $A \subseteq X$ [10]. For a subset $A \subseteq X$, $cl^\star(A)$ and $int^\star(A)$ will, respectively, denote the closure and the interior of $A$ in $(X, \tau^\star)$.

**Proposition 2.6** ([1]). *Let $(X, \tau, \mathcal{I})$ be an ideal topological space and $H$ a subset of $X$. If $\mathcal{I} = \{\phi\}$(resp. $\mathbb{P}(X)$), then $H^\star = cl(H)$(resp. $\phi$).*

**Lemma 2.7** ([10]). *Let $(X, \tau, \mathcal{I})$ be an ideal topological space and $A$, $B$ subsets of $X$. Then the following properties hold:*

*(1).* $A \subseteq B \Rightarrow A^\star \subseteq B^\star$,

*(2).* $A^\star = cl(A^\star) \subseteq cl(A)$,

*(3).* $(A^\star)^\star \subseteq A^\star$,

*(4).* $(A \cup B)^\star = A^\star \cup B^\star$,

*(5).* $(A \cap B)^\star \subseteq A^\star \cap B^\star$.

**Definition 2.8.** *A subset $G$ of an ideal topological space $(X, \tau, \mathcal{I})$ is said to be*

*(1).* $\mathcal{I}_g$*-closed [6, 14] if* $G^\star \subseteq H$ *whenever* $G \subseteq H$ *and* $H$ *is open in* $(X, \tau, \mathcal{I})$.

*(2).* $pre^\star_{\mathcal{I}}$*-open [7] if* $G \subseteq int^\star(cl(G))$.

*(3).* $pre^\star_{\mathcal{I}}$*-closed [7] if* $X \backslash G$ *is* $pre^\star_{\mathcal{I}}$*-open (or)* $cl^\star(int(G)) \subseteq G$.

*(4).* $\mathcal{I}_g$-$\omega$*-closed [5] if* $G^\star \subseteq H$ *whenever* $G \subseteq H$ *and* $H$ *is* $\omega$*-open in* $(X, \tau, \mathcal{I})$.

*(5).* $\mathcal{I}$-$R$ *closed [2] if* $G = cl^\star(int(G))$.

*(6).* $\star$*-closed [10] if* $G = cl^\star(G)$ *or* $G^\star \subseteq G$.

**Remark 2.9** ([8]). *In any ideal topological space, every $\mathcal{I}$-$R$ closed set is $\star$-closed but not conversely.*

**Definition 2.10** ([15]). *In an ideal topological space $(X, \tau, \mathcal{I})$, a subset $G$ of $X$ is said to be weakly $\mathcal{I}_g$-$\omega$-closed if $(int(G))^\star \subseteq H$ whenever $G \subseteq H$ and $H$ is $\omega$-open in $X$.*

**Example 2.11** ([15]). *In $\mathbb{R}$ with the topology $\tau = \{\phi, \mathbb{R}, \mathbb{Q}\}$ and $\mathcal{I} = \{\phi\}$,*

*(1). For $G = \mathbb{R} - \mathbb{Q}$, if $H$ is any $\omega$-open subset of $\mathbb{R}$ such that $G \subseteq H$, then $(int(G))^\star = \phi^\star = \phi \subseteq H$ and hence $G$ is weakly $\mathcal{I}_g$-$\omega$-closed in $X$.*

*(2). $K = \mathbb{Q} \subseteq \mathbb{Q}$, $\mathbb{Q}$ being $\omega$-open whereas $(int(\mathbb{Q}))^\star = \mathbb{Q}^\star = cl(\mathbb{Q}) = \mathbb{R} \nsubseteq \mathbb{Q}$ which implies $K = \mathbb{Q}$ is not weakly $\mathcal{I}_g$-$\omega$-closed in $X$.*

**Definition 2.12** ([15]). *A subset $G$ in an ideal topological space $(X, \tau, \mathcal{I})$ is said to be weakly $\mathcal{I}_g$-$\omega$-open if $X \backslash G$ is weakly $\mathcal{I}_g$-$\omega$-closed.*

**Theorem 2.13** ([15]). *In an ideal topological space $(X, \tau, \mathcal{I})$, a subset $G$ of $X$ is weakly $\mathcal{I}_g$-$\omega$-closed $\Leftrightarrow (int(G))^\star \subseteq G$.*

**Proposition 2.14** ([15]). *In an ideal topological space $(X, \tau, \mathcal{I})$, every $\mathcal{I}_g$-$\omega$-closed set is weakly $\mathcal{I}_g$-$\omega$-closed but not conversely.*

**Theorem 2.15** ([5]). *In an ideal topological space $(X, \tau, \mathcal{I})$, a subset $G$ is $\star$-closed if and only if it is $\mathcal{I}_g$-$\omega$-closed.*

**Theorem 2.16** ([5]). *In an ideal topological space $(X, \tau, \mathcal{I})$, every $\mathcal{I}_g$-$\omega$-closed set is $\mathcal{I}_g$-closed but not conversely.*

## 3. Properties of Weakly $\mathcal{I}_g$-$\omega$-closed Sets

**Theorem 3.1.** *In an ideal topological space $(X, \tau, \mathcal{I})$, for a subset $G$ of $X$, the following properties are equivalent.*

*(1). $G$ is weakly $\mathcal{I}_g$-$\omega$-closed;*

*(2). $(int(G))^\star \backslash G = \phi$;*

*(3). $cl^\star(int(G)) \backslash G = \phi$;*

*(4). $cl^\star(int(G)) \subseteq G$;*

*(5). $G$ is $pre^\star_{\mathcal{I}}$-closed.*

*Proof.*

(1) $\Leftrightarrow$ (2) $G$ is weakly $\mathcal{I}_g$-$\omega$-closed $\Leftrightarrow$ $(int(G))^\star \subseteq G$ by Theorem 2.13 $\Leftrightarrow$ $(int(G))^\star \backslash G = \phi$.

(2) $\Leftrightarrow$ (3) $(int(G))^\star \backslash G = \phi \Leftrightarrow cl^\star(int(G)) \backslash G = ((int(G))^\star \cup int(G)) \backslash G = [(int(G))^\star \backslash G] \cup [int(G) \backslash G] = int(G) \backslash G = \phi$.

(3) $\Leftrightarrow$ (4) $cl^\star(int(G)) \backslash G = \phi \Leftrightarrow cl^\star(int(G)) \subseteq G$.

(4) $\Leftrightarrow$ (5) $cl^\star(int(G)) \subseteq G \Leftrightarrow G$ is $pre^\star_{\mathcal{I}}$-closed by (3) of Definition 2.8. □

**Theorem 3.2.** *In an ideal topological space $(X, \tau, \mathcal{I})$, if $G$ is weakly $\mathcal{I}_g$-$\omega$-closed, then $G \cup (X - (int(G))^\star)$ is weakly $\mathcal{I}_g$-$\omega$-closed.*

*Proof.* Since $G$ is weakly $\mathcal{I}_g$-$\omega$-closed, $(int(G))^\star \subseteq G$ by Theorem 2.13. Then $X - G \subseteq X - (int(G))^\star$ and $G \cup (X - G) \subseteq G \cup (X - (int(G))^\star)$. Thus $X \subseteq G \cup (X - (int(G))^\star)$ and so $G \cup (X - (int(G))^\star) = X$. Hence $G \cup (X - (int(G))^\star)$ is weakly $\mathcal{I}_g$-$\omega$-closed. □

**Theorem 3.3.** *In an ideal topological space $(X, \tau, \mathcal{I})$, the following properties are equivalent:*

*(1). $G$ is a $\star$-closed and an open set,*

*(2). $G$ is a $\mathcal{I}$-$R$ closed and an open set,*

*(3). $G$ is a weakly $\mathcal{I}_g$-$\omega$-closed and an open set.*

*Proof.*

(1) $\Rightarrow$ (2): Since $G$ is $\star$-closed and open, $G = cl^\star(G)$ and $G = int(G)$. Thus $G = cl^\star(int(G))$ and $G = int(G)$. Hence $G$ is $\mathcal{I}$-$R$ closed and open.

(2) $\Rightarrow$ (3): Since $G$ is $\mathcal{I}$-$R$ closed and open, $G = cl^\star(int(G)) = (int(G))^\star \cup int(G) = (int(G))^\star \cup G$. Thus $(int(G))^\star \subseteq G$. By Theorem 2.13, $G$ is weakly $\mathcal{I}_g$-$\omega$-closed and open.

(3) $\Rightarrow$ (1): Since $G$ is weakly $\mathcal{I}_g$-$\omega$-closed, $(int(G))^\star \subseteq G$ by Theorem 2.13. Again $G$ is open implies $G^\star = (int(G))^\star \subseteq G$. Thus $G$ is $\star$-closed and open. □

**Theorem 3.4.** *In an ideal topological space $(X, \tau, \mathcal{I})$, every closed set is weakly $\mathcal{I}_g$-$\omega$-closed.*

*Proof.* If A is closed, then A is $\star$-closed and $\mathcal{I}_g$-$\omega$-closed by Theorem 2.15. By Proposition 2.14, A is weakly $\mathcal{I}_g$-$\omega$-closed. □

**Remark 3.5.** *The converse of Theorem 3.4 is not true follows from the following example.*

**Example 3.6.** *In $\mathbb{R}$ with the topology $\tau = \{\phi, \mathbb{R}, \{1\}\}$ and the ideal $\mathcal{I} = \mathbb{P}(\mathbb{R})$, for $G = \{1\}$, if $H$ is any $\omega$-open subset of $\mathbb{R}$ such that $G \subseteq H$, then $(int(G))^\star = G^\star = \phi \subseteq H$ and hence $G$ is weakly $\mathcal{I}_g$-$\omega$-closed. But $G$ is not closed for $cl(G) = \mathbb{R} \nsubseteq G$.*

**Remark 3.7.** *The following example shows that the concepts of $\mathcal{I}_g$-closedness and weakly $\mathcal{I}_g$-$\omega$-closedness are independent of each other.*

**Example 3.8.** *In $\mathbb{R}$ with the topology $\tau = \{\phi, \mathbb{R}, \mathbb{R}-\mathbb{Q}\}$ and the ideal $\mathcal{I} = \{\phi\}$,*

*(1). for $G = (\mathbb{R}-\mathbb{Q})_+$ = the set of positive irrationals, $int(G) = \phi$. So $(int(G))^\star = \phi^\star = \phi \subseteq G$ and thus $G$ is weakly $\mathcal{I}_g$-$\omega$-closed by Theorem 2.13. But $G$ is not $\mathcal{I}_g$-closed for $G \subseteq (\mathbb{R}-\mathbb{Q}) \in \tau$ whereas $G^\star = cl(G) = \mathbb{R} \nsubseteq \mathbb{R}-\mathbb{Q}$.*

*(2). for $G = (\mathbb{R}-\mathbb{Q}) \cup \{1\}$, $\mathbb{R}$ is the only open set containing $G$. Hence $G$ is $\mathcal{I}_g$-closed. But $G$ is not weakly $\mathcal{I}_g$-$\omega$-closed for $(int(G))^\star = (\mathbb{R}-\mathbb{Q})^\star = cl(\mathbb{R}-\mathbb{Q}) = \mathbb{R} \nsubseteq G$.*

**Remark 3.9.** *In an ideal topological space $(X, \tau, \mathcal{I})$, the following relations hold for a subset $G$ of $X$.*

$$\begin{array}{ccccc} \text{closed} & \longrightarrow & \text{weakly } \mathcal{I}_g\text{-}\omega\text{-closed} & & \\ \downarrow & & \uparrow & & \\ \star\text{-closed} & \longleftrightarrow & \mathcal{I}_g\text{-}\omega\text{-closed} & \longrightarrow & \mathcal{I}_g\text{-closed} \end{array}$$

*Where $A \longleftrightarrow B$ means $A$ implies and is implied by $B$ and $A \rightarrow B$ means $A$ implies $B$ but not conversely.*

**Theorem 3.10.** *In an ideal topological space $(X, \tau, \mathcal{I})$, $A^\star$ is always weakly $\mathcal{I}_g$-$\omega$-closed for every subset $A$ of $X$.*

*Proof.* Since $(A^\star)^\star \subseteq A^\star$ [10], $A^\star$ is $\star$-closed. Hence $A^\star$ is $\mathcal{I}_g$-$\omega$-closed by Theorem 2.15 and weakly $\mathcal{I}_g$-$\omega$-closed by Proposition 2.14. □

## 4. Further Properties

**Theorem 4.1.** *In an ideal topological space $(X, \tau, \mathcal{I})$, if $G$ is weakly $\mathcal{I}_g$-$\omega$-closed and $H$ is a subset such that $G \subseteq H \subseteq cl^\star(int(G))$, then $H$ is weakly $\mathcal{I}_g$-$\omega$-closed.*

*Proof.* Since $G$ is weakly $\mathcal{I}_g$-$\omega$-closed, $cl^\star(int(G)) \subseteq G$ by (4) of Theorem 3.1. Thus by assumption, $G \subseteq H \subseteq cl^\star(int(G)) \subseteq G$. Then $G = H$ and so $H$ is weakly $\mathcal{I}_g$-$\omega$-closed. □

**Corollary 4.2.** *Let $(X, \tau, \mathcal{I})$ be an ideal topological space. If $G$ is a weakly $\mathcal{I}_g$-$\omega$-closed set and an open set, then $cl^\star(G)$ is weakly $\mathcal{I}_g$-$\omega$-closed.*

*Proof.* Since $G$ is open in X, $G \subseteq cl^\star(G) = cl^\star(int(G))$. $G$ is weakly $\mathcal{I}_g$-$\omega$-closed implies $cl^\star(G)$ is weakly $\mathcal{I}_g$-$\omega$-closed by Theorem 4.1. □

**Theorem 4.3.** *In an ideal topological space $(X, \tau, \mathcal{I})$, a nowhere dense subset is weakly $\mathcal{I}_g$-$\omega$-closed.*

*Proof.* If $G$ is a nowhere dense subset in $X$ then $int(cl(G)) = \phi$. Since $int(G) \subseteq int(cl(G))$, $int(G) = \phi$. Hence $(int(G))^\star = \phi^\star = \phi \subseteq G$. Thus, $G$ is weakly $\mathcal{I}_g$-$\omega$-closed in $(X, \tau, \mathcal{I})$ by Theorem 2.13. □

**Remark 4.4.** *The converse of Theorem 4.3 is not true in general as shown in the following example.*

**Example 4.5.** *In $\mathbb{R}$ with the topology $\tau = \{\phi, \mathbb{R}, \mathbb{R}-\mathbb{Q}\}$ and $\mathcal{I} = \mathbb{P}(\mathbb{R})$, for $G = \mathbb{R}-\mathbb{Q}$, $(int(G))^\star = G^\star = \phi \subseteq G$. Hence $G = \mathbb{R}-\mathbb{Q}$ is weakly $\mathcal{I}_g$-$\omega$-closed by Theorem 2.13. On the other hand, $int(cl(G)) = int(\mathbb{R}) = \mathbb{R} \neq \phi$ and thus $G = \mathbb{R}-\mathbb{Q}$ is not nowhere dense in $X$.*

**Remark 4.6.** *In an ideal topological space $(X, \tau, \mathcal{I})$, the intersection of two weakly $\mathcal{I}_g$-$\omega$-closed subsets is weakly $\mathcal{I}_g$-$\omega$-closed.*

*Proof.* Let A and B be weakly $\mathcal{I}_g$-$\omega$-closed subsets in $(X, \tau, \mathcal{I})$. Then $(int(A))^\star \subseteq A$ and $(int(B))^\star \subseteq B$ by Theorem 2.13. Also $[int(A \cap B)]^\star \subseteq [int(A)]^\star \cap [int(B)]^\star \subseteq A \cap B$. This implies that $A \cap B$ is weakly $\mathcal{I}_g$-$\omega$-closed by Theorem 2.13. □

**Remark 4.7.** *In an ideal topological space $(X, \tau, \mathcal{I})$, the union of two weakly $\mathcal{I}_g$-$\omega$-closed subsets need not be weakly $\mathcal{I}_g$-$\omega$-closed.*

**Example 4.8.** *In $\mathbb{R}$ with the topology $\tau = \{\phi, \mathbb{R}, \mathbb{R} - \mathbb{Q}\}$ and ideal $\mathcal{I} = \{\phi\}$, for $A = (\mathbb{R} - \mathbb{Q})_+ =$ the set of positive irrationals and $B = (\mathbb{R} - \mathbb{Q})_- =$ the set of negative irrationals, $int(A) = \phi$ and $int(B) = \phi$ respectively. So $(int(A))^\star = \phi^\star = \phi \subseteq A$ and thus $A$ is weakly $\mathcal{I}_g$-$\omega$-closed by Theorem 2.13. Similarly $B$ is also weakly $\mathcal{I}_g$-$\omega$-closed. But $int(A \cup B) = int(\mathbb{R} - \mathbb{Q}) = \mathbb{R} - \mathbb{Q}$. So $[int(A \cup B)]^\star = (\mathbb{R} - \mathbb{Q})^\star = cl(\mathbb{R} - \mathbb{Q}) = \mathbb{R} \nsubseteq \mathbb{R} - \mathbb{Q} = A \cup B$. Hence $A \cup B$ is not weakly $\mathcal{I}_g$-$\omega$-closed.*

**Theorem 4.9.** *In an ideal topological space $(X, \tau, \mathcal{I})$, a subset $G$ is weakly $\mathcal{I}_g$-$\omega$-open if and only if $G \subseteq int^\star(cl(G))$.*

*Proof.* $G$ is weakly $\mathcal{I}_g$-$\omega$-open $\Leftrightarrow$ $X \backslash G$ is weakly $\mathcal{I}_g$-$\omega$-closed $\Leftrightarrow$ $X \backslash G$ is $pre^\star_{\mathcal{I}}$-closed by (5) of Theorem 3.1 $\Leftrightarrow$ $G$ is $pre^\star_{\mathcal{I}}$-open $\Leftrightarrow$ $G \subseteq int^\star(cl(G))$. □

**Theorem 4.10.** *In an ideal topological space $(X, \tau, \mathcal{I})$, if the subset $G$ is weakly $\mathcal{I}_g$-$\omega$-closed, then $cl^\star(int(G)) \backslash G$ is weakly $\mathcal{I}_g$-$\omega$-open in $(X, \tau, \mathcal{I})$.*

*Proof.* Since $G$ is weakly $\mathcal{I}_g$-$\omega$-closed, $cl^\star(int(G)) \backslash G = \phi$ by (3) of Theorem 3.1. Thus $cl^\star(int(G)) \backslash G$ is weakly $\mathcal{I}_g$-$\omega$-open in $(X, \tau, \mathcal{I})$. □

**Theorem 4.11.** *In an ideal topological space $(X, \tau, \mathcal{I})$, if $G$ is weakly $\mathcal{I}_g$-$\omega$-open, then $int^\star(cl(G)) \cup (X - G) = X$.*

*Proof.* Since $G$ is weakly $\mathcal{I}_g$-$\omega$-open, $G \subseteq int^\star(cl(G))$ by Theorem 4.9. So $(X - G) \cup G \subseteq (X - G) \cup int^\star(cl(G))$ which implies $X = (X - G) \cup int^\star(cl(G))$. □

**Theorem 4.12.** *In an ideal topological space $(X, \tau, \mathcal{I})$, if $G$ is weakly $\mathcal{I}_g$-$\omega$-open and $H$ is a subset such that $int^\star(cl(G)) \subseteq H \subseteq G$, then $H$ is weakly $\mathcal{I}_g$-$\omega$-open.*

*Proof.* Since $G$ is weakly $\mathcal{I}_g$-$\omega$-open, $G \subseteq int^\star(cl(G))$ by Theorem 4.9. By assumption $int^\star(cl(G)) \subseteq H \subseteq G$. This implies $G \subseteq int^\star(cl(G)) \subseteq H \subseteq G$. Thus $G = H$ and so $H$ is weakly $\mathcal{I}_g$-$\omega$-open. □

**Corollary 4.13.** *In an ideal topological space $(X, \tau, \mathcal{I})$, if $G$ is a weakly $\mathcal{I}_g$-$\omega$-open set and a closed set, then $int^\star(G)$ is weakly $\mathcal{I}_g$-$\omega$-open.*

*Proof.* Let $G$ be a weakly $\mathcal{I}_g$-$\omega$-open set and a closed set in $(X, \tau, \mathcal{I})$. Then $int^\star(cl(G)) = int^\star(G) \subseteq int^\star(G) \subseteq G$. Thus, by Theorem 4.12, $int^\star(G)$ is weakly $\mathcal{I}_g$-$\omega$-open in $(X, \tau, \mathcal{I})$. □

**Definition 4.14.** *A subset $H$ of an ideal topological space $(X, \tau, \mathcal{I})$ is called a $W_{\mathcal{I}}$-set if $H = M \cup N$ where $M$ is $\omega$-closed and $N$ is $pre^\star_{\mathcal{I}}$-open.*

**Proposition 4.15.** *Every $pre^\star_{\mathcal{I}}$-open (resp. $\omega$-closed) set is a $W_{\mathcal{I}}$-set.*

**Remark 4.16.** *The separate converses of Proposition 4.15 are not true in general as shown in the following example.*

**Example 4.17.**

*(1). Let $\mathbb{R}, \tau$ and $\mathcal{I}$ be as in Example 2.11 and $G = \mathbb{R} - \mathbb{Q}$. Since $G$ is closed, it is $\omega$-closed and hence a $W_{\mathcal{I}}$-set. But $int^\star(cl(G)) = int^\star(G) = \mathbb{R} \backslash cl^\star(\mathbb{Q}) = \mathbb{R} \backslash cl(\mathbb{Q}) = \mathbb{R} \backslash \mathbb{R} = \phi \nsupseteq G$. Hence $G = \mathbb{R} - \mathbb{Q}$ is not $pre^\star_{\mathcal{I}}$-open.*

*(2). In Example 4.5, for $G = \mathbb{R} - \mathbb{Q}$, $int^\star(cl(G)) = int^\star(\mathbb{R}) = \mathbb{R} \supseteq G$. Thus $G$ is $pre^\star_\mathcal{I}$-open and hence a $W_\mathcal{I}$-set. But $G$ is not $\omega$-closed for any $x \in \mathbb{Q}$ is a condensation point of $G$ and $x \notin G$.*

**Remark 4.18.** *The following example shows that the concepts of $pre^\star_\mathcal{I}$-openness and $\omega$-closedness are independent of each other.*

**Example 4.19.** *In Example 4.17(1), $G = \mathbb{R} - \mathbb{Q}$ is $\omega$-closed but not $pre^\star_\mathcal{I}$-open.*

*In Example 4.17(2), $G = \mathbb{R} - \mathbb{Q}$ is $pre^\star_\mathcal{I}$-open but not $\omega$-closed.*

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