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Fixed Point Theorems in Generalized Intuitionistic Fuzzy Metric Spaces by Occasionally Weakly Compatible Maps

Research Article

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Abstract: In this paper we proved fixed point theorems in generalized intuitionistic fuzzy metric spaces by using occasionally weakly compatible maps.

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1. Introduction

The concept of fuzzy sets was introduced by Zadeh [12] following the concept of fuzzy sets, fuzzy metric spaces have been introduced by Kramosil and Michlek [4] and George and Veeramani [3] modified the notion of fuzzy metric spaces with the help of continuous t-norms. As a generalization of fuzzy sets, Atanassove [2] introduced and studied the concept of intuitionistic fuzzy sets. Park [6] using the idea of intuitionistic fuzzy sets defined the notion of intuitionistic fuzzy metric spaces with the help of continuous t-norm and continuous t-conorms as a generalized of fuzzy metric spaces, George and Veeramani [3] showed that every metric induces an intuitionistic fuzzy metric, every fuzzy metric space in an intuitionsitic fuzzy metric space. The concept of compatible maps introduced by Kramosil and Michalek [4] and weakly compatible maps in fuzzy metric space is generalized by A. Al. Thagafi and Nasser Shahzad [9] by introducing the concept of occasionally weakly compatible mappings (OWC). In this paper we have proved fixed point theorems in generalized intuitionistic fuzzy metric spaces by using occasionally weakly compatible maps.

2. Preliminaries

Definition 2.1. *A binary operation* $* : [0,1] \times [0,1]$ *is a continuous t-norm if it satisfies the following condition.*

(1). $*$ *is associative and commutative,*

(2). $*$ *is continuous,*

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(3). $a * 1 = a$ *for all* $a \in [0, 1]$,

(4). $a * b < c * d$ *whenever* $a \leq c$ *and* $b \leq d$ *for each* $a, b, c, d \in [0, 1]$.

Two typical example of a continuous t-norm are $a * b = ab$ and $a * b = \min\{a, b\}$.

Definition 2.2. *A binary operation* $\diamondsuit : [0, 1] \times [0, 1] \longrightarrow [0, 1]$ *is a continuous t-conorm if it satisfies the following conditions:*

(1). $\diamondsuit$ *is associative and commutative,*

(2). $\diamondsuit$ *is continuous,*

(3). $a \diamondsuit 0 = a$ *for all* $a \in [0, 1]$,

(4). $a \diamondsuit b \leq c \diamondsuit d$ *whenever* $a \leq c$ *and* $b \leq d$, *for each* $a, b, c, d \in [0, 1]$.

Two typical examples of a continuous t-conorm are $a \diamondsuit b = \min\{1, a + b\}$ and $a \diamondsuit b = \max\{a, b\}$.

Definition 2.3. *A 5-tuple* $(X,\ \mathcal{M},\ \mathcal{N},\ *,\ \diamondsuit)$ *is called an generalized intuitionistic fuzzy metric space if* X *is an arbitrary (non-empty) set,* $*$ *is a continuous t-norm, a continuous t-conorm and* $\mathcal{M}$, $\mathcal{N}$ *are fuzzy sets on* $X^3 \times (0, \infty)$, *satisfying the following conditions:*
For each $x, y, z, a \in X$ *and* $t, s > 0$.

(a) $\mathcal{M}(x, y, z, t) + \mathcal{N}(x, y, z, t) \leq 1$,

(b) $\mathcal{M}(x, y, z, t) > 0$,

(c) $\mathcal{M}(x, y, z, t) = 1$ *if and only if* $x = y = z$,

(d) $\mathcal{M}(x, y, z, t) = \mathcal{M}(p\{x, y, z\}, t)$, *where* p *is a permutation function,*

(e) $\mathcal{M}(x, y, z, a, t) * \mathcal{M}(a, z, z, s) \leq M(x, y, z, t + s)$,

(f) $\mathcal{M}(x, y, z, .) : (0, \infty) \to [0, 1]$ *is continuous,*

(g) $\mathcal{N}(x, y, z, t) > 0$,

(h) $\mathcal{N}(x, y, z, t) = 0$, *if and only if* $x = y = z$,

(i) $\mathcal{N}(x, y, z, t) = \mathcal{N}(p\{x, y, z\}, t)$ *where* p *is a permutation function,*

(j) $\mathcal{N}(x, y, z, a, t) \diamondsuit \mathcal{N}(a, z, z, s) \geq \mathcal{N}(x, y, z, t + s)$,

(k) $\mathcal{N}(x, y, z) : (0, \infty) \to [0, 1]$ *is continuous.*

Then $(\mathcal{M},\ \mathcal{N})$ *is called an generalized intuitionistic fuzzy metric on* X.

Lemma 2.4. *Let* $(X,\ \mathcal{M},\ \mathcal{N},\ *,\ \diamondsuit)$ *be an generalized intuitionistic fuzzy metric space. Then* $\mathcal{M}(x, y, z, t)$ *and* $\mathcal{N}(x, y, z, t)$ *are non-decreasing with respect to* t, *for all* x, y, z *in* X.

Definition 2.5. *Let* $(X,\ \mathcal{M},\ \mathcal{N}, *,\ \diamondsuit)$ *be a generalized intuitionistic fuzzy metric space and* $\{x_n\}$ *be a sequence in* X.

(a). $\{x_n\}$ *is said to be converges to a point* $x \in X$ *if* $\lim_{n\to\infty} \mathcal{M}(x, x, x_m, t) = 1$ *and* $\lim_{n\to\infty} \mathcal{N}(x, x, x_m, t) = 0$ *for all* $t > 0$.

(b). $\{x_n\}$ *is called Cauchy sequence if* $\lim_{n\to\infty} \mathcal{M}(x_{n+P},\ x_{n+P},\ x_n,\ t) = 1$ *and* $\mathcal{N}(x_{n+P},\ x_{n+P},\ x_n,\ t) = 0$ *for all* $t > 0$ *and* $P > 0$.

(c). A Intuitionistic fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

Definition 2.6. *Two self mappings f and g of intuitionistic fuzzy metric space $(X,\ \mathcal{M},\ \mathcal{N},\ *,\ \diamondsuit)$ are called compatible if $\lim_{n\to\infty} \mathcal{M}(fgx_n,\ gfx_n,\ \ gfx_n,\ t) = 1$ and $\lim_{n\to\infty} \mathcal{N}(fgx_n,\ gfx_n,\ gfx_n,\ t) = 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = x$, for some $x \in X$.*

Definition 2.7. *Two self maps A and B are called occasionally weakly compatible if there is point $x \in X$ which is a coincidence point of A and B at which A and B commute.*

Lemma 2.8. *Let X be a set A and B OWC self maps of X. If A and B have a unique point of coincidence $w = Ax = Bx$, then w is the unique common fixed point of A and B.*

3. Complete Generalized Intuitionistic Fuzzy Metric Spaces

Theorem 3.1. *Let $(X,\ \mathcal{M},\ \mathcal{N}, *,\ \diamondsuit)$ be the complete generalized intuitionistic fuzzy metric space and let A, B, S, T be self mapping of X. Let the pairs (A, S) and (B, T) be OWC and $k > 1$, then*

$$\mathcal{M}(Ax,\ By,\ By,\ kt) \le \min\left\{ \begin{array}{c} \left(\begin{array}{c} \mathcal{M}(Sx,\ Ty,\ Ty,\ t),\ \mathcal{M}(Sx,\ Ax,\ Ax,\ t),\mathcal{M}(Ty,\ By,\ By,\ t), \\ \mathcal{M}(Sx,\ By,\ By,\ t),\mathcal{M}(Ty,\ Ax,\ Ax,\ t) \end{array} \right) \\ \frac{(a\ \mathcal{M}(Ax,\ Ty,\ Ty,\ t) + b\ \mathcal{M}(By,\ Sx,\ Sx,\ t) + c\ \mathcal{M}(Sx,\ Ty,\ Ty,\ t))}{a+b+c} \\ \cdot \frac{1+\mathcal{M}(Ax,\ Sx,\ Sx,\ t)}{2} \end{array} \right\}$$

$$\mathcal{N}(Ax,\ By,\ By,\ kt) \ge \max\left\{ \begin{array}{c} \left(\begin{array}{c} \mathcal{N}(Sx,\ Ty,\ Ty,\ t),\mathcal{N}(Sx,\ Ax,\ Ax,\ t),\mathcal{N}(Ty,\ By,\ By,\ t), \\ \mathcal{N}(Sx,\ By,\ By,\ t),\mathcal{N}(Ty,\ Ax,\ Ax,\ t) \end{array} \right) \\ \frac{(a\ \mathcal{N}(Ax,\ Ty,\ Ty,\ t) + b\ \mathcal{N}(By,\ Sx,\ Sx,\ t) + c\ \mathcal{N}(Sx,\ Ty,\ Ty,\ t))}{a+b+c} \\ \cdot \frac{1+\mathcal{N}(Ax,\ Sx,\ Sx,\ t)}{2} \end{array} \right\} \quad (1)$$

for all $x, y \in X$ and $t > 0$ such that $Aw = Sw = w$ and a unique point $z \in X$ such that $Bz = Tz = z$. Moreover $z = w$, so that there is a unique common fixed point of A, B, S and T.

Proof. Let the pairs (A, S) and (B, T) are OWC so there are points $x, y \in X$ such that $Ax = Sx$ and $By = Ty$, We claim that $Ax = By$. If not then by inequality (1)

$$\mathcal{M}(Ax,\ By,\ By,\ kt)\ \le \min\left\{ \begin{array}{c} \left(\begin{array}{c} \mathcal{M}(Sx,\ Ty,\ Ty,\ t),\ \ \mathcal{M}(Sx,\ Ax,\ Ax,\ t),\mathcal{M}(Ty,\ By,\ By,\ t), \\ \mathcal{M}(Sx,\ By,\ By,\ t)\,\mathcal{M}(Ty,\ Ax,\ Ax,\ t) \end{array} \right) \\ \frac{(a\ \mathcal{M}(Ax,\ Ty,\ Ty,\ t) + b\ \mathcal{M}(By,\ Sx,\ Sx,\ t) + c\ \mathcal{M}(Sx,\ Ty,\ Ty,\ t))}{a+b+c} \\ \cdot \frac{1+\mathcal{M}\ (Ax,\ Sx,\ Sx,\ t)}{2} \end{array} \right\}$$

$$\mathcal{M}(Ax,\ By,\ By,\ kt) \le \min\{\mathcal{M}(Ax,\ Ty,\ Ty,\ t),\quad 1,\quad 1,\quad \mathcal{M}(Ax,\ By,\ By,\ t),\mathcal{M}(By,\ Ax,\ Ax,\ t),\ 1\}$$

$$\mathcal{M}(Ax,\ By,\ By,\ kt) \le\ \mathcal{M}(Ax, By, By, t) \quad \text{and}$$

$$\mathcal{N}(Ax,\ By,\ By,\ kt)\ \ge\ \max\left\{ \begin{array}{c} \left(\begin{array}{c} \mathcal{N}(Sx,\ Ty,\ Ty,\ t),\mathcal{N}(Sx,\ Ax,\ Ax,\ t),\mathcal{N}(Ty,\ By,\ By,\ t), \\ \mathcal{N}(Sx,\ By,\ By,\ t),\mathcal{N}(Ty,\ Ax,\ Ax,\ t) \end{array} \right) \\ \frac{(a\ \mathcal{N}(Ax,\ Ty,\ Ty,\ t) + b\ \mathcal{N}(By,\ Sx,\ Sx,\ t) + c\ \mathcal{N}(Sx,\ Ty,\ Ty,\ t))}{a+b+c} \\ \cdot \frac{1+\mathcal{N}(Ax,\ Sx,\ Sx,\ t)}{2} \end{array} \right\}$$

$$\mathcal{N}(Ax,\ By,\ By,\ kt)\ \geq\ \max\{\mathcal{N}(Ax,\ Ty,\ Ty,\ t),\quad 0,\quad 0,\ \ \mathcal{N}(Ax,\ By,\ By,\ t),\mathcal{N}(By,\ Ax,\ Ax,\ t),\ 0\}$$

$$\mathcal{N}(Ax,\ By,\ By,\ kt)\ \geq \mathcal{N}(Ax,By,By,t).$$

Then by Lemma 2.8, $Ax = By$. Suppose that there is another point z such that $Az = Sz$. Then by inequality (1), we have $Az = Sz = By = Ty$ so $Ax = Az$ and $w = Ax = Sx$ is the unique point of coincidence of A and S. By Lemma 2.8, w is the only common point of A and S. Similarly, there is a unique point $z \in X$ such that $z = Bz = Tz$. Assume that $w \neq z$, then by (1), $\mathcal{M}(w,\ z,\ z,\ kt) = \mathcal{M}(Aw,\ Bz,\ Bz,\ kt)$

$$\mathcal{M}(Ax,By,By,\ kt) \leq\ \min\left\{\begin{array}{c}\left(\begin{array}{c}\mathcal{M}(Sx,Ty,\ Ty,\ t),\ \ \mathcal{M}(Sx,\ Ax,\ Ax,\ t),\\ \mathcal{M}(Ty,\ \ By,\ \ By,\ \ t),\mathcal{M}(Ty,\ \ Ax,\ \ Ax,\ \ t)\end{array}\right)\\ \dfrac{(a\ \mathcal{M}(Ax,\ Ty,\ Ty,\ t)+b\ \mathcal{M}(By,\ Sx,\ Sx,\ t)+c\ \mathcal{M}(Sx,\ Ty,\ Ty,\ t))}{\frac{a+b+c}{\cdot\ \frac{1+\mathcal{M}(Ax,\ Sx,\ Sx,\ t)}{2}}}\end{array}\right\}$$

$$\mathcal{M}(Aw,Bz,Bz,\ kt) \leq\ \min\left\{\begin{array}{c}\left(\begin{array}{c}\mathcal{M}(Sw,Tz,\ Tz,\ \ t),\ \ \ \mathcal{M}(Sw,\ Aw,\ Aw,\ t),\mathcal{M}(Tz,\ Bz,\ Bz,\ t),\\ \mathcal{M}(Sw,Bz,Bz,\ t)\,\mathcal{M}(Tz,Aw,Aw,t)\end{array}\right)\\ \dfrac{(a\ \mathcal{M}(Aw,Tz,Tz,\ t)+b\ \mathcal{M}(Bz,Sw,Sw,t)+c\ \mathcal{M}(Sw,\ Tz,\ Tz,\ t))}{\frac{a+b+c}{\cdot\ \frac{1+\mathcal{M}(Aw,\ Sw,\ Sw,\ t)}{2}}}\end{array}\right\}$$

$$\mathcal{M}\ \ (w,\ z,\ z,kt) \leq\ \min\{\mathcal{M}(w,\ z,\ z,\ t),\ 1,\ 1,\mathcal{M}(Aw,Bz,Bz,t),\mathcal{M}(Bz,Aw,Aw,t),1\}$$

$\mathcal{M}(w,\ z,\ z,\ kt) \leq \mathcal{M}(w,z,z,t)$ and $\mathcal{N}(w,\ z,\ z,\ kt) = \mathcal{N}(Aw,\ Bz,\ Bz,\ kt)$

$$\mathcal{N}(Ax,By,By,\ kt) \geq\ \max\left\{\begin{array}{c}\left(\begin{array}{c}\mathcal{N}(Sx,Ty,\ Ty,\ t),\ \ \mathcal{N}(Sx,\ Ax,\ Ax,\ t),\\ \mathcal{N}(Ty,\ \ By,\ \ By,\ \ t),\ \ \mathcal{N}(Ty,\ \ Ax,\ \ Ax,\ \ t)\end{array}\right)\\ \dfrac{(a\ \mathcal{N}(Ax,\ Ty,\ Ty,\ t)+b\ \mathcal{N}(By,\ Sx,\ Sx,\ t)+c\ \mathcal{N}(Sx,\ Ty,\ Ty,\ t))}{\frac{a+b+c}{\cdot\ \frac{1+\mathcal{N}(Ax,\ Sx,\ Sx,\ t)}{2}}}\end{array}\right\}$$

$$\mathcal{N}(Aw,Bz,Bz,\ kt) \geq\ \max\left\{\begin{array}{c}\left(\begin{array}{c}\mathcal{N}(Sw,Tz,\ Tz,\ \ t),\ \ \ \mathcal{N}(Sw,\ Aw,\ Aw,\ t),\mathcal{N}(Tz,\ Bz,\ Bz,\ t),\\ \mathcal{N}(Sw,Bz,Bz,\ t),\mathcal{N}(Tz,Aw,Aw,t)\end{array}\right)\\ \dfrac{(a\ \mathcal{N}(Aw,Tz,Tz,\ t)+b\ \mathcal{N}(Bz,Sw,Sw,t)+c\ \mathcal{N}(Sw,\ Tz,\ Tz,\ t))}{\frac{a+b+c}{\cdot\ \frac{1+\mathcal{N}(Aw,\ Sw,\ Sw,\ t)}{2}}}\end{array}\right\}$$

$$\mathcal{N}\ \ (w,\ z,\ z,\ kt) \geq\ \max\{\mathcal{N}(w,\ z,\ z,\ t),\ 0,\ 0,\mathcal{N}(Aw,\ Bz,\ Bz,\ t),\mathcal{N}(Bz,\ Aw,\ Aw,\ t),0\}$$

$$\mathcal{N}(w,\ z,\ z,\ kt) \geq \mathcal{N}(w,\ z,\ z,\ t).$$

Then by Lemma 2.8. Therefore $w = z$, z is a common fixed point of A, B, S and T.

Uniqueness: Let u be another common fixed point of A, B, S and T. Then put $x = z$ and $y = u$ in (1),

$$\mathcal{M}(Az,\ Bu,\ Bu,\ kt) \leq \min\left\{\begin{array}{c}\left(\begin{array}{c}\mathcal{M}(Sz,\ Tu,\ Tu,\ t),\mathcal{M}(Sz,\ Az,\ Az,\ t),\mathcal{M}(Tu,\ Bz,\ Bz,\ t),\\ \mathcal{M}(Sz,\ Bu,\ Bu,\ t),\ \mathcal{M}(Tu,\ Az,\ Az,\ t)\end{array}\right)\\ \dfrac{(a\ \mathcal{M}(Az,\ Tu,\ Tu,\ t)+b\ \mathcal{M}(Bu,\ Sz,\ Sz,\ t)+c\ \mathcal{M}(Sz,\ Tu,\ Tu,\ t))}{\frac{a+b+c}{\cdot\ \frac{1+\mathcal{M}(Az,\ Sz,\ Sz,\ t)}{2}}}\end{array}\right\}$$

$$\mathcal{M}(z,\ u,\ u,\ kt) \leq\ \min\{\mathcal{M}(z,\ u,\ u,\ t),\ \ 1,\ \ 1,\ \ \mathcal{M}(z,\ u,\ u,\ t),\ \ \ \mathcal{M}(u,\ z,\ z,t),\ 1\}$$

$\mathcal{M}(Az,\ Bu,\ Bu,\ kt) \leq \mathcal{M}(z,\ u,\ u,t)$ and

$$\mathcal{N}(Az, Bu, Bu, kt) \geq \max\left\{\begin{array}{c}\left(\begin{array}{c}\mathcal{N}(Sz, Tu, Tu, t), \mathcal{N}(Sz, Az, Az, t), \mathcal{N}(Tu, Bz, Bz, t), \\ \mathcal{N}(Sz, Bu, Bu, t), \mathcal{N}(Tu, Az, Az, t)\end{array}\right) \\ \frac{(a\,\mathcal{N}(Az, Tu, Tu, t) + b\,\mathcal{N}(Bu, Sz, Sz, t) + c\,\mathcal{N}(Sz, Tu, Tu, t))}{a+b+c} \\ \cdot \frac{1+\mathcal{N}(Az, Sz, Sz, t)}{2}\end{array}\right\}$$

$$\mathcal{N}(z, u, u, kt) \geq \max\{\mathcal{N}(z, u, u, t), 0, 0, \mathcal{N}(z, u, u, t), \mathcal{N}(u, z, z, t), 0\}$$

$\mathcal{N}(Az, Bu, Bu, kt) \geq \mathcal{N}(z, u, u, t)$. Then by Lemma 2.8, $z = u$. □

Theorem 3.2. *Let $(X, \mathcal{M}, \mathcal{N}, *, \diamondsuit)$ be the complete generalized intuitionistic fuzzy metric space and let A, B, S, T be self mapping of X. Let the pairs (A, S) and (B, T) be OWC and $k > 1$ and $\alpha + \beta = 1$, then*

$$\mathcal{M}(Ax, By, By, kt) \leq \min\left\{\begin{array}{c}\mathcal{M}(Sx, Ty, Ty, t), \\ \mathcal{M}(Sx, Ax, Ax, t), \\ \mathcal{M}(Ty, By, By, t), \\ \mathcal{M}(Sx, By, By, t), \\ \mathcal{M}(Ty, Ax, Ax, t), \\ \left\{\begin{array}{c}\alpha\,\mathcal{M}(Sx, By, By, t) + \\ \beta min\left\{\begin{array}{c}\mathcal{M}(By, Ty, Ty, t), \\ \mathcal{M}(Sx, Ax, Ax, t), \\ \mathcal{M}(Sx, Ty, Ty, t)\end{array}\right\}\end{array}\right\}\end{array}\right\} \text{ and}$$

$$\mathcal{N}(Ax, By, By, kt) \geq \max\left\{\begin{array}{c}\mathcal{N}(Sx, Ty, Ty, t), \\ \mathcal{N}(Sx, Ax, Ax, t), \\ \mathcal{N}(Ty, By, By, t), \\ \mathcal{N}(Sx, By, By, t), \\ \mathcal{N}(Ty, Ax, Ax, t), \\ \left\{\begin{array}{c}\alpha\,\mathcal{N}(Sx, By, By, t) + \\ \beta max\left\{\begin{array}{c}\mathcal{N}(By, Ty, Ty, t), \\ \mathcal{N}(Sx, Ax, Ax, t), \\ \mathcal{N}(Sx, Ty, Ty, t)\end{array}\right\}\end{array}\right\}\end{array}\right\} \quad (2)$$

for all $x, y \in X$ and $t > 0$ such that $Aw = Sw = w$ and a unique point $z \in X$ such that $Bz = Tz = z$. Moreover $z = w$, so that there is unique fixed point of A, B, S and T

Proof. Let the pairs (A, S) and (B, T) are OWC so there are points x, y∈ X such that Ax = Sx and By = Ty. We claim that Ax = By. If not then by inequality (2),

$$\mathcal{M}(Ax, By, By, kt) \leq \min\left\{\begin{array}{c}\mathcal{M}(Ax, By, By, t), \mathcal{M}(Ax, Ax, Ax, t), \mathcal{M}(By, By, By, t), \\ \mathcal{M}(Ax, By, By, t), \mathcal{M}(By, Ax, Ax, t), \\ \left\{\alpha\,(Sx, By, By, t) + \beta min\left\{\begin{array}{c}\mathcal{M}(By, By, By, t), \mathcal{M}(Ax, Ax, Ax, t), \\ \mathcal{M}(Ax, By, By, t)\end{array}\right\}\right\}\end{array}\right\}$$

$$\mathcal{M}(Ax, By, By, kt) \leq \min\left\{\begin{array}{c}\mathcal{M}(Ax, By, By, t), 1, 1, \mathcal{M}(Ax, By, By, t), \mathcal{M}(By, Ax, Ax, t), \\ \alpha\,\mathcal{M}(Ax, By, By, t) + \beta min\{1, 1, \mathcal{M}(Ax, By, By, t)\}\end{array}\right\}$$

$\mathcal{M}(Ax,\ By,\ By,\ kt) \leq \mathcal{M}(Ax,\ By,\ By,\ t)$ and

$$\mathcal{N}(Ax,\ By,\ By,\ kt) \geq \max\left\{\begin{array}{c}\mathcal{N}(Ax, By, By, t), \mathcal{N}(Ax,\ Ax,\ Ax\ , t), \mathcal{N}(By,\ By,\ By,\ t), \\ \mathcal{N}(Ax,\ By,\ By,\ t), \mathcal{N}(By,\ Ax,\ Ax,\ t), \\ \left\{\alpha\, \mathcal{N}(Sx,\ By,\ By,\ t) + \beta max\left\{\begin{array}{c}\mathcal{N}\ (By,\ By,\ By,\ t), \mathcal{N}(Ax,\ Ax,\ Ax,\ t), \\ \mathcal{N}(Ax,\ By,\ By,\ t)\end{array}\right\}\right\}\end{array}\right\}$$

$$\mathcal{N}\ (Ax,\ By,\ By,\ kt) \geq \max\left\{\begin{array}{c}\mathcal{N}(Ax, By, By, t),\ \ 0,\ \ 0, \mathcal{N}(Ax,\ By,\ By\ , t), \mathcal{N}(By,\ Ax,\ Ax,\ t), \\ \alpha\, \mathcal{N}(Ax,\ By,\ By,\ t) + \beta max\{0,\ 0, \mathcal{N}\ (Ax,\ By,\ By,\ t)\}\end{array}\right\}$$

$\mathcal{N}(Ax,\ By,\ By,\ kt) \geq \mathcal{N}(Ax,\ By,\ By,\ t)$. Then by Lemma 2.8, $Ax = By$. Suppose that there is another point z such that $Az = Sz$. Then by inequality (2), we have $Az = Sz = By = Ty$, So, $Ax = Az$ and $w = Ax = Sx$ is the unique point of coincidence of A and S. By Lemma 2.8, w is the only common point of A and S. Similarly there is a unique point $z \in X$ such that $z = Bz = Tz$. Assume that $w \neq z$, then by (2), $\mathcal{M}(w,\ z,\ z,\ kt) = \mathcal{M}(Aw,\ Bz,\ Bz,\ kt)$

$$\mathcal{M}(Ax,\ By,\ By,\ kt) \leq \min\left\{\begin{array}{c}\mathcal{M}(Sx,\ Ty,\ Ty,\ t), \mathcal{M}(Sx,\ Ax,\ Ax,\ t), \mathcal{M}(Ty,\ By,\ By,\ t), \\ \mathcal{M}(Sx, By,\ By,\ t), \mathcal{M}(Ty,\ Ax,\ Ax,\ t), \\ \left\{a\ \mathcal{M}(Ax,\ By,\ t) + min\left\{\begin{array}{c}\mathcal{M}(By,\ Ty,\ Ty,\ t), \\ \mathcal{M}\ (Sx,\ Ax,\ Ax,\ t), \\ \mathcal{M}(Sx,\ Ty,\ Ty,\ t)\end{array}\right\}\right\}\end{array}\right\} \text{ and}$$

$$\mathcal{N}(w,\ z,\ z,\ kt) = \mathcal{N}(Aw,\ Bz,\ Bz,\ kt)$$

$$\mathcal{N}(Ax,\ By,\ By,\ kt) \geq \max\left\{\begin{array}{c}\mathcal{N}(Sx,\ Ty,\ Ty,\ t), \mathcal{N}(Sx,\ Ax,\ Ax,\ t), \mathcal{N}(Ty,\ By,\ By,\ t), \\ \mathcal{N}(Sx, By,\ By,\ t), \mathcal{N}(Ty,\ Ax,\ Ax,\ t), \\ \left\{a\ \mathcal{N}(Ax,\ By,\ t) + max\left\{\begin{array}{c}\mathcal{N}(By,\ Ty,\ Ty,\ t), \\ \mathcal{N}\ (Sx,\ Ax,\ Ax,\ t), \\ \mathcal{N}(Sx,\ Ty,\ Ty,\ t)\end{array}\right\}\right\}\end{array}\right\}$$

Put $x = w$ and $y = z$ in inequality (2),

$$\mathcal{M}(Aw,\ Bz,\ Bz,\ kt) \leq \min\left\{\begin{array}{c}\mathcal{M}(Sw,\ Tz,\ Tz,\ t), \mathcal{M}(Sw,\ Aw,\ Aw,\ t), \mathcal{M}(Tz,\ Bz,\ Bz,\ t), \\ \mathcal{M}(Sw, Bz,\ Bz,\ t), \mathcal{M}(Tz,\ Aw,\ Aw,\ t), \\ \left\{a\ \mathcal{M}\ (Sw,\ Bz,\ Bz,\ t) + min\left\{\begin{array}{c}\mathcal{M}(Bz,\ Tz,\ Tz,\ t), \\ \mathcal{M}\ (Sw,\ Aw,\ Aw,\ t), \\ \mathcal{M}(Sw,\ Tz,\ Tz,\ t)\end{array}\right\}\right\}\end{array}\right\}$$

$$\mathcal{M}(w,\ z,\ z,\ kt) \leq \min\left\{\begin{array}{c}\mathcal{M}(w,\ z,\ z,\ t), \mathcal{M}(w,\ w,\ w,\ t), \mathcal{M}(z, z, z,\ t), \\ \mathcal{M}(w,\ z,\ z,\ t), \mathcal{M}(z,\ w,\ w,\ t), \\ \left\{a\ \mathcal{M}(w, z, z,\ t) + min\left\{\begin{array}{c}\mathcal{M}(z, z, z,\ t), \\ \mathcal{M}(w, w, w,\ t), \\ \mathcal{M}(w, z,\ z,\ t)\end{array}\right\}\right\}\end{array}\right\}$$

$$\mathcal{M}(w,\ z,\ z,\ kt) \leq \min\left\{\begin{array}{c}\mathcal{M}(w,\ z,\ z,\ t),\ \ 1,\ 1, \mathcal{M}(z,\ w,\ w,\ t), \\ \{a\ \mathcal{M}(w,\ z,\ z,\ t) + min\ \{1,\ 1, \mathcal{M}(w,\ z,\ z,\ t)\}\ \}\end{array}\right\}$$

$\mathcal{M}(w,\ z,\ z,\ kt) = \mathcal{M}(w,\ z,\ z,\ kt)$ and

$$\mathcal{N}(Aw,\ Bz,\ Bz,\ kt) \geq \max\left\{\begin{array}{c}\mathcal{N}(Sw,\ Tz,\ Tz,\ t),\mathcal{N}(Sw,\ Aw,\ Aw,\ t),\mathcal{N}(Tz,\ Bz,\ Bz,\ t),\\ \mathcal{N}(Sw, Bz,\ Bz,\ t),\mathcal{N}(Tz,\ Aw,\ Aw,\ t),\\ \left\{a\,\mathcal{N}\ (Sw,\ Bz,\ Bz,\ t)+max\left\{\begin{array}{c}\mathcal{N}(Bz,\ Tz,\ Tz,\ t),\\ \mathcal{N}\ (Sw,\ Aw,\ Aw,\ t),\\ \mathcal{N}(Sw,\ Tz,\ Tz,\ t)\end{array}\right\}\right\}\end{array}\right\}$$

$$\mathcal{N}\ (w,\ z,\ z,\ kt) \geq \max\left\{\begin{array}{c}\mathcal{N}(w,\ z,\ z,\ t),\mathcal{N}(w,\ w,\ w,\ t),\mathcal{N}(z,z,z,\ t),\\ \mathcal{N}(w,\ z,\ z,\ t),\mathcal{N}(z,\ w,\ w,\ t),\\ \left\{a\,\mathcal{N}(w,z,z,\ t)+max\left\{\begin{array}{c}\mathcal{N}(z,z,z,\ t),\\ \mathcal{N}(w,w,w,\ t),\\ \mathcal{N}(w,z,\ z,\ t)\end{array}\right\}\right\}\end{array}\right\}$$

$$\mathcal{N}\ (w,\ z,\ z,\ kt) \geq \max\left\{\begin{array}{c}\mathcal{N}(w,\ z,\ z,\ t),\ \ 0,\ \ 0,\ \mathcal{N}(z,\ w,\ w,\ t),\\ \{a\,\mathcal{N}(w,\ z,\ z,\ t)+max\ \{0\ ,\ 0\ ,\mathcal{N}(w,\ z,\ z,\ t)\}\ \}\end{array}\right\}$$

$$\mathcal{N}\ (w,\ z,\ z,\ kt)\ \geq\mathcal{N}(w,\ z,\ z,\ kt).$$

Then by Lemma 2.8, we get $w = z$.

Uniqueness: Let u be another common fixed point of A, B, S and T. Then put $x = z$ and $y = u$ in (2),

$$\mathcal{M}(Ax,\ By,\ By,\ kt) \leq \min\left\{\begin{array}{c}\mathcal{M}(Sx,\ Ty,\ Ty,\ t),\mathcal{M}(Sx,\ Ax,\ Ax,\ t),\mathcal{M}(Ty,\ By,\ By,\ t),\\ \mathcal{M}(Sx, By,\ By,\ t),\mathcal{M}(Ty,\ Ax,\ Ax,\ t),\\ \left\{a\,\mathcal{M}(Sx,\ By,\ By,\ t)+min\left\{\begin{array}{c}\mathcal{M}(By,\ Ty,\ Ty,\ t),\\ \mathcal{M}\ (Sx,\ Ax,\ Ax,\ t),\\ \mathcal{M}(Sx,\ Ty,\ Ty,\ t)\end{array}\right\}\right\}\end{array}\right\}$$

$$\mathcal{M}(Az,\ Bu,\ Bu,\ kt) \leq \min\left\{\begin{array}{c}\mathcal{M}(Sz,\ Tu,\ Tu,\ t),\mathcal{M}(Sz,\ Az,\ Az,\ t),\mathcal{M}(Tu,\ Bu,\ Bu,\ t),\\ \mathcal{M}(Sz, Bu,\ Bu,\ t),\mathcal{M}(Tu,\ Az,\ Az,\ t),\\ \left\{a\,\mathcal{M}\ (Sz,\ Bu,\ Bu,\ t)+min\left\{\begin{array}{c}\mathcal{M}(Bu,\ Tu,\ Tu,\ t),\\ \mathcal{M}\ (Sz,\ Az,\ Az,\ t),\\ \mathcal{M}(Sz,\ Tu,\ Tu,\ t)\end{array}\right\}\right\}\end{array}\right\}$$

$$\mathcal{M}(z,\ u,\ u,\ kt) \leq \min\left\{\begin{array}{c}\mathcal{M}(z,\ u,\ u,\ t),\mathcal{M}(z,\ z,\ z,\ t),\mathcal{M}(u,u,\ u,\ t),\\ \mathcal{M}(z,\ u,\ u,\ t),\mathcal{M}(u,\ z,z,\ t),\\ \left\{a\,\mathcal{M}(z,\ u,\ u,\ t)+min\left\{\begin{array}{c}\mathcal{M}(u,\ u,\ u,\ t),\\ \mathcal{M}\ (z,\ z,\ z,\ t),\\ \mathcal{M}(z,\ u,\ u,\ t)\end{array}\right\}\right\}\end{array}\right\}$$

$$\mathcal{M}(z,\ u,\ u,\ kt) \leq \min\left\{\begin{array}{c}\mathcal{M}(z,\ u,\ u,\ t),\ 1,\ 1,\mathcal{M}(z,\ u,\ u,\ t),\mathcal{M}(u,\ z,\ z,\ t),\\ \{a\,\mathcal{M}(z,\ u,\ u,\ t)+min\{1,\ 1,\ \mathcal{M}(z,\ u,\ u,\ t)\}\ \}\end{array}\right\}$$

$$\mathcal{M}(z,\ u,\ u,\ kt) \leq \mathcal{M}(z,\ u,\ u,\ t)\ \text{ and}$$

$$\mathcal{N}(Ax,\ By,\ By,\ kt) \geq \max\left\{\begin{array}{c}\mathcal{N}\ (Sx,\ Ty,\ Ty,\ t),\mathcal{N}\ (Sx,\ Ax,\ Ax,\ t),\mathcal{N}\ (Ty,\ By,\ By,\ t),\\ \mathcal{N}\ (Sx, By,\ By,\ t),\mathcal{N}\ (Ty,\ Ax,\ Ax,\ t),\\ \left\{a\,\mathcal{N}\ (Sx,\ By,\ By,\ t)+max\left\{\begin{array}{c}\mathcal{N}\ (By,\ Ty,\ Ty,\ t),\\ \mathcal{N}\ (Sx,\ Ax,\ Ax,\ t),\\ \mathcal{N}\ (Sx,\ Ty,\ Ty,\ t)\end{array}\right\}\right\}\end{array}\right\}$$

$$\mathcal{N}(Az,\ Bu,\ Bu,\ kt) \geq \max\left\{\begin{array}{c} \mathcal{N}\ (Sz,\ Tu,\ Tu,\ t)\,,\mathcal{N}\ (Sz,\ Az,\ Az,\ t)\,,\mathcal{N}\ (Tu,\ Bu,\ Bu,\ t)\,, \\ \mathcal{N}\ (Sz, Bu,\ Bu,\ t)\,,\mathcal{N}\ (Tu,\ Az,\ Az,\ t)\,, \\ \left\{ a\,\mathcal{N}\ (Sz,\ Bu,\ Bu,\ t) + max\left\{\begin{array}{c} \mathcal{N}\ (Bu,\ Tu,\ Tu,\ t)\,, \\ \mathcal{N}\ \ (Sz,\ Az,\ Az,\ t)\,, \\ \mathcal{N}\ (Sz,\ Tu,\ Tu,\ t) \end{array}\right\}\right\} \end{array}\right\}$$

$$\mathcal{N}(z,\ u,\ u,\ kt) \geq \max\left\{\begin{array}{c} \mathcal{N}\ (z,\ u,\ u,\ t)\,,\mathcal{N}\ (z,\ z,\ z,\ t)\,,\mathcal{N}\ (u, u,\ u,\ t)\,, \\ \mathcal{N}\ (z,\ u,\ u,\ t)\,,\mathcal{N}\ (u,\ z, z,\ t)\,, \\ \left\{ a\,\mathcal{N}\ (z,\ u,\ u,\ t) + max\left\{\begin{array}{c} \mathcal{N}\ (u,\ u,\ u,\ t)\,, \\ \mathcal{N}\ \ (z,\ z,\ z,\ t)\,, \\ \mathcal{N}\ (z,\ u,\ u,\ t) \end{array}\right\}\right\} \end{array}\right\}$$

$$\mathcal{N}(z,\ u,\ u,\ kt) \geq \max\left\{\begin{array}{c} \mathcal{N}\ (z,\ u,\ u,\ t)\,,\ 0,\ \ 0, \mathcal{N}(z,\ u,\ u,\ t)\,,\mathcal{N}\ (u,\ z,\ z,\ t)\,, \\ \{a\,\mathcal{N}\ (z,\ u,\ u,\ t) + max\,\{0,\ \ 0,\ \ \mathcal{N}\ (z,\ u,\ u,\ t)\}\ \} \end{array}\right\}$$

$\mathcal{N}(z,\ u,\ u,\ kt) \geq \mathcal{N}(z,\ u,\ u,\ t)$. Then by Lemma 2.8, we get $z = u$. □

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