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Intuitionistic Fuzzy g'''-Closed Sets

Research Article

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Abstract: In this paper, we introduce the concepts of intuitionistic fuzzy g'''-closed sets and intuitionistic fuzzy g'''-open sets. Further, we study some of their properties.

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1. Introduction

The concept of fuzzy sets was introduced by Zadeh [18] and later Atanassov [1] generalized this idea to intuitionistic fuzzy sets using the notion of fuzzy sets. On the other hand Coker [3] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. The concept of generalized closed sets in topological spaces was introduced by Levine [8]. In this paper we introduce intuitionistic fuzzy g'''-closed sets and intuitionistic fuzzy g'''-open sets. The relations between intuitionistic fuzzy g'''-closed sets and other intuitionistic fuzzy generalized closed sets are given.

2. Preliminaries

Definition 2.1 ([1]). *Let X be a non empty set. An intuitionistic fuzzy set (IFS in short) A in X can be described in the form*

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$$

where the function $\mu_A : X \to [0, 1]$ is called the membership function and $\mu_A(x)$ denotes the degree to which $x \in A$ and the function $\nu_A : X \to [0, 1]$ is called the non-membership function and $\nu_A(x)$ denotes the degree to which $x \notin A$ and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote IFS(X), the set of all intuitionistic fuzzy sets in X.

Throughout the paper, X denotes a non empty set.

Definition 2.2 ([1]). *Let A and B be any two IFSs of the form $A=\{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ and $B=\{\langle x, \mu_B(x), \nu_B(x)\rangle \mid x \in X\}$. Then*

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(1). $A \subseteq B$ *if and only if* $\mu_A(x) \leq \mu_B(x)$ *and* $\nu_A(x) \geq \nu_B(x)$ *for all* $x \in X$,

(2). $A = B$ *if and only if* $A \subseteq B$ *and* $B \subseteq A$,

(3). $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle \mid x \in X\}$,

(4). $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle \mid x \in X\}$,

(5). $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle \mid x \in X\}$.

Definition 2.3 ([1]). *The intuitionistic fuzzy sets* $0_\sim = \{\langle x, 0, 1\rangle \mid x \in X\}$ *and* $1_\sim = \{\langle x, 1, 0\rangle \mid x \in X\}$ *are called the empty set and the whole set of X respectively.*

Definition 2.4 ([1]). *Let A and B be any two IFSs of the form* $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ *and* $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle \mid x \in X\}$. *Then*

(1). $A \subseteq B$ *and* $A \subseteq C \Rightarrow A \subseteq B \cap C$,

(2). $A \subseteq C$ *and* $B \subseteq C \Rightarrow A \cup B \subseteq C$,

(3). $A \subseteq B$ *and* $B \subseteq C \Rightarrow A \subseteq C$,

(4). $(A \cup B)^c = A^c \cap B^c$ *and* $(A \cap B)^c = A^c \cup B^c$,

(5). $((A)^c)^c = A$,

(6). $(1_\sim)^c = 0_\sim$ *and* $(0_\sim)^c = 1_\sim$.

Definition 2.5 ([3]). *An intuitionistic fuzzy topology (IFT in short) on X is a family* τ *of IFSs in X satisfying the following axioms :*

(1). $0_\sim, 1_\sim \in \tau$,

(2). $G_1 \cap G_2 \in \tau$ *for any* $G_1, G_2 \in \tau$,

(3). $\cup G_i \in \tau$ *for any family* $\{G_i \mid i \in J\} \subseteq \tau$.

In this case the pair (X, τ) *is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in* τ *is known as an intuitionistic fuzzy open set (IFOS in short) in X.*
The complement A^c *of an IFOS A in an IFTS* (X, τ) *is called an intuitionistic fuzzy closed set (IFCS in short) in X.*

Definition 2.6 ([3]). *Let* (X, τ) *be an IFTS and* $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ *be an IFS in X. Then the intuitionistic fuzzy interior and the intuitionistic fuzzy closure are defined as follows:*

$$int(A) = \cup \{G \mid G \text{ is an IFOS in } X \text{ and } G \subseteq A\},$$
$$cl(A) = \cap \{K \mid K \text{ is an IFCS in } X \text{ and } A \subseteq K\}.$$

Proposition 2.7 ([3]). *For any IFSs A and B in* (X, τ), *we have*

(1). $int(A) \subseteq A$,

(2). $A \subseteq cl(A)$,

(3). *A is an IFCS in X* $\Leftrightarrow cl(A) = A$,

(4). A is an IFOS in X $\Leftrightarrow$ $int(A) = A$,

(5). $A \subseteq B \Rightarrow int(A) \subseteq int(B)$ *and* $cl(A) \subseteq cl(B)$,

(6). $int(int(A)) = int(A)$,

(7). $cl(cl(A)) = cl(A)$,

(8). $cl(A \cup B) = cl(A) \cup cl(B)$,

(9). $int(A \cap B) = int(A) \cap int(B)$.

Proposition 2.8 ([3]). *For any IFS A in* (X, τ), *we have*

(1). $int(0_\sim) = 0_\sim$ *and* $cl(0_\sim) = 0_\sim$,

(2). $int(1_\sim) = 1_\sim$ *and* $cl(1_\sim) = 1_\sim$,

(3). $(int(A))^c = cl(A^c)$,

(4). $(cl(A))^c = int(A^c)$.

Proposition 2.9 ([3]). *If A is an IFCS in* (X, τ) *then* $cl(A) = A$ *and if A is an IFOS in* (X, τ) *then* $int(A) = A$. *The arbitrary union of IFCSs is an IFCS in* (X, τ).

Definition 2.10. *An IFS A in an IFTS* (X, τ) *is said to be an*

(1). intuitionistic fuzzy α*-closed set (IF*α*CS in short) if* $cl(int(cl(A))) \subseteq A$, [5]

(2). intuitionistic fuzzy semi closed set (IFSCS in short) if $int(cl(A)) \subseteq A$, [4]

(3). intuitionistic fuzzy pre closed set (IFPCS in short) if $cl(int(A) \subseteq A$, [4]

(4). intuitionistic fuzzy semi pre closed set (IFSPCS in short) if there exists an IFPCS B such that $int(B) \subseteq A \subseteq B$. [17]

The family of all IFα*CSs (resp. IFSCSs, IFPCSs, IFSPCSs) of* (X, τ) *is denoted by IF*α*C(X) (resp. IFSC(X), IFPC(X), IFSPC(X)).*

Definition 2.11. *An IFS A in an IFTS* (X, τ) *is said to be an*

(1). intuitionistic fuzzy α*-open set (IF*α*OS in short) if* $A \subseteq int(cl(int(A)))$, [5]

(2). intuitionistic fuzzy semi open set (IFSOS in short) if $A \subseteq cl(int(A))$, [5]

(3). intuitionistic fuzzy preopen set (IFPOS in short) if $A \subseteq int(cl(A))$, [5]

(4). intuitionistic fuzzy semi pre open set (IFSPOS in short) if $A \subseteq cl(int(cl(A)))$. [17]

The family of all IFα*OSs (resp. IFSOSs, IFPOSs, IFSPOSs) of* (X, τ) *is denoted by IF*α*O(X) (resp. IFSO(X), IFPO(X), IFSPO(X)).*

Remark 2.12 ([7]).

$$IFCS \rightarrow IF\alpha CS \rightarrow IFSCS \rightarrow IFSPCS$$

None of the above implications are reversible.

Definition 2.13 ([13]). *Let A be an IFS in an IFTS (X, τ). Then the α-interior of A ($\alpha int(A)$ in short) and the α-closure of A ($\alpha cl(A)$ in short) are defined as follows:*

$$\alpha int(A) = \cup \{G \mid G \text{ is an IF}\alpha OS \text{ in } (X, \tau) \text{ and } G \subseteq A\},$$
$$\alpha cl(A) = \cap \{K \mid K \text{ is an IF}\alpha CS \text{ in } (X, \tau) \text{ and } A \subseteq K\}.$$

$sint(A)$, $scl(A)$, $spint(A)$ and $spcl(A)$ are similarly defined. For any IFS A in (X, τ), we have $\alpha cl(A^c) = (\alpha int(A))^c$ and $\alpha int(A^c) = (\alpha cl(A))^c$.

Remark 2.14 ([13]). *Let A be an IFS in an IFTS (X, τ). Then*

(1). $\alpha cl(A) = A \cup cl(int(cl(A)))$,

(2). $\alpha int(A) = A \cap int(cl(int(A)))$.

Definition 2.15. *An IFS A in (X, τ) is said to be an*

(1). intuitionistic fuzzy generalized closed set (IFGCS in short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ), [15]

(2). intuitionistic fuzzy generalized semi closed set (IFGSCS in short) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ), [9]

(3). intuitionistic fuzzy α generalized closed set (IFαGCS in short) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ), [13]

(4). intuitionistic fuzzy α generalized semi closed set (IFαGSCS in short) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFSOS in (X, τ), [6]

(5). intuitionistic fuzzy ω closed set (IFωCS in short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFSOS in (X, τ), [14]

(6). intuitionistic fuzzy generalized semi pre closed set (IFGSPCS in short) if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ). [11]

The complement of an IFGCS (resp. an IFGSCS, an IFαGCS, an IFαGSCS, an IFωCS, an IFGSPCS) is called an IFGOS (resp. an IFGSOS, an IFαGOS, an IFαGSOS, an IFωOS, an IFGSPOS).

The family of all IFGCSs (resp. IFGSCSs, IFαGCSs, IFωCSs, IFGSPCSs) of (X, τ) is denoted by IFGC(X), IFGSC(X), IFαGC(X), IFωC(X), IFGSPC(X)).

The family of all IFGOSs (resp. IFGSOSs, IFαGOSs, IFωOSs, IFGSPOSs) of (X, τ) is denoted by IFGO(X), IFGSO(X), IFαGO(X), IFωO(X), IFGSPC(X)).

Remark 2.16 ([14]).

(1). Every IFOS is an IFGSOS,

(2). Every IFSOS is an IFGSOS.

Definition 2.17 ([16]). *Two IFSs A and B are said to be q-coincident (AqB in short) if and only if there exists an element $x \in X$ such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$. For any two IFS A and B of (X, τ), $A\bar{q}B$ if and only if $A \subseteq B^c$.*

3. Intuitionistic Fuzzy g'''-closed Sets

In this section we introduce intuitionistic fuzzy g'''-closed sets and study some of its properties.

Definition 3.1. *An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy g'''-closed set (IFG$'''$CS in short) if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U $\in$ IFGSO(X). The collection of all intuituionistic fuzzy g'''-closed sets in X is denoted by IFG$'''$C(X).*

Example 3.2. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.2, 0.3), (0.7, 0.6)\rangle$. We have $\mu_G(a) = 0.2$, $\mu_G(b) = 0.3$, $\nu_G(a) = 0.7$ and $\nu_G(b) = 0.6$. Here IFGSO(X) $= \{0_\sim, A, 1_\sim\}$ where $0_\sim \subset A \subset 1_\sim$ and IFG$'''$C(X) $= \{0_\sim, G^c, 1_\sim\}$. Consider an IFS $A = \langle x, (0.7, 0.6), (0.2, 0.3)\rangle$. Then A is an IFG$'''$CS in (X, τ).*

Example 3.3. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.2, 0.3), (0.7, 0.6)\rangle$. We have $\mu_G(a) = 0.2$, $\mu_G(b) = 0.3$, $\nu_G(a) = 0.7$ and $\nu_G(b) = 0.6$. Here IFGSO(X) $= \{0_\sim, A, 1_\sim\}$ where $0_\sim \subset A \subset 1_\sim$ and IFG$'''$C(X) $= \{0_\sim, G^c, 1_\sim\}$. Consider an IFS $A = \langle x, (0.3, 0.4), (0.6, 0.5)\rangle$. Then A is not an IFG$'''$CS in (X, τ).*

Theorem 3.4. *Every IFCS is an IFG$'''$CS.*

Proof. Let A be an IFCS in (X, τ). Let U $\in$ IFGSO(X) such that A $\subseteq$ U. Since A is an IFCS, cl(A) = A. Thus we have cl(A) $\subseteq$ U, whenever A $\subseteq$ U and U $\in$ IFGSO(X). Therefore A is an IFG$'''$CS in (X, τ). Hence every IFCS is an IFG$'''$CS. □

The converse of Theorem 3.4 need not be true as seen from the following Example.

Example 3.5. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.7, 0.6), (0.2, 0.3)\rangle$. We have $\mu_G(a) = 0.7$, $\mu_G(b) = 0.6$, $\nu_G(a) = 0.2$ and $\nu_G(b) = 0.3$. Here IFGSO(X) $= \{0_\sim, A, G, B, 1_\sim\}$ where $0_\sim \subset A \subset G^c$ and $G \subset B \subset 1_\sim$ and IFG$'''$C(X) $= \{0_\sim, A, G^c, 1_\sim\}$ where $0_\sim \subset A \subset G^c$. Consider an IFS $A = \langle x, (0.1, 0.2), (0.8, 0.7)\rangle$. Then the IFS A is an IFG$'''$CS but not an IFCS in (X, τ).*

Theorem 3.6. *Every IFG$'''$CS is an IFGSPCS.*

Proof. Let A $\subseteq$ U where U is an IFOS in (X, τ). Since every IFOS is an IFGSOS and since A is an IFG$'''$CS in (X, τ), we have cl(A) $\subseteq$ U, whenever A $\subseteq$ U and U $\in$ IFGSO(X). We have cl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is an IFOS in (X, τ). Since spcl(A) $\subseteq$ cl(A), we have spcl(A) $\subseteq$ U. Therefore A is an IFGSPCS in (X, τ). Hence every IFG$'''$CS is an IFGSPCS. □

The converse of Theorem 3.6 need not be true as seen from the following Example.

Example 3.7. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.2, 0.3), (0.7, 0.6)\rangle$. We have $\mu_G(a) = 0.2$, $\mu_G(b) = 0.3$, $\nu_G(a) = 0.7$ and $\nu_G(b) = 0.6$. Consider an IFS $A = \langle x, (0.3, 0.4), (0.6, 0.5)\rangle$. Here IFG$'''$C(X) $= \{0_\sim, G^c, 1_\sim\}$. Therefore A is not an IFG$'''$CS in (X, τ). And we have spcl(A) = A. Therefore A is an IFGSPCS in (X, τ).*

Theorem 3.8. *Every IFG$'''$CS is an IFωCS.*

Proof. Let A $\subseteq$ U where U is an IFSOS in (X, τ). Since every IFSOS is an IFGSOS and since A is an IFG$'''$CS in (X, τ), we have cl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is an IFGSOS in (X, τ). We have cl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is an IFSOS in (X, τ). Therefore A is an IFωCS in (X, τ). Hence every IFG$'''$CS is an IFωCS. □

The converse of Theorem 3.8 need not be true as seen from the following Example.

Example 3.9. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.3, 0.4), (0.6, 0.5)\rangle$. We have $\mu_G(a) = 0.3$, $\mu_G(b) = 0.4$, $\nu_G(a) = 0.6$ and $\nu_G(b) = 0.5$. Consider an IFS $A = \langle x, (0.7, 0.6), (0.2, 0.3)\rangle$. Here IFG$'''$C(X) $= \{0_\sim, G^c, 1_\sim\}$. Therefore A is not an IFG$'''$CS in (X, τ). And here IFSO(X) $= \{0_\sim, G, A, G^c, 1_\sim\}$ where $G \subset A \subset G^c$. Therefore A is an IFωCS in (X, τ).*

Theorem 3.10. *Every IFG$'''$CS is an IFGCS.*

Proof. Let A ⊆ U where U is an IFOS in (X, τ). Since every IFOS is an IFGSOS and since A is an IFG$'''$CS in (X, τ), we have cl(A) ⊆ U, whenever A ⊆ U and U is an IFGSOS in (X, τ). We have cl(A) ⊆ U, whenever A ⊆ U and U is an IFOS in (X, τ). Therefore A is an IFGCS in (X, τ). Hence every IFG$'''$CS is an IFGCS. □

The converse of Theorem 3.10 need not be true as seen from the following Example.

Example 3.11. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.6, 0.5), (0.3, 0.4)\rangle$. We have $\mu_G(a) = 0.6$, $\mu_G(b) = 0.5$, $\nu_G(a) = 0.3$ and $\nu_G(b) = 0.4$. Consider an IFS $A = \langle x, (0.7, 0.6), (0.2, 0.3)\rangle$. Here IFG$'''$C(X) $= \{0_\sim, A, G^c, 1_\sim\}$ where $0_\sim \subset A \subset G^c$. Therefore A is not an IFG$'''$CS in (X, τ). And here $U = 1_\sim$ is the only IFOS which contains A. Therefore A is an IFGCS in (X, τ).*

Theorem 3.12. *Every IFG$'''$CS is an IFαGCS.*

Proof. Let A ⊆ U where U is an IFOS in (X, τ). Since every IFOS is an IFGSOS and since A is an IFG$'''$CS in (X, τ), we have cl(A) ⊆ U, whenever A ⊆ U and U is an IFGSOS in (X, τ). We have cl(A) ⊆ U, whenever A ⊆ U and U is an IFOS in (X, τ). Since αcl(A) ⊆ cl(A), we have αcl(A) ⊆ U. Therefore A is an IFαGCS in (X, τ). Hence every IFG$'''$CS is an IFαGCS. □

The converse of Theorem 3.12 need not be true as seen from the following Example.

Example 3.13. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.6, 0.7), (0.3, 0.2)\rangle$. We have $\mu_G(a) = 0.6$, $\mu_G(b) = 0.7$, $\nu_G(a) = 0.3$ and $\nu_G(b) = 0.2$. Consider an IFS $A = \langle x, (0.6, 0.7), (0.3, 0.2)\rangle$. Here IFG$'''$C(X) $= \{0_\sim, A, G^c, 1_\sim\}$ where $0_\sim \subset A \subset G^c$. Therefore A is not an IFG$'''$CS in (X, τ). And here $U = 1_\sim$ is the only IFOS which contains A. Therefore A is an IFαGCS in (X, τ).*

Theorem 3.14. *Every IFG$'''$CS is an IFGSCS.*

Proof. Let A ⊆ U where U is an IFOS in (X, τ). Since every IFOS is an IFGSOS and since A is an IFG$'''$CS in (X, τ), we have cl(A) ⊆ U, whenever A ⊆ U and U is an IFGSOS in (X, τ). We have cl(A) ⊆ U, whenever A ⊆ U and U is an IFOS in (X, τ). Since scl(A) ⊆ cl(A), we have scl(A) ⊆ U. Therefore A is an IFGSCS in (X, τ). Hence every IFG$'''$CS is an IFGSCS. □

The converse of Theorem 3.14 need not be true as seen from the following Example.

Example 3.15. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.2, 0.3), (0.7, 0.6)\rangle$. We have $\mu_G(a) = 0.2$, $\mu_G(b) = 0.3$, $\nu_G(a) = 0.7$ and $\nu_G(b) = 0.6$. Consider an IFS $A = \langle x, (0.6, 0.7), (0.3, 0.2)\rangle$. Here IFG$'''$C(X) $= \{0_\sim, G^c, 1_\sim\}$. Therefore A is not an IFG$'''$CS in (X, τ). And here $U = 1_\sim$ is the only IFOS which contains A. Therefore A is an IFGSCS in (X, τ).*

Definition 3.16. *An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy g_s'''-closed set (IFG_S'''CS in short) if scl(A) ⊆ U whenever A ⊆ U and U is an IFGSOS in (X, τ). The complement of an intuitionistic fuzzy g_s'''-closed set is called an intuitionistic fuzzy g_s'''-open set (IFG_S'''OS in short).*

Theorem 3.17. *Every IFG'''CS is an IFG'''_SCS.*

Proof. Let A ⊆ U where U is an IFGSOS in (X, τ). Since A is an IFG'''CS in (X, τ), we have cl(A) ⊆ U, whenever A ⊆ U and U is an IFGSOS in (X, τ). Since scl(A) ⊆ cl(A), we have scl(A) ⊆ U. Therefore A is an IFG'''_SCS in (X, τ). Hence every IFG'''CS is an IFG'''_SCS. □

The converse of Theorem 3.17 need not be true as seen from the following Example.

Example 3.18. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.2, 0.3), (0.7, 0.6)\rangle$. We have $\mu_G(a) = 0.2$, $\mu_G(b) = 0.3$, $\nu_G(a) = 0.7$ and $\nu_G(b) = 0.6$. Consider an IFS $A = \langle x, (0.3, 0.4), (0.6, 0.5)\rangle$. Here IF$G'''$C(X) $= \{0_\sim, G^c, 1_\sim\}$. Therefore A is not an IFG'''CS in (X, τ). And here IFSC(X) $= \{0_\sim, G, A, G^c, 1_\sim\}$ where $G \subset A \subset G^c$. Therefore A is an IFG'''_SCS in (X, τ).*

Theorem 3.19. *Every IFG'''CS is an IFαGSCS.*

Proof. Let A ⊆ U where U is an IFSOS in (X, τ). Since every IFSOS is an IFGSOS and since A is an IFG'''CS in (X, τ), we have cl(A) ⊆ U, whenever A ⊆ U and U is an IFGSOS in (X, τ). We have cl(A) ⊆ U, whenever A ⊆ U and U is an IFSOS in (X, τ). Since αcl(A) ⊆ cl(A), we have αcl(A) ⊆ U. Therefore A is an IFαGSCS in (X, τ). Hence every IFG'''CS is an IFαGSCS. □

The converse of Theorem 3.19 need not be true as seen from the following Example.

Example 3.20. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.4, 0.3), (0.5, 0.6)\rangle$. We have $\mu_G(a) = 0.4$, $\mu_G(b) = 0.3$, $\nu_G(a) = 0.5$ and $\nu_G(b) = 0.6$. Consider an IFS $A = \langle x, (0.6, 0.7), (0.3, 0.2)\rangle$. Here IF$G'''$C(X) $= \{0_\sim, G^c, 1_\sim\}$. Therefore A is not an IFG'''CS in (X, τ). And here IFSO(X) $= \{0_\sim, G, A, G^c, 1_\sim\}$ where $G \subset A \subset G^c$. Therefore A is an IFαGSCS in (X, τ).*

Definition 3.21. *An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy g'''_α-closed set (IFG'''_αCS in short) if αcl(A) ⊆ U whenever A ⊆ U and U is an IFGSOS in (X, τ). The complement of an intuitionistic fuzzy g'''_α-closed set is called an intuitionistic fuzzy g'''_α-open set (IFG'''_αOS in short).*

Theorem 3.22. *Every IFG'''CS is an IFG'''_αCS.*

Proof. Let A be an IFG'''CS in (X, τ). Then we have cl(A) ⊆ U, whenever A ⊆ U and U is an IFGSOS in (X, τ). Since αcl(A) ⊆ cl(A), we have αcl(A) ⊆ U, whenever A ⊆ U and U is an IFGSOS in (X, τ). Therefore A is an IFG'''_αCS in (X, τ). Hence every IFG'''CS is an IFG'''_αCS. □

The converse of Theorem 3.22 need not be true as seen from the following Example.

Example 3.23. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.3, 0.6), (0.6, 0.3)\rangle$. We have $\mu_G(a) = 0.3$, $\mu_G(b) = 0.6$, $\nu_G(a) = 0.6$ and $\nu_G(b) = 0.3$. Consider an IFS $A = \langle x, (0.2, 0.3), (0.7, 0.6)\rangle$. Then A is an IF$G'''_\alpha$CS but not an IF$G'''$CS in (X, τ).*

Remark 3.24. *IFαCS and IFG'''CS are independent.*

Example 3.25. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.2, 0.3), (0.8, 0.7)\rangle$. We have $\mu_G(a) = 0.2$, $\mu_G(b) = 0.3$, $\nu_G(a) = 0.8$ and $\nu_G(b) = 0.7$. Consider an IFS $A = \langle x, (0.1, 0.4), (0.9, 0.6)\rangle$. Then A is an IF$\alpha$CS but not an IF$G'''$CS in (X, τ).*

Example 3.26. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ be an IFT on X, where $G_1 = \langle x, (0.3, 0.2), (0.6, 0.7)\rangle$ and $G_2 = \langle x, (0.3, 0.4), (0.6, 0.5)\rangle$. We have $\mu_{G_1}(a) = 0.3$, $\mu_{G_1}(b) = 0.2$, $\nu_{G_1}(a) = 0.6$, $\nu_{G_1}(b) = 0.7$, $\mu_{G_2}(a) = 0.3$, $\mu_{G_2}(b) = 0.4$, $\nu_{G_2}(a) = 0.6$ and $\nu_{G_2}(b) = 0.5$. Consider an IFS $A = \langle x, (0.65, 0.75), (0.25, 0.15)\rangle$. Then A is an IFG$'''$CS but not an IF$\alpha$CS in (X, τ).*

Remark 3.27. *IFSCS and IFG$'''$CS are independent.*

Example 3.28. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.3, 0.4), (0.6, 0.5)\rangle$. We have $\mu_G(a) = 0.3$, $\mu_G(b) = 0.4$, $\nu_G(a) = 0.6$ and $\nu_G(b) = 0.5$. Consider an IFS $A = \langle x, (0.4, 0.5), (0.5, 0.4)\rangle$. Then A is an IFSCS but not an IFG$'''$CS in (X, τ).*

Example 3.29. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ be an IFT on X, where $G_1 = \langle x, (0.2, 0.3), (0.7, 0.6)\rangle$ and $G_2 = \langle x, (0.3, 0.4), (0.6, 0.5)\rangle$. We have $\mu_{G_1}(a) = 0.2$, $\mu_{G_1}(b) = 0.3$, $\nu_{G_1}(a) = 0.7$, $\nu_{G_1}(b) = 0.6$, $\mu_{G_2}(a) = 0.3$, $\mu_{G_2}(b) = 0.4$, $\nu_{G_2}(a) = 0.6$ and $\nu_{G_2}(b) = 0.5$. Consider an IFS $A = \langle x, (0.8, 0.7), (0.1, 0.2)\rangle$. Then A is an IFG$'''$CS but not an IFSCS in (X, τ).*

Theorem 3.30. *If A and B are IFG$'''$CSs in an IFTS (X, τ), then $A \cup B$ is also an IFG$'''$CS in (X, τ).*

Proof. If $A \cup B \subseteq G$ where G is IFGSOS, then $A \subseteq G$ and $B \subseteq G$. Since A and B are IFG$'''$CSs, $cl(A) \subseteq G$ and $cl(B) \subseteq G$ and hence $cl(A) \cup cl(B) = cl(A \cup B) \subseteq G$. Thus $A \cup B$ is an IFG$'''$CS in (X, τ). □

Remark 3.31. *The intersection of two IFG$'''$CSs in an IFTS (X, τ) need not be an IFG$'''$CS in (X, τ).*

Example 3.32. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ be an IFT on X, where $G_1 = \langle x, (0.2, 0.3), (0.7, 0.6)\rangle$ and $G_2 = \langle x, (0.3, 0.4), (0.6, 0.5)\rangle$. We have $\mu_{G_1}(a) = 0.2$, $\mu_{G_1}(b) = 0.3$, $\nu_{G_1}(a) = 0.7$, $\nu_{G_1}(b) = 0.6$, $\mu_{G_2}(a) = 0.3$, $\mu_{G_2}(b) = 0.4$, $\nu_{G_2}(a) = 0.6$ and $\nu_{G_2}(b) = 0.5$. Consider the two IFSs $A = \langle x, (0.1, 0.7), (0.8, 0.2)\rangle$ and $B = \langle x, (0.8, 0.2), (0.1, 0.7)\rangle$. Then A and B are IFG$'''$CSs. But $A \cap B = \langle x, (0.1, 0.2), (0.8, 0.7)\rangle$ is not an IFG$'''$CS in (X, τ).*

Theorem 3.33. *If A is an IFG$'''$CS in an IFTS (X, τ) and $A \subseteq B \subseteq cl(A)$, then B is an IFG$'''$CS in (X, τ).*

Proof. Let $B \subseteq U$ where U is an IFGSOS in (X, τ). Since $A \subseteq B$, $A \subseteq U$. Since A is an IFG$'''$CS in (X, τ), $cl(A) \subseteq U$. Since $B \subseteq cl(A)$, $cl(B) \subseteq cl(A) \subseteq U$. Therefore B is an IFG$'''$CS in (X, τ). □

Theorem 3.34. *Let A be an IFS in an IFTS (X, τ). Then A is an IFG$'''$CS if and only if $A\bar{q}F$ implies $cl(A)\bar{q}F$ for every IFGSCS F in (X, τ).*

Proof. Necessary Part: Let F be an IFGSCS in (X, τ) and let $A\bar{q}F$. Then $A \subseteq F^c$, where F^c is an IFGSOS in (X, τ). Therefore by hypothesis $cl(A) \subseteq F^c$. Hence $cl(A)\bar{q}F$.

Sufficient Part: Let F be an IFGSCS in (X, τ) and let A be an IFS in (X, τ). By hypothesis, $A\bar{q}F$ implies $cl(A)\bar{q}F$. Then $cl(A) \subseteq F^c$ whenever $A \subseteq F^c$ and F^c is an IFGSOS in (X, τ). Hence A is an IFG$'''$CS in (X, τ). □

Theorem 3.35. *Let (X, τ) be an IFTS. Then IFC(X) = IFG$'''$C(X) if every IFS in (X, τ) is an IFGSOS in X, where IFC(X) denotes the collection of IFCSs of an IFTS (X, τ).*

Proof. Suppose that every IFS in (X, τ) is an IFGSOS in X. Let $A \in$ IFG$'''$C(X). Then $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFGSOS in X. Since every IFS is an IFGSOS, A is also an IFGSOS and $A \subseteq A$. Therefore $cl(A) \subseteq A$. Hence cl(A) = A. Therefore $A \in$ IFC(X). Hence IFG$'''$C(X) $\subseteq$ IFC(X) $\rightarrow$ (1). Let $A \in$ IFC(X). Then by Theorem 3.4, $A \in$ IFG$'''$C(X). Hence IFC(X) $\subseteq$ IFG$'''$C(X) $\rightarrow$ (2). From (1) and (2), we have IFC(X) = IFG$'''$C(X). □

Proposition 3.36. *If A is an IFGSOS and IFG$'''$CS in an IFTS (X, τ), then A is an IFCS in (X, τ).*

Proof. Since A is an IFGSOS and IFG$'''$CS, $cl(A) \subseteq A$. Hence A is an IFCS in (X, τ). □

4. Intuitionistic Fuzzy g'''-open Sets

In this section we introduce intuitionistic fuzzy g'''-open sets and study some of its properties.

Definition 4.1. *An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy g'''-open set (IFG'''OS in short) if A^c is an intuitionistic fuzzy g'''-closed set in (X, τ). The collection of all intuitionistic fuzzy g'''-open sets in X is denoted by IFG'''O(X).*

Example 4.2. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.2, 0.3), (0.7, 0.6)\rangle$. We have $\mu_G(a) = 0.2$, $\mu_G(b) = 0.3$, $\nu_G(a) = 0.7$ and $\nu_G(b) = 0.6$. Consider an IFS $A = \langle x, (0.2, 0.3), (0.7, 0.6)\rangle$. Then by Example 3.2, A^c is an IFG'''CS in (X, τ). Hence A is an IFG'''OS in (X, τ).*

Example 4.3. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.2, 0.3), (0.7, 0.6)\rangle$. We have $\mu_G(a) = 0.2$, $\mu_G(b) = 0.3$, $\nu_G(a) = 0.7$ and $\nu_G(b) = 0.6$. Consider an IFS $A = \langle x, (0.6, 0.5), (0.3, 0.4)\rangle$. Then by Example 3.3, A^c is not an IFG'''CS in (X, τ). Hence A is not an IFG'''OS in (X, τ).*

Theorem 4.4. *An IFS A in an IFTS (X, τ) is IFG'''OS if and only if $F \subseteq int(A)$ whenever $F \subseteq A$ and F^c is an IFGSOS.*

Proof. Necessary Part: Let A be an IFG'''OS in (X, τ). Let F^c be an IFGSOS such that $F \subseteq A$. Then $A^c \subseteq F^c$ where A^c is an IFG'''CS. Hence $cl(A^c) \subseteq F^c$. This implies $(int(A))^c \subseteq F^c$. Thus we have $F \subseteq int(A)$ whenever $F \subseteq A$ and F^c is an IFGSOS.

Sufficient Part: Let $F \subseteq int(A)$ whenever $F \subseteq A$ and F^c is an IFGSOS in (X, τ). This implies $(int(A))^c \subseteq F^c$ whenever $A^c \subseteq F^c$ and F^c is an IFGSOS. That is $cl(A^c) \subseteq F^c$ whenever $A^c \subseteq F^c$ and F^c is an IFGSOS. Therefore A^c is an IFG'''CS. Hence A is an IFG'''OS in (X, τ). □

Theorem 4.5. *Every IFOS is an IFG'''OS.*

Proof. Let A be an IFOS in (X, τ). Therefore A^c is an IFCS in (X, τ). Then by Theorem 3.4, A^c is an IFG'''CS in (X, τ). Therefore A is an IFG'''OS in (X, τ). □

The converse of Theorem 4.5 need not be true as seen from the following Example.

Example 4.6. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.7, 0.6), (0.2, 0.3)\rangle$. We have $\mu_G(a) = 0.7$, $\mu_G(b) = 0.6$, $\nu_G(a) = 0.2$ and $\nu_G(b) = 0.3$. Consider an IFS $A = \langle x, (0.8, 0.7), (0.1, 0.2)\rangle$. Then by Example 3.5, A^c is an IFG'''CS but not an IFCS in (X, τ). Hence A is an IFG'''OS but not an IFOS in (X, τ).*

Theorem 4.7. *Every IFG'''OS is an IFGSPOS.*

Proof. Let A be an IFG'''OS in (X, τ). Therefore A^c is an IFG'''CS in (X, τ). Then by Theorem 3.6, A^c is an IFGSPCS in (X, τ). Therefore A is an IFGSPOS in (X, τ). □

The converse of Theorem 4.7 need not be true as seen from the following Example.

Example 4.8. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.2, 0.3), (0.7, 0.6)\rangle$. We have $\mu_G(a) = 0.2$, $\mu_G(b) = 0.3$, $\nu_G(a) = 0.7$ and $\nu_G(b) = 0.6$. Consider an IFS $A = \langle x, (0.6, 0.5), (0.3, 0.4)\rangle$. Then by Example 3.7, A^c is an IFGSPCS but not an IFG'''CS in (X, τ). Hence A is an IFGSPOS but not an IFG'''OS in (X, τ).*

Theorem 4.9. *Every IFG'''OS is an IFωOS.*

Proof. Let A be an IFG$'''$OS in (X, τ). Therefore A^c is an IFG$'''$CS in (X, τ). Then by Theorem 3.8, A^c is an IFωCS in (X, τ). Therefore A is an IFωOS in (X, τ). □

The converse of Theorem 4.9 need not be true as seen from the following Example.

Example 4.10. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.3, 0.4), (0.6, 0.5)\rangle$. We have $\mu_G(a) = 0.3$, $\mu_G(b) = 0.4$, $\nu_G(a) = 0.6$ and $\nu_G(b) = 0.5$. Consider an IFS $A = \langle x, (0.2, 0.3), (0.7, 0.6)\rangle$. Then by Example 3.9, A^c is an IFωCS but not an IFG$'''$CS in (X, τ). Hence A is an IFωOS but not an IFG$'''$OS in (X, τ).*

Theorem 4.11. *Every IFG$'''$OS is an IFGOS.*

Proof. Let A be an IFG$'''$OS in (X, τ). Therefore A^c is an IFG$'''$CS in (X, τ). Then by Theorem 3.10, A^c is an IFGCS in (X, τ). Therefore A is an IFGOS in (X, τ). □

The converse of Theorem 4.11 need not be true as seen from the following Example.

Example 4.12. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.6, 0.5), (0.3, 0.4)\rangle$. We have $\mu_G(a) = 0.6$, $\mu_G(b) = 0.5$, $\nu_G(a) = 0.3$ and $\nu_G(b) = 0.4$. Consider an IFS $A = \langle x, (0.2, 0.3), (0.7, 0.6)\rangle$. Then by Example 3.11, A^c is an IFGCS but not an IFG$'''$CS in (X, τ). Hence A is an IFGOS but not an IFG$'''$OS in (X, τ).*

Theorem 4.13. *Every IFG$'''$OS is an IFαGOS.*

Proof. Let A be an IFG$'''$OS in (X, τ). Therefore A^c is an IFG$'''$CS in (X, τ). Then by Theorem 3.12, A^c is an IFαGCS in (X, τ). Therefore A is an IFαGOS in (X, τ). □

The converse of Theorem 4.13 need not be true as seen from the following Example.

Example 4.14. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.6, 0.7), (0.3, 0.2)\rangle$. We have $\mu_G(a) = 0.6$, $\mu_G(b) = 0.7$, $\nu_G(a) = 0.3$ and $\nu_G(b) = 0.2$. Consider an IFS $A = \langle x, (0.3, 0.2), (0.6, 0.7)\rangle$. Then by Example 3.13, A^c is an IFαGCS but not an IFG$'''$CS in (X, τ). Hence A is an IFαGOS but not an IFG$'''$OS in (X, τ).*

Theorem 4.15. *Every IFG$'''$OS is an IFGSOS.*

Proof. Let A be an IFG$'''$OS in (X, τ). Therefore A^c is an IFG$'''$CS in (X, τ). Then by Theorem 3.14, A^c is an IFGSCS in (X, τ). Therefore A is an IFGSOS in (X, τ). □

The converse of Theorem 4.15 need not be true as seen from the following Example.

Example 4.16. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.2, 0.3), (0.7, 0.6)\rangle$. We have $\mu_G(a) = 0.2$, $\mu_G(b) = 0.3$, $\nu_G(a) = 0.7$ and $\nu_G(b) = 0.6$. Consider an IFS $A = \langle x, (0.3, 0.2), (0.6, 0.7)\rangle$. Then by Example 3.15, A^c is an IFGSCS but not an IFG$'''$CS in (X, τ). Hence A is an IFGSOS but not an IFG$'''$OS in (X, τ).*

Theorem 4.17. *Every IFG$'''$OS is an IFG_S'''OS.*

Proof. Let A be an IFG$'''$OS in (X, τ). Therefore A^c is an IFG$'''$CS in (X, τ). Then by Theorem 3.17, A^c is an IFG_S'''CS in (X, τ). Therefore A is an IFG_S'''OS in (X, τ). □

The converse of Theorem 4.17 need not be true as seen from the following Example.

Example 4.18. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.2, 0.3), (0.7, 0.6)\rangle$. We have $\mu_G(a) = 0.2$, $\mu_G(b) = 0.3$, $\nu_G(a) = 0.7$ and $\nu_G(b) = 0.6$. Consider an IFS $A = \langle x, (0.6, 0.5), (0.3, 0.4)\rangle$. Then by Example 3.18, A^c is an IFG_S'''CS but not an IFG$'''$CS in (X, τ). Hence A is an IFG_S'''OS but not an IFG$'''$OS in (X, τ).*

Theorem 4.19. *Every IFG'''OS is an IFαGSOS.*

Proof. Let A be an IFG'''OS in (X, τ). Therefore A^c is an IFG'''CS in (X, τ). Then by Theorem 3.19, A^c is an IFαGSCS in (X, τ). Therefore A is an IFαGSOS in (X, τ). □

The converse of Theorem 4.19 need not be true as seen from the following Example.

Example 4.20. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.4, 0.3), (0.5, 0.6)\rangle$. We have $\mu_G(a) = 0.4$, $\mu_G(b) = 0.3$, $\nu_G(a) = 0.5$ and $\nu_G(b) = 0.6$. Consider an IFS $A = \langle x, (0.3, 0.2), (0.6, 0.7)\rangle$. Then by Example 3.20, A^c is an IFαGSCS but not an IFG'''CS in (X, τ). Hence A is an IFαGSOS but not an IFG'''OS in (X, τ).*

Theorem 4.21. *Every IFG'''OS is an IFG'''_αOS.*

Proof. Let A be an IFG'''OS in (X, τ). Therefore A^c is an IFG'''CS in (X, τ). Then by Theorem 3.22, A^c is an IFG'''_αCS in (X, τ). Therefore A is an IFG'''_αOS in (X, τ). □

The converse of Theorem 4.21 need not be true as seen from the following Example.

Example 4.22. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.3, 0.6), (0.6, 0.3)\rangle$. We have $\mu_G(a) = 0.3$, $\mu_G(b) = 0.6$, $\nu_G(a) = 0.6$ and $\nu_G(b) = 0.3$. Consider an IFS $A = \langle x, (0.7, 0.6), (0.2, 0.3)\rangle$. Then A is an IF$G'''_\alpha$OS but not an IF$G'''$OS in (X, τ).*

Remark 4.23. *IFαOS and IFG'''OS are independent.*

Example 4.24. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.2, 0.3), (0.8, 0.7)\rangle$. We have $\mu_G(a) = 0.2$, $\mu_G(b) = 0.3$, $\nu_G(a) = 0.8$ and $\nu_G(b) = 0.7$. Consider an IFS $A = \langle x, (0.9, 0.6), (0.1, 0.4)\rangle$. Then A is an IF$\alpha$OS but not an IF$G'''$OS in (X, τ).*

Example 4.25. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ be an IFT on X, where $G_1 = \langle x, (0.3, 0.2), (0.6, 0.7)\rangle$ and $G_2 = \langle x, (0.3, 0.4), (0.6, 0.5)\rangle$. We have $\mu_{G_1}(a) = 0.3$, $\mu_{G_1}(b) = 0.2$, $\nu_{G_1}(a) = 0.6$, $\nu_{G_1}(b) = 0.7$, $\mu_{G_2}(a) = 0.3$, $\mu_{G_2}(b) = 0.4$, $\nu_{G_2}(a) = 0.6$ and $\nu_{G_2}(b) = 0.5$. Consider an IFS $A = \langle x, (0.25, 0.15), (0.65, 0.75)\rangle$. Then A is an IF$G'''$OS but not an IF$\alpha$OS in (X, τ).*

Remark 4.26. *IFSOS and IFG'''OS are independent.*

Example 4.27. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.3, 0.4), (0.6, 0.5)\rangle$. We have $\mu_G(a) = 0.3$, $\mu_G(b) = 0.4$, $\nu_G(a) = 0.6$ and $\nu_G(b) = 0.5$. Consider an IFS $A = \langle x, (0.5, 0.4), (0.4, 0.5)\rangle$. Then A is an IFSOS but not an IFG'''OS in (X, τ).*

Example 4.28. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G_1, G_2\ 1_\sim\}$ be an IFT on X, where $G_1 = \langle x, (0.2, 0.3), (0.7, 0.6)\rangle$ and $G_2 = \langle x, (0.3, 0.4), (0.6, 0.5)\rangle$. We have $\mu_{G_1}(a) = 0.2$, $\mu_{G_1}(b) = 0.3$, $\nu_{G_1}(a) = 0.7$, $\nu_{G_1}(b) = 0.6$, $\mu_{G_2}(a) = 0.3$, $\mu_{G_2}(b) = 0.4$, $\nu_{G_2}(a) = 0.6$ and $\nu_{G_2}(b) = 0.5$. Consider an IFS $A = \langle x, (0.1, 0.2), (0.8, 0.7)\rangle$. Then A is an IF$G'''$OS but not an IFSOS in (X, τ).*

Theorem 4.29. *If A and B are IFG'''OSs in an IFTS (X, τ), then $A \cap B$ is also an IFG'''OS in (X, τ).*

Proof. Let A and B be IFG'''OSs in (X, τ). Therefore A^c and B^c are IFG'''CSs in (X, τ). By Theorem 3.30, (A$^c \cup$ B^c) is an IFG'''CS in (X, τ). Since (A$^c \cup$ B^c) = (A $\cap$ B)^c, $A \cap B$ is an IFG'''OS in (X, τ). □

Remark 4.30. *The union of two IFG'''OSs in an IFTS (X, τ) need not be an IFG'''OS in (X, τ).*

Example 4.31. *Let $X = \{a, b\}$. Let $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ be an IFT on X, where $G_1 = \langle x, (0.2, 0.3), (0.7, 0.6)\rangle$ and $G_2 = \langle x, (0.3, 0.4), (0.6, 0.5)\rangle$. We have $\mu_{G_1}(a) = 0.2$, $\mu_{G_1}(b) = 0.3$, $\nu_{G_1}(a) = 0.7$, $\nu_{G_1}(b) = 0.6$, $\mu_{G_2}(a) = 0.3$, $\mu_{G_2}(b) = 0.4$, $\nu_{G_2}(a) = 0.6$ and $\nu_{G_2}(b) = 0.5$. Consider the two IFSs $A = \langle x, (0.1, 0.7), (0.8, 0.2)\rangle$ and $B = \langle x, (0.8, 0.2), (0.1, 0.7)\rangle$. Then A and B are IFG$'''$OSs. But $A \cup B = \langle x, (0.8, 0.7), (0.1, 0.2)\rangle$ is not an IFG$'''$OS in (X, τ).*

Theorem 4.32. *If A is an IFG$'''$OS in an IFTS (X, τ) such that int(A) $\subseteq$ B $\subseteq$ A, then B is IFG$'''$OS in (X, τ).*

Proof. Let A be an IFG$'''$OS in (X, τ) such that int(A) $\subseteq$ B $\subseteq$ A. It implies $A^c \subseteq B^c \subseteq cl(A^c)$ where A^c is an IFG$'''$CS in (X, τ). By Theorem 3.33, B^c is an IFG$'''$CS in (X, τ). Therefore B is an IFG$'''$OS in (X, τ). □

Theorem 4.33. *Let (X, τ) be an IFTS. Then IFO(X) = IFG$'''$O(X) if every IFS in (X, τ) is an IFGSOS in X, where IFO(X) denotes the collection of IFOSs of an IFTS (X, τ).*

Proof. Suppose that every IFS in (X, τ) is an IFGSOS in X. Then by Theorem 3.35, we have IFC(X) = IFG$'''$C(X). Therefore IFO(X) = IFG$'''$O(X). □

References

[1] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy sets and Systems, 20(1986), 87-96, doi: 10.1016/S0165-0114(86)80034-3.

[2] K. K. Azad, On fuzzy semi continuity, fuzzy almost continuity and fuzzy weakly continuity, Jour. Math. Anal. Appl., 82(1981), 14-32.

[3] D. Coker, An introduction to intuitionistic fuzzy topological spaces, Fuzzy sets and Systems, 88(1997), 81-89, doi: 10.1016/S0165-0114(96)00076-0.

[4] H. Gurcay, D. Coker and Es. A. Haydar, On fuzzy continuity in intuitionistic fuzzy topological spaces, J. Fuzzy Math., 5(1997), 365-378.

[5] K. Hur and Y. B. Jun, On intuitionistic fuzzy alpha continuous mappings, Honam Math. J., 25(2003), 131-139.

[6] M. Jeyaraman, A. Yuvarani and O. Ravi, Intuitionistic fuzzy α-generalized semi continuous and irresolute mappings, International Journal of Analysis and Applications, 3(2)(2013), 93-103.

[7] Joung Kon Jeon, Young Bae Jun and Jin Han Park, Intuitionistic fuzzy alpha continuity and intuitionistic fuzzy pre continuity, Int. J. Mathematics and Mathematical Sciences, 19(2005), 3091-3101, doi: 10.1155/IJMMS.2005.3091.

[8] N. Levine, Generalized closed sets in topology, Rend. Circ. Mat. Palermo, 19(1970), 89-96, doi: 10.1007/BF02843888.

[9] P. Rajarajeswari and R. Krishna Moorthy, Intuitionistic fuzzy weakly generalized closed mappings, Journal of Advanced Studies in Topology, 4(2012), 20-27.

[10] K. Sakthivel, Intuitionistic fuzzy alpha generalized continuous mappings and intuitionistic fuzzy alpha generalized irresolute mappings, Applied Mathematics Sciences, 4(2010), 1831-1842.

[11] R. Santhi and D. Jayanthi, Intuitionistic fuzzy generalized semipreclosed mappings, NIFS, 16(2010), 28-39.

[12] R. Santhi and K. Sakthivel, Intuitionistic fuzzy generalized semi continuous mappings, Advances in Theoretical and Applied Mathematics, 5(2009), 73-82.

[13] R. Santhi and K. Arun Prakash, On intuitionistic fuzzy semi generalized closed sets and its applications, Int. J. Contemp. Math. Sci., 5(2010), 1677-1688.

[14] S. S. Thakur and Jyoti Pandey Bajpai, Intuitionistic fuzzy w-closed sets and intuitionistic fuzzy w-continuity, International Journal of Contemporary Advanced Mathematics, 1(2010), 1-15.

[15] S. S. Thakur and Rekha Chaturvedi, Generalized closed sets in intuitionistic fuzzy topology, The Journal of Fuzzy Mathematics, 16(2008), 559-572.

[16] S. S. Thakur and Rekha Chaturvedi, Regular generalized closed sets in intuitionistic fuzzy topological spaces, Universitatea Din Bacau Studii Si Cercetar Stiintifice, 6(2006), 257-272.

[17] Young Bae Jun and Seok-Zun Song, Intuitionistic fuzzy semi preopen sets and Intuitionistic semi pre continuous mappings, J. Appl. Math. & Computing, 19(2005), 467-474.

[18] L. A. Zadeh, Fuzzy sets, Information and Control, 8(1965), 338-353, doi: 10.1016/S0019-9958 (65)90241-X.