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$\mathcal{I}_{mg^{\#}}$-closed Sets

Research Article

O.Ravi[1]*, K.Indirani[2] and A.R.Thilagavathi[3]

1 Department of Mathematics, P. M. Thevar College, Usilampatti, Madurai, Tamil Nadu, India.
2 Department of Mathematics, Nirmala College for Women, Coimbatore, Tamil Nadu, India.
3 Department of Mathematics, Sri G. V. G Visalatkshi College for Women, Udumalpet, Thirupur District, Tamil Nadu, India.

Abstract: In this paper, we introduce the notion of $\mathcal{I}_{mg^{\#}}$-closed sets. In Sections 3 and 4, we obtain some basic properties and characterizations of $\mathcal{I}_{mg^{\#}}$-closed sets. In the last section, we define several new subsets in ideal topological spaces which lie between $\star$-closed sets and $\mathcal{I}_{g^\star}$-closed sets.

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1. Introduction

In 1970, Levine [6] introduced the notion of generalized closed (briefly g-closed) sets in topological spaces. Veerakumar [18] introduced the notion of $g^\star$-closed sets in topological spaces. Recently, many variations or generalizations of g-closed sets [1, 2, 7, 12] and $g^\star$-closed sets [3, 9, 15–17] are introduced and investigated. By combining a topological space (X, τ) and an ideal $\mathcal{I}$ on (X, τ), Ravi et al. [14] introduced the notion of $\mathcal{I}_{g^\star}$-closed sets and investigated the properties of $\mathcal{I}_{g^\star}$-closed sets. In this paper, we introduce the notion of $\mathcal{I}_{mg^{\#}}$-closed sets. In Sections 3 and 4, we obtain some basic properties and characterizations of $\mathcal{I}_{mg^{\#}}$-closed sets. In the last section, we define several new subsets in ideal topological spaces which lie between $\star$-closed sets and $\mathcal{I}_{g^\star}$-closed sets.

2. Preliminaries

Definition 2.1 ([13]). *A subfamily $m_X \subseteq \wp(X)$ is said to be a minimal structure (briefly, m-structure) on X if $\emptyset$, $X \in m_X$. The pair (X, m_X) is called a minimal space (briefly m-space). Each member of m_X is said to be m-open and the complement of an m-open set is said to be m-closed.*
Notice that (X, m_X, $\mathcal{I}$) is called an ideal m-space.

Remark 2.2. *Let (X, τ) be a topological space. Then $m_X = \tau$, SO(X) and SPO(X) are minimal structures on X.*

Definition 2.3. *Let (X, m_X) be an m-space. For a subset A of X, the m-closure of A and the m-interior of A are defined in [8] as follows:*

* *E-mail: siingam@yahoo.com*

(1). m-cl(A) $= \cap\{F : A \subseteq F,\ F^c \in m_X\}$,

(2). m-int(A) $= \cup\{U : U \subseteq A,\ U \in m_X\}$.

Lemma 2.4 ([5]). *Let* $(X, \tau, \mathcal{I})$ *be an ideal topological space and A, B subsets of X. Then the following properties hold:*

(1). $A \subseteq B \Rightarrow A^\star \subseteq B^\star$,

(2). $A^\star = cl(A^\star) \subseteq cl(A)$,

(3). $(A^\star)^\star \subseteq A^\star$,

(4). $(A \cup B)^\star = A^\star \cup B^\star$,

(5). $(A \cap B)^\star \subseteq A^\star \cap B^\star$.

Theorem 2.5 ([14]). *Let* $(X, \tau, \mathcal{I})$ *be an ideal topological space. Then every* $g^\star$*-closed set is an* $\mathcal{I}_{g^\star}$*-closed set but not conversely.*

Definition 2.6 ([10]). *Let* (X, τ) *be a topological space and* m_X *an m-structure on* X*. A subset A of X is said to be*

(1). $mg^\star$*-closed if* $cl(A) \subseteq U$ *whenever* $A \subseteq U$ *and* $U \in m_X$,

(2). $mg^\star$*-open if its complement is* $mg^\star$*-closed.*

The family of all $mg^\star$*-open sets in X is an m-structure on X and it is denoted by* $mg^\star O(X)$.

3. $\mathcal{I}_{mg^{\#}}$-closed Sets

In this section, let (X, τ, $\mathcal{I}$) be an ideal topological space and m_X an m-structure on X. We obtain several basic properties of $\mathcal{I}_{mg^{\#}}$-closed sets.

Definition 3.1. *Let* $(X, \tau, \mathcal{I})$ *be an ideal topological space and* m_X *an m-structure on X. A subset A of X is said to be*

(1). $\mathcal{I}_{mg^{\#}}$*-closed if* $A^\star \subseteq U$ *whenever* $A \subseteq U$ *and U is* $mg^\star$*-open,*

(2). $\mathcal{I}_{mg^{\#}}$*-open if its complement is* $\mathcal{I}_{mg^{\#}}$*-closed.*

Remark 3.2. *Let* $(X, \tau, \mathcal{I})$ *be an ideal topological space and A a subset of X. If* $mg^\star O(X) = gO(X)$ *(resp.* τ, $\pi O(X)$, $RO(X)$*) and A is* $\mathcal{I}_{mg^{\#}}$*-closed, then A is said to be* $\mathcal{I}_{g^\star}$*-closed (resp.* $\mathcal{I}_g$*-closed,* $\mathcal{I}_{\pi g}$*-closed,* $\mathcal{I}_{rg}$*-closed).*

Proposition 3.3. *Every* $mg^{\#}$*-closed set is* $\mathcal{I}_{mg^{\#}}$*-closed but not conversely.*

Proof. Let A be an $mg^{\#}$-closed, then cl(A)$\subseteq$U whenever A$\subseteq$U and U is $mg^\star$-open. By Lemma 2.4, $A^\star \subseteq$cl(A). Hence A is $\mathcal{I}_{mg^{\#}}$-closed. □

Example 3.4. *Let* $X=\{a, b, c\}$, $\tau=\{\phi, X, \{b\}, \{a, c\}\}$, $\mathcal{I}=\{\phi, \{c\}\}$ *and* $m_X=\{\phi, X, \{c\}\}$*. Then* $mg^{\#}$*-closed sets are* ϕ, X, $\{b\}$, $\{a, b\}$ *and* $\{a, c\}$*; and* $\mathcal{I}_{mg^{\#}}$*-closed sets are* ϕ, X, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{a, c\}$ *and* $\{b, c\}$*. It is clear that* $\{c\}$ *is* $\mathcal{I}_{mg^{\#}}$*-closed set but it is not* $mg^{\#}$*-closed.*

Proposition 3.5. *Let* $gO(X) \subseteq mg^\star O(X)$*. Then every* $\mathcal{I}_{mg^{\#}}$*-closed set is* $\mathcal{I}_{g^\star}$*-closed but not conversely.*

Proof. Suppose that A is an $\mathcal{I}_{mg^{\#}}$-closed set. Let A$\subseteq$U and U$\in g$O(X). Since gO(X)$\subseteq$ mg*O(X), $A^\star\subseteq$U and hence A is $\mathcal{I}_{g^\star}$-closed. □

Example 3.6. *Let $X = \{a, b, c\}$, $\tau=\{\phi, X, \{c\}, \{b, c\}\}$, $\mathcal{I}=\{\phi, \{a\}\}$ and $m_X=\{\phi, X\}$. Then $mg^{\star}$-open sets are the power set of X; g-open sets are ϕ, X, $\{b\}$, $\{c\}$ and $\{b, c\}$; $\mathcal{I}_{mg\#}$-closed sets are ϕ, X, $\{a\}$ and $\{a, b\}$; and $\mathcal{I}_{g^{\star}}$-closed sets are ϕ, X, $\{a\}$, $\{a, b\}$ and $\{a, c\}$. It is clear that $\{a, c\}$ is $\mathcal{I}_{g^{\star}}$-closed set but it is not $\mathcal{I}_{mg\#}$-closed.*

Remark 3.7. *Let $gO(X)\subseteq mg^{\star}O(X)$. Then we have the following implications for the subsets stated above.*

$$\begin{array}{ccccc} \textit{closed} & \longrightarrow & mg^{\#}\textit{-closed} & \longrightarrow & g^{\star}\textit{-closed} \\ \downarrow & & \downarrow & & \downarrow \\ \star\textit{-closed} & \longrightarrow & \mathcal{I}_{mg\#}\textit{-closed} & \longrightarrow & \mathcal{I}_{g^{\star}}\textit{-closed} \end{array}$$

The implications in the first line are known in [11]. The three vertical implications follow from Proposition 3.3, Theorem 2.5 and Lemma 2.4(2). It is obvious that every $\star$-closed set is $\mathcal{I}_{mg\#}$-closed and by Proposition 3.5, every $\mathcal{I}_{mg\#}$-closed set is $\mathcal{I}_{g^{\star}}$-closed.

Lemma 3.8 ([4])**.** *Let $\{A_\lambda : \lambda\in\wedge\}$ be a locally finite family of sets in $(X, \tau, \mathcal{I})$. Then $\cup_{\lambda\in\wedge} A_\lambda^{\star}=(\cup_{\lambda\in\wedge}A_\lambda)^{\star}$.*

Proposition 3.9. *If $\{A_\lambda : \lambda\in\wedge\}$ is a locally finite family of sets in $(X, \tau, \mathcal{I})$ and A_λ is $\mathcal{I}_{mg\#}$-closed for each $\lambda\in\wedge$, then $(\cup_{\lambda\in\wedge}A_\lambda)$ is $\mathcal{I}_{mg\#}$-closed.*

Proof. Let $(\cup_{\lambda\in\wedge}A_\lambda)\subseteq U$ where U is $mg^{\star}$-open. Then $A_\lambda\subseteq U$ for each $\lambda\in\wedge$. Since A_λ is $\mathcal{I}_{mg\#}$-closed for each $\lambda\in\wedge$, we have $A_\lambda^{\star}\subseteq U$ and hence $\cup_{\lambda\in\wedge}A_\lambda^{\star}\subseteq U$. By Lemma 3.8, $(\cup_{\lambda\in\wedge}A_\lambda)^{\star}\subseteq U$. Hence $(\cup_{\lambda\in\wedge}A_\lambda)$ is $\mathcal{I}_{mg\#}$-closed. □

Corollary 3.10. *If A and B are $\mathcal{I}_{mg\#}$-closed sets in $(X, \tau, \mathcal{I})$, then $A\cup B$ is $\mathcal{I}_{mg\#}$-closed.*

Proof. Let $A\cup B\subseteq U$ where U is $mg^{\star}$-open. Then $A\subseteq U$ and $B\subseteq U$. Since A and B are $\mathcal{I}_{mg\#}$-closed, then $A^{\star}\subseteq U$ and $B^{\star}\subseteq U$ and so $A^{\star}\cup B^{\star}\subseteq U$. By Lemma 2.4, $(A\cup B)^{\star}=A^{\star}\cup B^{\star}$. Hence $A\cup B$ is $\mathcal{I}_{mg\#}$-closed. □

Definition 3.11 ([11])**.** *An m-structure $mg^{\star}O(X)$ on a nonempty set X is said to have property $\mathcal{J}$ if the union of any family of subsets belonging to $mg^{\star}O(X)$ belongs to $mg^{\star}O(X)$.*

Example 3.12. *Let $X=\{a, b, c\}$, $\tau=\{\phi, X, \{c\}, \{b, c\}\}$ and $m_X = \{\phi, X, \{c\}\}$. Then $mg^{\star}$-open sets are ϕ, X, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, c\}$ and $\{b, c\}$. It is shown that $mg^{\star}O(X)$ does not have property $\mathcal{J}$.*

Proposition 3.13. *Let $gO(X)\subseteq mg^{\star}O(X)$ and $mg^{\star}O(X)$ have property $\mathcal{J}$. If A is $\mathcal{I}_{mg\#}$-closed in $(X, \tau, \mathcal{I})$ and B is closed in (X, τ), then $A\cap B$ is $\mathcal{I}_{mg\#}$-closed.*

Proof. Let $A\cap B\subseteq U$ where U is $mg^{\star}$-open. Then we have $A\subseteq U\cup(X-B)$. Since $\tau\subseteq gO(X) \subseteq mg^{\star}O(X)$ and so $U\cup(X-B)$ is $mg^{\star}$-open. Since A is $\mathcal{I}_{mg\#}$-closed, then $A^{\star}\subseteq U\cup(X-B)$ and hence $A^{\star}\cap B\subseteq U\cap B\subseteq U$. By Lemma 2.4, $(A\cap B)^{\star}\subseteq A^{\star}\cap B^{\star}$. Since $\tau\subseteq\tau^{\star}$, B is $\star$-closed and $B^{\star}\subseteq B$. Therefore, we obtain $(A\cap B)^{\star}\subseteq A^{\star}\cap B^{\star}\subseteq A^{\star}\cap B\subseteq U$. This shows that $A\cap B$ is $\mathcal{I}_{mg\#}$-closed. □

Proposition 3.14. *If A is $\mathcal{I}_{mg\#}$-closed and $A\subseteq B\subseteq cl^{\star}(A)$, then B is $\mathcal{I}_{mg\#}$-closed.*

Proof. Let $B\subseteq U$ where U is $mg^{\star}$-open. Then $A\subseteq U$ and A is $\mathcal{I}_{mg\#}$-closed. Therefore $A^{\star}\subseteq U$ and $B^{\star}\subseteq cl^{\star}(B)\subseteq cl^{\star}(A)=A\cup A^{\star}\subseteq U$. Hence B is $\mathcal{I}_{mg\#}$-closed. □

Proposition 3.15. *A subset A of X is $\mathcal{I}_{mg\#}$-open if and only if $F\subseteq int^{\star}(A)$ whenever $F\subseteq A$ and F is $mg^{\star}$-closed.*

Proof. Suppose that A is $\mathcal{I}_{mg\#}$-open. Let F⊆A and F be mg*-closed. Then X−A⊆X−F and X−F is m$g^\star$-open. Since X−A is $\mathcal{I}_{mg\#}$-closed, then (X−A)*⊆X−F and X−int*(A) =cl*(X−A)=(X−A)∪(X−A)*⊆X−F and hence F⊆int*(A). Conversely, let X−A⊆G where G is m$g^\star$-open. Then X−G⊆A and X−G is m$g^\star$-closed. By the hypothesis, we have X−G⊆int*(A) and hence (X−A)*⊆cl*(X−A)= X−int*(A)⊆G. Therefore, X−A is $\mathcal{I}_{mg\#}$-closed and A is $\mathcal{I}_{mg\#}$-open. □

Corollary 3.16. *Let $gO(X)$⊆$mg^\star O(X)$ and $mg^\star O(X)$ have property $\mathcal{J}$. Then the following properties hold:*

(1). Every ⋆-open set is $\mathcal{I}_{mg\#}$-open and every $\mathcal{I}_{mg\#}$-open set is $\mathcal{I}_{g^\star}$-open,

(2). If A and B are $\mathcal{I}_{mg\#}$-open, then A∩B is $\mathcal{I}_{mg\#}$-open,

(3). If A is $\mathcal{I}_{mg\#}$-open and B is open in (X, τ), then A∪B is $\mathcal{I}_{mg\#}$-open,

(4). If A is $\mathcal{I}_{mg\#}$-open and int*(A)⊆B⊆A, then B is $\mathcal{I}_{mg\#}$-open.

Proof. This follows from Remark 3.7, Propositions 3.13 and 3.14 and Corollary 3.10. □

Lemma 3.17 ([11]). *Let (X, $mg^\star O(X)$) be an m-space and A a subset of X. Then x∈$mg^\star$-cl(A) if and only if U∩A≠ϕ for every U∈$mg^\star O(X)$ containing x.*

Lemma 3.18 ([11]). *Let X be a nonempty set, $mg^\star O(X)$ an m-structure on X and $mg^\star O(X)$ have property $\mathcal{J}$. For a subset A of X, the following properties hold:*

(1). A∈$mg^\star O(X)$ if and only if $mg^\star$-int(A)=A,

(2). A is $mg^\star$-closed if and only if $mg^\star$-cl(A)=A,

(3). $mg^\star$-int(A)∈$mg^\star O(X)$ and $mg^\star$-cl(A) is $mg^\star$-closed.

4. Characterizations of $\mathcal{I}_{mg\#}$-closed Sets

In this section, let (X, τ, $\mathcal{I}$) be an ideal topological space and mg*O(X) an m-structure on X. We obtain several characterizations of $\mathcal{I}_{mg\#}$-closed sets.

Theorem 4.1. *For a subset A of X, the following properties are equivalent:*

(1). A is $\mathcal{I}_{mg\#}$-closed,

(2). cl*(A)⊆U whenever A⊆U and U is $mg^\star$-open,

(3). cl*(A)∩F=ϕ whenever A∩F=ϕ and F is $mg^\star$-closed.

Proof. (1)⇒(2) Let A⊆U where U is mg*-open. Then by (1), $A^\star$ ⊆U and cl*(A)=A∪$A^\star$ ⊆U.
(2)⇒(3) Let A∩F=ϕ and F be mg*-closed. Then A⊆X−F and X−F is m$g^\star$-open. By (2), cl*(A)⊆X−F. Hence cl*(A)∩F=ϕ.
(3)⇒(1) Let A⊆U where U is mg*-open. Then A∩(X−U)=ϕ and X−U is mg*-closed. By (3), cl*(A)∩(X−U)=ϕ and so $A^\star$⊆cl*(A)⊆U. Hence A is $\mathcal{I}_{mg\#}$-closed. □

Definition 4.2. *Let (X, τ) be a topological space, $mg^\star O(X)$ an m-structure on X and A a subset of X. The subset $\wedge_{mg^\star}(A)$ is defined as follows: $\wedge_{mg^\star}(A)$=∩{U : A⊆U, U∈ $mg^\star O(X)$}.*

Theorem 4.3. *A subset A of X is $\mathcal{I}_{mg\#}$-closed if and only if cl*(A)⊆$\wedge_{mg^\star}(A)$.*

Proof. Suppose that A is $\mathcal{I}_{mg\#}$-closed. If x$\notin\wedge_{mg^\star}$(A), then there exists U$\in$mg*O(X) such that A$\subseteq$U and x$\notin$U. Since A is $\mathcal{I}_{mg\#}$-closed, by Theorem 4.1, cl*(A)$\subseteq$U and hence x$\notin$ cl*(A). Hence we obtain cl*(A)$\subseteq\wedge_{mg^\star}$(A).
Conversely, suppose that cl*(A)$\subseteq\wedge_{mg^\star}$(A). Let A$\subseteq$U and U$\in$mg*O(X). Then cl*(A)$\subseteq$ $\wedge_{mg^\star}$(A)$\subseteq$U. By Theorem 4.1, A is $\mathcal{I}_{mg\#}$-closed. □

Theorem 4.4. *Let $gO(X)\subseteq mg^\star O(X)$ and $mg^\star O(X)$ have property $\mathcal{J}$. For a subset A of X, the following properties are equivalent:*

(1). A is $\mathcal{I}_{mg\#}$-closed,

(2). $A^\star-A$ contains no nonempty $mg^\star$-closed set,

(3). $A^\star-A$ is $\mathcal{I}_{mg\#}$-open,

(4). $A\cup(X-A^\star)$ is $\mathcal{I}_{mg\#}$-closed,

(5). $cl^\star(A)-A$ contains no nonempty $mg^\star$-closed set,

(6). $mg^\star$-$cl(\{x\})\cap A\neq\phi$ for each $x\in cl^\star(A)$.

Proof. (1)$\Rightarrow$(2) Suppose that A is $\mathcal{I}_{mg\#}$-closed. Let F$\subseteq A^\star-$A and F be mg*-closed. Then F$\subseteq A^\star$ and F$\nsubseteq$A. We have A$\subseteq$X$-$F and X$-$F is mg*-open. Therefore $A^\star\subseteq$X$-$F and so F$\subseteq$X$-A^\star$. Hence F$\subseteq A^\star\cap$(X$-A^\star$)=ϕ.
(2)$\Rightarrow$(3) Let F $\subseteq$ $A^\star$ $-$A and F be mg*-closed. By (2), we have F=ϕ and so F$\subseteq$int*($A^\star$ $-$A). By Proposition 3.15, $A^\star-$A is $\mathcal{I}_{mg\#}$-open.
(3)$\Rightarrow$(1) Let A$\subseteq$U where U is mg*-open. Then X$-$U$\subseteq$X$-$A $\Rightarrow$ $A^\star\cap$(X$-$U)$\subseteq A^\star\cap$(X$-$A) =$A^\star-$A. Since $A^\star$ is closed in (X, τ) and hence $A^\star$ is g-closed in (X, τ). Since every g-closed set is mg*-closed and so $A^\star$ is mg*-closed. Since mg*O(X) has property $\mathcal{J}$, then $A^\star\cap$(X$-$U) is mg*-closed and by (3), $A^\star-$A is $\mathcal{I}_{mg\#}$-open. Therefore by Proposition 3.15, $A^\star\cap$(X$-$U)$\subseteq$int*($A^\star-$A) = int*($A^\star\cap$(X$-$A)) = int*($A^\star$)$\cap$int*(X$-$A) = int*($A^\star$)$\cap$(X$-$cl*(A))$\subseteq A^\star\cap$(A$\cup A^\star$)^c = $A^\star\cap$(A$^c\cap$($A^\star$)^c) = ϕ and hence $A^\star\subseteq$U. Hence A is $\mathcal{I}_{mg\#}$-closed.
(3)$\Leftrightarrow$(4) This follows from the fact that X$-$($A^\star-$A)=X$\cap$($A^\star\cap$A^c)^c = X$\cap$(($A^\star$)$^c\cup$A) = (X$\cap$($A^\star$)^c)$\cup$(X$\cap$A) = A$\cup$(X$-A^\star$).
(2)$\Leftrightarrow$(5) This follows from the fact that cl*(A)$-$A=(A$\cup A^\star$)$-$A = (A$\cup A^\star$) $\cap$A^c = (A$\cap$A^c)$\cup$($A^\star\cap$A^c) = $A^\star\cap$A^c = $A^\star-$A.
(1)$\Rightarrow$(6) Suppose that A is $\mathcal{I}_{mg\#}$-closed and mg*-cl({x})$\cap$A=ϕ for some x$\in$cl*(A). By Lemma 3.18, mg*-cl({x}) is mg*-closed. We have A$\subseteq$X$-$(mg*-cl({x})) and X$-$(mg*-cl({x})) is mg*-open. Therefore by Theorem 4.1, cl*(A)$\subseteq$X$-$(mg*-cl({x})) $\subseteq$X$-${x}. This contradicts that x$\in$cl*(A). Hence mg*-cl({x})$\cap$A$\neq\phi$ for each x$\in$cl*(A).
(6)$\Rightarrow$(1) Suppose mg*-cl({x})$\cap$A$\neq\phi$ for each x$\in$cl*(A). We have to prove that A is $\mathcal{I}_{mg\#}$-closed. Suppose A is not $\mathcal{I}_{mg\#}$-closed. Then, by Theorem 4.1, $\phi\neq$cl*(A)$-$U for some mg*-open set U containing A. There exists x$\in$cl*(A)$-$U. Since x$\notin$U, by Lemma 3.17, mg*-cl({x})$\cap$U=ϕ and hence mg*-cl({x})$\cap$ A $\subseteq$ mg*-cl({x})$\cap$U=ϕ. This shows that mg*-cl({x})$\cap$A=ϕ for some x$\in$ cl*(A). This is a contradiction. Hence A is $\mathcal{I}_{mg\#}$-closed. □

Corollary 4.5. *Let $gO(X)\subseteq mg^\star O(X)$ and $mg^\star O(X)$ have property $\mathcal{J}$. For a subset A of X, the following properties are equivalent:*

(1). A is $\mathcal{I}_{mg\#}$-open,

(2). $A-int^\star(A)$ contains no nonempty $mg^\star$-closed set,

(3). $mg^\star$-$cl(\{x\})\cap(X-A)\neq\phi$ for each $x\in X-int^\star(A)$.

Proof. This follows from Theorem 4.4. □

Theorem 4.6. *Let $gO(X) \subseteq mg^\star O(X)$ and $mg^\star O(X)$ have property $\mathcal{J}$. A subset A of X is $\mathcal{I}_{mg\#}$-closed if and only if A=F−N where F is ⋆-closed and N contains no nonempty $mg^\star$-closed set.*

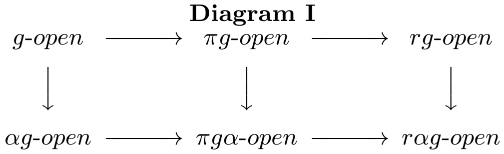
Proof. If A is $\mathcal{I}_{mg\#}$-closed, then by Theorem 4.4, N=$A^\star$−A contains no nonempty mg*-closed set. If F=cl*(A), then A∪$A^\star$=cl*(A)=F and by Lemma 2.4, we obtain F*=(A∪$A^\star$)*=$A^\star$∪$(A^\star)^\star$⊆$A^\star$∪A=F. Therefore F is ⋆-closed such that F−N=(A∪$A^\star$)− ($A^\star$−A)=(A∪$A^\star$)∩($A^\star$∩A^c)^c = (A∪$A^\star$)∩(A∪$(A^\star)^c$)=A∪($A^\star$∩$(A^\star)^c$)=A.

Conversely, suppose A=F−N where F is ⋆-closed and N contains no nonempty mg*-closed set. Let U be an mg*-open set such that A⊆U. Then F−N⊆U ⇒F∩(X−U)⊆N. Since $A^\star$ is mg*-closed and hence $A^\star$∩(X−U) is mg*-closed. Since A⊆F and F*⊆F, then $A^\star$∩(X−U)⊆F*∩(X−U)⊆F∩(X−U)⊆N. Therefore, $A^\star$∩(X−U)=ϕ and so $A^\star$⊆U. Hence A is $\mathcal{I}_{mg\#}$-closed. □

5. New Forms of Closed Sets in Ideal Topological Spaces

By gO(X) (resp. πGO(X), RGO(X), αGO(X), πGαO(X), RαGO(X)), we denote the collection of all g-open (resp. πg-open, rg-open, αg-open, πgα-open, rαg-open) sets of a topological space (X, τ). These collections are m-structures on X. By the definitions, we obtain the following diagram:

Diagram I

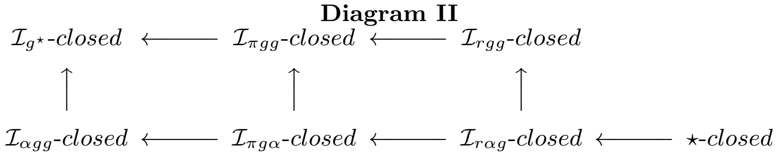


For subsets of an ideal topological space (X, τ, $\mathcal{I}$), we can define new types of closed sets as follows:

Definition 5.1. *A subset A of an ideal topological space (X, τ, $\mathcal{I}$) is said to be $\mathcal{I}_{g^\star}$-closed (resp. $\mathcal{I}_{\pi gg}$-closed, $\mathcal{I}_{rgg}$-closed, $\mathcal{I}_{\alpha gg}$-closed, $\mathcal{I}_{\pi g\alpha}$-closed, $\mathcal{I}_{r\alpha g}$-closed) if $A^\star \subseteq U$ whenever $A \subseteq U$ and U is g-open (resp. πg-open, rg-open, αg-open, $\pi g\alpha$-open, rαg-open) in (X, τ).*

By Diagram I and Definition 5.1, we have the following diagram:

Diagram II



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