International Journal of Current Research in Science and Technology

Volume 2, Issue 4 (2016), 13–20.

ISSN: 2394-5745

Available Online: http://ijcrst.in/

International Journal of Current Research in Science and Technology

New Classes of Ideal Topological Quotient Maps

Research Article

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Abstract: The purpose of this paper is to study the concept of quotient maps in ideal topological spaces and study some of its stronger forms.

MSC: 54C08.

Keywords: Ideal topological space, $\alpha^\star$-$\mathcal{I}$-open or $\alpha^\star_\mathcal{I}$-open set, semi*-$\mathcal{I}$-open set, pre$^*_\mathcal{I}$-open set.

1. Introduction

Let (X, τ) be a topological space with no separation axioms assumed. For any A $\subseteq$ X, cl(A) and int(A) will denote the closure and interior of A in (X, τ), respectively. Njastad [9] introduced the concept of an α-sets and Mashhour et al. [8] introduced α-continuous maps in topological spaces. The topological notions of semi-open sets and semi-continuity, and preopen sets and precontinuity were introduced by Levine [6] and Mashhour et al. [7] respectively. After advent of these notions, Reilly [11] and Lellis Thivagar [5] obtained many interesting and important results on α-continuity and α-irresolute maps in topological spaces. Lellis Thivagar [5] introduced the concepts of α-quotient maps and α^*-quotient maps in topological spaces.

A nonempty collection $\mathcal{I}$ of subsets of a set X is said to be an ideal on X if it satisfies the following two properties:

(1). A$\in\mathcal{I}$ and B$\subseteq$A imply B$\in\mathcal{I}$;

(2). A$\in\mathcal{I}$ and B$\in\mathcal{I}$ imply A$\cup$B$\in\mathcal{I}$. [4]

A topological space (X, τ) with an ideal $\mathcal{I}$ on X is called an ideal topological space (an ideal space) and is denoted by (X, τ, $\mathcal{I}$). For an ideal space (X, τ, $\mathcal{I}$) and a subset A$\subseteq$X, A*($\mathcal{I}$)={x$\in$X : U$\cap$A$\notin\mathcal{I}$ for every U$\in\tau$(x)}, is local function [4] of A with respect to $\mathcal{I}$ and τ. It is well known that cl*(A)=A$\cup$A* defines a Kuratowski closure operator for a topology τ^* finer than τ [12]. int$^*(A)$ will denote the interior of A in $(X, \tau^*, \mathcal{I})$.

Quite recently, Viswanathan and Jayasudha [13] introduced and studied the notion of $\alpha^\star$-$\mathcal{I}$-open or $\alpha^\star_\mathcal{I}$-open [10] sets. Ekici and Noiri [2] introduced and studied the notion of semi*-$\mathcal{I}$-open sets. In [3], they studied further properties of semi*-$\mathcal{I}$-open sets. Ekici [1] introduced and studied the notion of pre$^*_\mathcal{I}$-open sets.

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In this paper, we introduce new classes of ideal topological maps called $(\mathcal{I},\mathcal{J})$-$\alpha$-quotient maps and $(\mathcal{I},\mathcal{J})$-$\alpha^*$-quotient maps in ideal topological spaces. At every places the new notions have been substantiated with suitable examples.

2. Preliminaries

Definition 2.1 ([10, 13]). *A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is said to be $\alpha^\star$-$\mathcal{I}$-open or $\alpha^\star_{\mathcal{I}}$-open if $A \subseteq int^\star(cl(int^\star(A)))$.*

Definition 2.2 ([2, 3]). *A subset K of an ideal topological space $(X, \tau, \mathcal{I})$ is said to be*

(1). semi*-$\mathcal{I}$-open if $K \subseteq cl(int^*(K))$,

(2). semi*-$\mathcal{I}$-closed if its complement is semi*-$\mathcal{I}$-open.

Definition 2.3 ([1]). *A subset G of an ideal topological space $(X, \tau, \mathcal{I})$ is said to be*

(1). pre$^*_{\mathcal{I}}$-open if $G \subseteq int^*(cl(G))$.

(2). pre$^*_{\mathcal{I}}$-closed if $X\backslash G$ is pre$^*_{\mathcal{I}}$-open.

The family of all $\alpha^\star_{\mathcal{I}}$-open [resp. semi*-$\mathcal{I}$-open, pre$^*_{\mathcal{I}}$-open] sets of (X, τ, $\mathcal{I}$) is denoted by $\alpha^\star_{\mathcal{I}}$O(X) [resp. semi*-$\mathcal{I}$O(X), pre$^*_{\mathcal{I}}$O(X)].

Theorem 2.4 ([13]). *Let $(X, \tau, \mathcal{I})$ be an ideal topological space. Then, $\alpha^\star_{\mathcal{I}}O(X) = semi^*\text{-}\mathcal{I}O(X) \cap pre^*_{\mathcal{I}}O(X)$.*

Remark 2.5 ([13]). *For a subset of an ideal topological space, the following holds.*

Every open set is $\alpha^\star_{\mathcal{I}}$-open but not conversely.

3. $(\mathcal{I},\mathcal{J})$-$\alpha$-irresolute Maps

Definition 3.1 ([13]). *Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ be a map. Then f is said to be $\alpha^\star$-$\mathcal{I}$-continuous [resp. semi*-$\mathcal{I}$-continuous, pre$^*_{\mathcal{I}}$-continuous] if the inverse image of each open set of Y is $\alpha^\star$-$\mathcal{I}$-open [resp. semi*-$\mathcal{I}$-open, pre$^*_{\mathcal{I}}$-open] in X.*

Definition 3.2. *A map $f : (X, \tau) \to (Y, \sigma, \mathcal{I})$ is called $\alpha^\star$-$\mathcal{I}$-open [resp. semi*-$\mathcal{I}$-open, pre$^*_{\mathcal{I}}$-open, open] if the image of each open set in X is an $\alpha^\star$-$\mathcal{I}$-open [resp. semi*-$\mathcal{I}$-open, pre$^*_{\mathcal{I}}$-open, open] set of Y.*

Theorem 3.3.

(1). A map $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is $\alpha^\star$-$\mathcal{I}$-continuous if and only if it is semi*-$\mathcal{I}$-continuous and pre$^*_{\mathcal{I}}$-continuous.

(2). A map $f : (X, \tau) \to (Y, \sigma, \mathcal{I})$ is $\alpha^\star$-$\mathcal{I}$-open if and only if it is semi*-$\mathcal{I}$-open and pre$^*_{\mathcal{I}}$-open.

Definition 3.4. *Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ be a map. Then f is said to be $(\mathcal{I},\mathcal{J})$-$\alpha$-irresolute (resp. $(\mathcal{I},\mathcal{J})$-semi-irresolute, $(\mathcal{I},\mathcal{J})$-preirresolute) if the inverse image of every $\alpha^\star$-$\mathcal{J}$-open [resp. semi*-$\mathcal{J}$-open, pre$^*_{\mathcal{J}}$-open] set in Y is an $\alpha^\star$-$\mathcal{I}$-open [resp. semi*-$\mathcal{I}$-open, pre$^*_{\mathcal{I}}$-open] set in X.*

Theorem 3.5. *A map $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ is $(\mathcal{I},\mathcal{J})$-semi-irresolute if and only if for every semi*-$\mathcal{J}$-closed subset A of Y, $f^{-1}(A)$ is semi*-$\mathcal{I}$-closed in X.*

Proof. If f is $(\mathcal{I},\mathcal{J})$-semi-irresolute, then for every semi*-$\mathcal{J}$-open subset B of Y, f^{-1}(B) is semi*-$\mathcal{I}$-open in X. If A is any semi*-$\mathcal{J}$-closed subset of Y, then Y−A is semi*-$\mathcal{J}$-open. Thus f^{-1}(Y−A) is semi*-$\mathcal{I}$-open but f^{-1}(Y−A)=X−f^{-1}(A) so that f^{-1}(A) is semi*-$\mathcal{I}$-closed in X.
Conversely, if, for all semi*-$\mathcal{J}$-closed subsets A of Y, f^{-1}(A) is semi*-$\mathcal{I}$-closed in X and if B is any semi*-$\mathcal{J}$-open subset of Y, then Y−B is semi*-$\mathcal{J}$-closed. Also f^{-1}(Y−B)=X− f^{-1}(B) is semi*-$\mathcal{I}$-closed. Thus f^{-1}(B) is semi*-$\mathcal{I}$-open in X. Hence f is $(\mathcal{I},\mathcal{J})$-semi-irresolute. □

Theorem 3.6. *Let f and g be two maps. If f : (X, τ, $\mathcal{I}$)→(Y, σ, $\mathcal{J}$) is $(\mathcal{I},\mathcal{J})$-semi-irresolute and g : (Y, σ, $\mathcal{J}$)→(Z, μ, $\mathcal{K}$) is $(\mathcal{J},\mathcal{K})$-semi-irresolute then gof : (X, τ, $\mathcal{I}$)→(Z, μ, $\mathcal{K}$) is $(\mathcal{I},\mathcal{K})$-semi-irresolute.*

Proof. If A⊆Z is semi*-$\mathcal{K}$-open, then g^{-1}(A) is semi*-$\mathcal{J}$-open set in Y because g is $(\mathcal{J},\mathcal{K})$-semi-irresolute. Consequently since f is $(\mathcal{I},\mathcal{J})$-semi-irresolute, $f^{-1}(g^{-1}(A))=(gof)^{-1}(A)$ is semi*-$\mathcal{I}$-open set in X. Hence gof is $(\mathcal{I},\mathcal{K})$-semi-irresolute. □

Corollary 3.7. *If f : (X, τ, $\mathcal{I}$)→(Y, σ, $\mathcal{J}$) is $(\mathcal{I},\mathcal{J})$-α-irresolute and g : (Y, σ, $\mathcal{J}$)→(Z, μ, $\mathcal{K}$) is $(\mathcal{J},\mathcal{K})$-α-irresolute then gof : (X, τ, $\mathcal{I}$)→(Z, μ, $\mathcal{K}$) is $(\mathcal{I},\mathcal{K})$-α-irresolute.*

Corollary 3.8. *If the map f : (X, τ, $\mathcal{I}$)→(Y, σ, $\mathcal{J}$) is $(\mathcal{I},\mathcal{J})$-α-irresolute and the map g : (Y, σ, $\mathcal{J}$)→(Z, μ) is $\alpha^\star$-$\mathcal{J}$-continuous then gof : (X, τ, $\mathcal{I}$)→(Z, μ) is $\alpha^\star$-$\mathcal{I}$-continuous.*

Corollary 3.9. *Let f : (X, τ, $\mathcal{I}$)→(Y, σ, $\mathcal{J}$) and g : (Y, σ, $\mathcal{J}$)→(Z, μ) be two maps. Then*

(1). if f is $(\mathcal{I},\mathcal{J})$-semi-irresolute and g is semi*-$\mathcal{J}$-continuous, then gof is semi*-$\mathcal{I}$-continuous.

(2). if f is $(\mathcal{I},\mathcal{J})$-preirresolute and g is $pre^*_{\mathcal{J}}$-continuous, then gof is $pre^*_{\mathcal{I}}$-continuous.

Theorem 3.10. *If the map f : (X, τ, $\mathcal{I}$)→(Y, σ, $\mathcal{J}$) is $(\mathcal{I},\mathcal{J})$-semi-irresolute and $(\mathcal{I},\mathcal{J})$-preirresolute then f is $(\mathcal{I},\mathcal{J})$-α-irresolute.*

4. $(\mathcal{I},\mathcal{J})$-α-quotient Maps

Definition 4.1. *Let f : (X, τ)→(Y, σ) be a surjective map. Then f is said to be quotient provided a subset S of Y is open in Y if and only if f^{-1}(S) is open in X.*

Definition 4.2. *Let f : (X, τ, $\mathcal{I}$)→(Y, σ, $\mathcal{J}$) be a surjective map. Then f is said to be*

(1). an $(\mathcal{I},\mathcal{J})$-α-quotient if f is $\alpha^\star$-$\mathcal{I}$-continuous and f^{-1}(V) is open in X implies V is an $\alpha^\star$-$\mathcal{J}$-open set in Y.

(2). a $(\mathcal{I},\mathcal{J})$-semi-quotient if f is semi*-$\mathcal{I}$-continuous and f^{-1}(V) is open in X implies V is a semi*-$\mathcal{J}$-open set in Y.

(3). a $(\mathcal{I},\mathcal{J})$-prequotient if f is $pre^*_{\mathcal{I}}$-continuous and f^{-1}(V) is open in X implies V is a $pre^*_{\mathcal{J}}$-open set in Y.

Example 4.3. *Let X={a, b, c}, τ={∅, X, {c}, {a, c}, {b, c}} and $\mathcal{I}$={∅, {a}}. We have $\alpha^\star_{\mathcal{I}}O(X)$ = semi*-$\mathcal{I}O(X)$ = $pre^*{}_{\mathcal{I}}O(X)$ = {∅, X, {c}, {a, c}, {b, c}}. Let Y = {p, q, r}, σ={∅, Y, {r}, {p, r}, {q, r}} and J={∅, {p}}. We have $\alpha^\star_{\mathcal{J}}O(Y)$ = semi*-$\mathcal{J}O(Y)$ = $pre^*{}_{\mathcal{J}}O(Y)$ = {∅, Y, {r}, {p, r}, {q, r}}.*
Define f : (X, τ, $\mathcal{I}$)→(Y, σ, $\mathcal{J}$) by f(a) = p; f(c) = q; f(c) = r. Since the inverse image of each open in Y is $\alpha^\star$-$\mathcal{I}$-open in X, clearly f is $\alpha^\star$-$\mathcal{I}$-continuous and an $(\mathcal{I},\mathcal{J})$-α-quotient map.

Theorem 4.4. *If the map f : (X, τ, $\mathcal{I}$)→(Y, σ, $\mathcal{J}$) is surjective, $\alpha^\star$-$\mathcal{I}$-continuous and $\alpha^\star$-$\mathcal{J}$-open then f is an $(\mathcal{I},\mathcal{J})$-α-quotient map.*

Proof. Suppose $f^{-1}(V)$ is any open set in X. Then $f(f^{-1}(V))$ is an $\alpha^{\star}$-$\mathcal{J}$-open set in Y as f is $\alpha^{\star}$-$\mathcal{J}$-open. Since f is surjective, $f(f^{-1}(V))=V$. Thus V is an $\alpha^{\star}$-$\mathcal{J}$-open set in Y. Hence f is $(\mathcal{I},\mathcal{J})$-$\alpha$-quotient map. □

Theorem 4.5. *If the map $f:(X,\tau,\mathcal{I})\rightarrow(Y,\sigma,\mathcal{J})$ is open surjective and $(\mathcal{I},\mathcal{J})$-$\alpha$-irresolute, and the map $g:(Y,\sigma,\mathcal{J})\rightarrow(Z,\mu,\mathcal{K})$ is an $(\mathcal{J},\mathcal{K})$-$\alpha$-quotient then $gof:(X,\tau,\mathcal{I})\rightarrow(Z,\mu,\mathcal{K})$ is an $(\mathcal{I},\mathcal{K})$-$\alpha$-quotient map.*

Proof. Let V be any open set in Z. Since g is $\alpha^{\star}$-$\mathcal{J}$-continuous, $g^{-1}(V)\in\alpha^{\star}_{\mathcal{J}}O(Y)$. Since f is $(\mathcal{I},\mathcal{J})$-$\alpha$-irresolute, $f^{-1}(g^{-1}(V))=(gof)^{-1}(V)\in\alpha^{\star}_{\mathcal{I}}O(X)$. Thus gof is $\alpha^{\star}$-$\mathcal{I}$-continuous. Also suppose $f^{-1}(g^{-1}(V))$ is open set in X. Since f is open, $f(f^{-1}(g^{-1}(V)))$ is open set in Y. Since f is surjective, $f(f^{-1}(g^{-1}(V)))=g^{-1}(V)$ and since g is $(\mathcal{J},\mathcal{K})$-$\alpha$-quotient, $V\in\alpha^{\star}_{\mathcal{K}}O(Z)$. Hence gof is an $(\mathcal{I},\mathcal{K})$-$\alpha$-quotient. □

Corollary 4.6. *If the map $f:(X,\tau,\mathcal{I})\rightarrow(Y,\sigma,\mathcal{J})$ is open surjective and $(\mathcal{I},\mathcal{J})$-semi-[$(\mathcal{I},\mathcal{J})$-pre] irresolute and the map $g:(Y,\sigma,\mathcal{J})\rightarrow(Z,\mu,\mathcal{K})$ is $(\mathcal{J},\mathcal{K})$-semi- [$(\mathcal{J},\mathcal{K})$-pre] quotient then $gof:(X,\tau,\mathcal{I})\rightarrow(Z,\mu,K)$ is $(\mathcal{I},\mathcal{K})$-semi-[$(\mathcal{I},\mathcal{K})$-pre] quotient map.*

Theorem 4.7. *A map $f:(X,\tau,\mathcal{I})\rightarrow(Y,\sigma,\mathcal{J})$ is an $(\mathcal{I},\mathcal{J})$-$\alpha$-quotient if and only if it is both $(\mathcal{I},\mathcal{J})$-semi-quotient and $(\mathcal{I},\mathcal{J})$-prequotient.*

Proof. Let V be any open set in Y. Since f is $\alpha^{\star}$-$\mathcal{I}$-continuous, $f^{-1}(V)\in\alpha^{\star}_{\mathcal{I}}O(X)=\text{semi}^{*}\text{-}\mathcal{I}O(X)\cap\text{pre}^{*}_{\mathcal{I}}O(X)$. Thus f is both semi*-$\mathcal{I}$-continuous and pre$^{*}_{\mathcal{I}}$-continuous. Also suppose $f^{-1}(V)$ is an open set in X. Since f is $(\mathcal{I},\mathcal{J})$-$\alpha$-quotient, $V\in\alpha^{\star}_{\mathcal{J}}O(Y)=\text{semi}^{*}\text{-}\mathcal{J}O(Y)\cap\text{pre}^{*}_{\mathcal{J}}O(Y)$. Thus V is both semi*-$\mathcal{J}$-open set and pre$^{*}_{\mathcal{J}}$-open set in Y. Hence f is both $(\mathcal{I},\mathcal{J})$-semi-quotient and $(\mathcal{I},\mathcal{J})$-prequotient.

Conversely, since f is both $(\mathcal{I},\mathcal{J})$-semi-quotient and $(\mathcal{I},\mathcal{J})$-prequotient, f is both semi*-$\mathcal{I}$-continuous and pre$^{*}_{\mathcal{I}}$-continuous. Hence f is $\alpha^{\star}$-$\mathcal{I}$-continuous. Also suppose $f^{-1}(V)$ is an open set in X. By Definition 4.2, $V\in\text{semi}^{*}\text{-}\mathcal{J}O(Y)\cap\text{pre}^{*}_{\mathcal{J}}O(Y)=\alpha^{\star}_{\mathcal{J}}O(Y)$. Thus f is $(\mathcal{I},\mathcal{J})$-$\alpha$-quotient. □

Definition 4.8.

(1). Let $f:(X,\tau,\mathcal{I})\rightarrow(Y,\sigma)$ be a surjective and $\alpha^{\star}$-$\mathcal{I}$-continuous map. Then f is said to be strongly $\mathcal{I}$-α-quotient provided a subset S of Y is open set in Y if and only if $f^{-1}(S)$ is an $\alpha^{\star}$-$\mathcal{I}$-open set in X.

(2). Let $f:(X,\tau,\mathcal{I})\rightarrow(Y,\sigma)$ be a surjective and semi*-$\mathcal{I}$-continuous map. Then f is said to be strongly $\mathcal{I}$-semi-quotient provided a subset S of Y is open set in Y if and only if $f^{-1}(S)$ is semi*-$\mathcal{I}$-open set in X.

(3). Let $f:(X,\tau,\mathcal{I})\rightarrow(Y,\sigma)$ be a surjective and pre$^{*}_{\mathcal{I}}$-continuous map. Then f is said to be strongly $\mathcal{I}$-prequotient provided a subset S of Y is open set in Y if and only if $f^{-1}(S)$ is pre$^{*}_{\mathcal{I}}$-open set in X.

Theorem 4.9. *If the map $f:(X,\tau,\mathcal{I})\rightarrow(Y,\sigma)$ is strongly $\mathcal{I}$-semi-quotient and strongly $\mathcal{I}$-prequotient then f is strongly $\mathcal{I}$-α-quotient.*

Proof. Since f is both semi*-$\mathcal{I}$-continuous and pre$^{*}_{\mathcal{I}}$-continuous, by Theorem 3.3, f is $\alpha^{\star}$-$\mathcal{I}$-continuous. Also let V be an open set in Y. By Definition 4.8, $f^{-1}(V)\in\text{semi}^{*}\text{-}\mathcal{I}O(X)\cap\text{pre}^{*}_{\mathcal{I}}O(X)=\alpha^{\star}_{\mathcal{I}}O(X)$.

Conversely, let $f^{-1}(V)\in\alpha^{\star}_{\mathcal{I}}O(X)$. Then $\alpha^{\star}_{\mathcal{I}}O(X)=\text{semi}^{*}\text{-}\mathcal{I}O(X)\cap\text{pre}^{*}_{\mathcal{I}}O(X)$. Since f is strongly $\mathcal{I}$-semi-quotient and strongly $\mathcal{I}$-prequotient, V is open set in Y. Hence f is strongly $\mathcal{I}$-α-quotient. □

5. $(\mathcal{I},\mathcal{J})$-$\alpha$*-quotient maps

Definition 5.1. *Let f : (X, τ, $\mathcal{I}$)→(Y, σ, $\mathcal{J}$) be a surjective map. Then f is said to be*

(1). $(\mathcal{I},\mathcal{J})$-$\alpha$-quotient if f is $(\mathcal{I},\mathcal{J})$-$\alpha$-irresolute and $f^{-1}(S)$ is $\alpha^\star$-$\mathcal{I}$-open set in X implies S is open set in Y.*

*(2). $(\mathcal{I},\mathcal{J})$-semi-*quotient if f is $(\mathcal{I},\mathcal{J})$-semi-irresolute and $f^{-1}(S)$ is semi*-$\mathcal{I}$-open set in X implies S is open set in Y.*

*(3). $(\mathcal{I},\mathcal{J})$-pre-*quotient if f is $(\mathcal{I},\mathcal{J})$-preirresolute and $f^{-1}(S)$ is pre$^*_\mathcal{I}$-open set in X implies S is open set in Y.*

Definition 5.2. *Let f : (X, τ, $\mathcal{I}$)→(Y, σ, $\mathcal{J}$) be a map. Then f is said to be strongly $(\mathcal{I},\mathcal{J})$-$\alpha$-open if the image of every $\alpha^\star$-$\mathcal{I}$-open set in X is an $\alpha^\star$-$\mathcal{J}$-open set in Y.*

Example 5.3. *Consider the Example 4.3. Clearly f is $(\mathcal{I},\mathcal{J})$-$\alpha$-irresolute and $(\mathcal{I},\mathcal{J})$-$\alpha$*-quotient.*

Example 5.4. *Consider the Example 4.3. Clearly f is strongly $(\mathcal{I},\mathcal{J})$-$\alpha$-open.*

Theorem 5.5. *Let the map f : (X, τ, $\mathcal{I}$)→(Y, σ, $\mathcal{J}$) be surjective strongly $(\mathcal{I},\mathcal{J})$-$\alpha$-open and $(\mathcal{I},\mathcal{J})$-$\alpha$-irresolute, and the map g : (Y, σ, $\mathcal{J}$)→(Z, μ, $\mathcal{K}$) be an $(\mathcal{J},\mathcal{K})$-$\alpha$*-quotient. Then gof : (X, τ, $\mathcal{I}$)→(Z, μ, $\mathcal{K}$) is an $(\mathcal{I},\mathcal{K})$-$\alpha$*-quotient map.*

Proof. Let V be any $\alpha^\star$-$\mathcal{K}$-open set in Z. Then g^{-1}(V) is an $\alpha^\star$-$\mathcal{J}$-open set in Y as g is an $(\mathcal{J},\mathcal{K})$-$\alpha$*-quotient map. Then $f^{-1}(g^{-1}(V))=(gof)^{-1}(V)$ is an $\alpha^\star$-$\mathcal{I}$-open set in X as f is $(\mathcal{I},\mathcal{J})$-$\alpha$-irresolute. This shows that gof is $(\mathcal{I},\mathcal{K})$-$\alpha$-irresolute. Also suppose $(gof)^{-1}(V)=f^{-1}(g^{-1}(V))$ is an $\alpha^\star$-$\mathcal{I}$-open set in X. Since f is strongly $(\mathcal{I},\mathcal{J})$-$\alpha$-open, $f(f^{-1}(g^{-1}(V)))$ is an $\alpha^\star$-$\mathcal{J}$-open set in Y. Since f is surjective, $f(f^{-1}(g^{-1}(V)))=g^{-1}(V)$ is an $\alpha^\star$-$\mathcal{J}$-open set in Y. Since g is an $(\mathcal{J},\mathcal{K})$-$\alpha$*-quotient map, V is open in Z. Hence the theorem. □

Theorem 5.6. *If the map f : (X, τ, $\mathcal{I}$)→(Y, σ, $\mathcal{J}$) is both $(\mathcal{I},\mathcal{J})$-semi-*quotient and $(\mathcal{I},\mathcal{J})$-pre-*quotient then f is $(\mathcal{I},\mathcal{J})$-$\alpha$*-quotient.*

Proof. Since f is both $(\mathcal{I},\mathcal{J})$-semi-*quotient and $(\mathcal{I},\mathcal{J})$-pre-*quotient, f is $(\mathcal{I},\mathcal{J})$-semi-irresolute and $(\mathcal{I},\mathcal{J})$-preirresolute. By Theorem 3.10, f is $(\mathcal{I},\mathcal{J})$-$\alpha$-irresolute. Also suppose $f^{-1}(V)\in\alpha^\star_\mathcal{I}O(X)$. Then $\alpha^\star_\mathcal{I}O(X)$=semi*-$\mathcal{I}$O(X)∩pre$^*_\mathcal{I}$O(X). Therefore f^{-1}(V) is semi*-$\mathcal{I}$-open in X and f^{-1}(V) is pre$^*_\mathcal{I}$-open in X. Since f is $(\mathcal{I},\mathcal{J})$-semi-*quotient and $(\mathcal{I},\mathcal{J})$-pre-*quotient, by Definition 5.1., V is open set in Y. Thus f is $(\mathcal{I},\mathcal{J})$-$\alpha$*-quotient. □

Theorem 5.7. *Let f : (X, τ, $\mathcal{I}$)→(Y, σ, $\mathcal{J}$) be a strongly $\mathcal{I}$-α-quotient and $(\mathcal{I},\mathcal{J})$-$\alpha$-irresolute map and g : (Y, σ, $\mathcal{J}$)→(Z, μ, K) be an $(\mathcal{J},\mathcal{K})$-$\alpha$*-quotient map then gof : (X, τ, $\mathcal{I}$) →(Z, μ, $\mathcal{K}$) is an $(\mathcal{I},\mathcal{K})$-$\alpha$*-quotient.*

Proof. Let $V\in\alpha^\star_\mathcal{K}O(Z)$. Since g is $(\mathcal{J},\mathcal{K})$-$\alpha$-irresolute, $g^{-1}(V)\in\alpha^\star_\mathcal{J}O(Y)$. Since f is $(\mathcal{I},\mathcal{J})$-$\alpha$-irresolute, $f^{-1}(g^{-1}(V))=(gof)^{-1}(V)\in\alpha^\star_\mathcal{I}O(X)$. Thus gof is $(\mathcal{I},\mathcal{K})$-$\alpha$-irresolute. Also suppose $(gof)^{-1}(V)=f^{-1}(g^{-1}(V))\in\alpha^\star_\mathcal{I}O(X)$. Since f is strongly $\mathcal{I}$-α-quotient, g^{-1}(V) is open set in Y. Then $g^{-1}(V)\in\alpha^\star_\mathcal{J}O(Y)$. Since g is $(\mathcal{J},\mathcal{K})$-$\alpha$*-quotient, V is open set in Z. Hence gof is $(\mathcal{I},\mathcal{K})$-$\alpha$*-$\mathcal{I}$-quotient. □

6. Comparison

Theorem 6.1. *Let f : (X, τ, $\mathcal{I}$)→(Y, σ, $\mathcal{J}$) be a surjective map. Then f is $(\mathcal{I},\mathcal{J})$-$\alpha$*-quotient if and only if it is strongly $\mathcal{I}$-α-quotient.*

Proof. Let V be an open set in Y. Then $V\in\alpha^\star_\mathcal{J}O(Y)$. Since f is $(\mathcal{I},\mathcal{J})$-$\alpha$*-quotient, $f^{-1}(V)\in\alpha^\star_\mathcal{I}O(X)$. Conversely, let $f^{-1}(V)\in\alpha^\star_\mathcal{I}O(X)$. Since f is $(\mathcal{I},\mathcal{J})$-$\alpha$*-quotient, V is open set in Y. Hence f is strongly $\mathcal{I}$-α-quotient map.

Conversely, let V be an open set in Y. Then V$\in\alpha^\star_{\mathcal{J}}$O(Y). Since f is strongly $\mathcal{I}$-α-quotient, f^{-1}(V)$\in\alpha^\star_{\mathcal{I}}$O(X). Thus f is $(\mathcal{I},\mathcal{J})$-$\alpha$-irresolute. Also since f is strongly $\mathcal{I}$-α-quotient, f^{-1}(V)$\in\alpha^\star_{\mathcal{I}}$O(X) implies V is open set in Y. Hence f is $(\mathcal{I},\mathcal{J})$-$\alpha$*-quotient map. □

Theorem 6.2. *If the map f : (X, τ, $\mathcal{I}$)$\to$(Y, σ, $\mathcal{J}$) is quotient then it is $(\mathcal{I},\mathcal{J})$-$\alpha$-quotient.*

Proof. Let V be an open set in Y. Since f is quotient, f^{-1}(V) is open set in X and f^{-1}(V)$\in\alpha^\star_{\mathcal{I}}$O(X). Hence f is $\alpha^\star$-$\mathcal{I}$-continuous. Suppose f^{-1}(V) is an open set in X. Since f is quotient, V is open set in Y. Then V$\in\alpha^\star_{\mathcal{J}}$O(Y). Hence f is $(\mathcal{I},\mathcal{J})$-$\alpha$-quotient map. □

Theorem 6.3. *If the map f : (X, τ, $\mathcal{I}$)$\to$(Y, σ, $\mathcal{J}$) is $(\mathcal{I},\mathcal{J})$-$\alpha$-irresolute then it is $\alpha^\star$-$\mathcal{I}$-continuous.*

Proof. Let A be open set in Y. Then A$\in\alpha^\star_{\mathcal{J}}$O(Y). Since f is $(\mathcal{I},\mathcal{J})$-$\alpha$-irresolute, f^{-1}(A)$\in\alpha^\star_{\mathcal{I}}$O(X). It shows that f is $\alpha^\star$-$\mathcal{I}$-continuous map. □

Theorem 6.4. *If the map f : (X, τ, $\mathcal{I}$)$\to$(Y, σ, $\mathcal{J}$) is $(\mathcal{I},\mathcal{J})$-$\alpha$*-quotient then it is $(\mathcal{I},\mathcal{J})$-$\alpha$-quotient.*

Proof. Let f be $(\mathcal{I},\mathcal{J})$-$\alpha$*-quotient. Then f is $(\mathcal{I},\mathcal{J})$-$\alpha$-irresolute. We have f is $\alpha^\star$-$\mathcal{I}$-continuous. Also suppose f^{-1}(V) is an open in X. Then f^{-1}(V)$\in\alpha^\star_{\mathcal{I}}$O(X). By assumption, V is open set in Y. Therefore V$\in\alpha^\star_{\mathcal{J}}$O(Y). Hence f is $(\mathcal{I},\mathcal{J})$-$\alpha$-quotient. □

Theorem 6.5. *Every $(\mathcal{I},\mathcal{J})$-$\alpha$*-quotient map is $(\mathcal{I},\mathcal{J})$-$\alpha$-irresolute.*

Proof. We obtain it from Definition 5.1. □

Theorem 6.6. *Every $(\mathcal{I},\mathcal{J})$-$\alpha$-quotient map is $\alpha^\star$-$\mathcal{I}$-continuous.*

Proof. We obtain it from Definition 4.2. □

Remark 6.7. *The converses of Theorems 4.9 and 5.6 are not true as per the following example.*

Example 6.8. *Let X={a, b, c}, τ={$\emptyset$, X, {c}, {a, c}, {b, c}}, Y={p, q, r}, σ={$\emptyset$, Y, {r}, {p, r}}, $\mathcal{I}$={$\emptyset$, {b}, {a, b}} and $\mathcal{J}$ = {$\emptyset$, {q}, {p, q}}. Define f : (X, τ, $\mathcal{I}$)$\to$(Y, σ, $\mathcal{J}$) by f(a) = p; f(b) = q and f(c) = r. Clearly f is α*$\mathcal{I}$-continuous and strongly $\mathcal{I}$-α-quotient. Since f^{-1}({q, r})={b, c}$\in$ semi*$\mathcal{I}$O(X) and {q, r} is not open set in Y, f is not strongly $\mathcal{I}$-semi-quotient. Moreover f is ($\mathcal{I}$, $\mathcal{J}$)-α-irresolute, ($\mathcal{I}$, $\mathcal{J}$)-α*-quotient and ($\mathcal{I}$, $\mathcal{J}$)-semi-irresolute. Since f^{-1}({q, r})={b, c}$\in$ semi*$\mathcal{I}$O(X) and {q, r} is not open set in Y, f is not ($\mathcal{I}$, $\mathcal{J}$)α-*quotient.*

Remark 6.9. *The converses of Theorems 6.4 and 6.5 are not true as per the following example.*

Example 6.10. *Consider the Example 6.8. Clearly f is ($\mathcal{I}$, $\mathcal{J}$)-α-irresolute and ($\mathcal{I}$, $\mathcal{J}$)-α-quotient maps. Since f^{-1}({q, r})={b, c}$\in$ $\alpha^\star_{\mathcal{I}}$O(X) and {p, q} is not open set in Y, f is neither strongly $\mathcal{I}$-α-quotient nor ($\mathcal{I}$, $\mathcal{J}$)-α*-quotient.*

Remark 6.11. *The converse of Theorem 6.2 is not true and a strongly $\mathcal{I}$-α-quotient map need not be quotient as per the following example.*

Example 6.12. *Let X={a, b, c}, τ={$\emptyset$, X, {a}}, Y={p, q, r}, σ={$\emptyset$, Y, {p}, {p, q}, {p, r}}, $\mathcal{I}$={$\emptyset$} and $\mathcal{J}$ = {$\emptyset$}. Define f : (X, τ, $\mathcal{I}$)$\to$(Y, σ, $\mathcal{J}$) by f(a)=p, f(b)=q and f(c)=r. Clearly f is $(\mathcal{I},\mathcal{J})$-$\alpha$-quotient and strongly $\mathcal{I}$-α-quotient map. Since f^{-1}({p, q})={a, b} is not open in X where {p, q} is open in Y, f is not quotient map.*

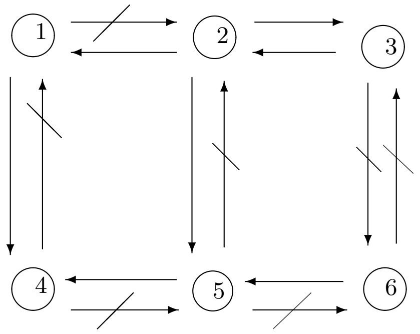
Remark 6.13. *A quotient map need not be strongly $\mathcal{I}$-α-quotient as per the following example.*

Example 6.14. *Let X={a, b, c}, τ={$\emptyset$, X, {a}, {a, b}}, Y={p, q, r}, σ={$\emptyset$, Y, {p}, {p, q}} and $\mathcal{I}$={$\emptyset$}. Define f : (X, τ, $\mathcal{I}$)$\to$(Y, σ) by f(a)=p; f(b)=q and f(c)=r. Clearly f is quotient but not strongly $\mathcal{I}$-α-quotient map.*

Remark 6.15. *The converses of Theorems 6.3 and 6.6 are not true as per the following example.*

Example 6.16. *Let* $X=\{a, b, c\}$, $\tau=\{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$, $Y=\{p, q, r\}$, $\sigma=\{\emptyset, Y, \{r\}, \{p, r\}\}$, $\mathcal{I}=\{\emptyset, \{b\}, \{a, b\}\}$ *and* $\mathcal{J} = \{\emptyset, \{q\}, \{p, q\}\}$. *Define* $f : (X, \tau, \mathcal{I})\to(Y, \sigma, \mathcal{J})$ *by* $f(a)=p$; $f(b)=q$ *and* $f(c)=r$. *Clearly f is* $\alpha^\star$-$\mathcal{I}$-*continuous. Since* $f^{-1}(\{q, r\})=\{b, c\}\notin\alpha^\star_{\mathcal{I}}O(X)$ *where* $\{q, r\}\in\alpha^\star_{\mathcal{J}}O(Y)$, *f is not* $(\mathcal{I},\mathcal{J})$-$\alpha$-*irresolute. Also, since* $f^{-1}(\{q\})=\{b\}$ *is open in X where* $\{q\}\notin\alpha^\star_{\mathcal{J}}O(Y)$, *f is not* $(\mathcal{I},\mathcal{J})$-$\alpha$-*quotient map.*

Remark 6.17. *We obtain the following diagram from the above discussions.*



Where A$\nrightarrow$B means that A does not necessarily imply B and, moreover,

(1) = $(\mathcal{I},\mathcal{J})$-$\alpha$-irresolute map.

(2) = $(\mathcal{I},\mathcal{J})$-$\alpha^*$-quotient map.

(3) = strongly $\mathcal{I}$-α-quotient map.

(4) = $\alpha^\star$-$\mathcal{I}$-continuous map.

(5) = $(\mathcal{I},\mathcal{J})$-$\alpha$-quotient map.

(6) = quotient map.

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