International Journal of Current Research in Science and Technology
Volume 2, Issue 4 (2016), 1–11.
ISSN: 2394-5745
Available Online: http://ijcrst.in/

International Journal of Current Research in Science and Technology

Intuitionistic Fuzzy $\ddot{g}$-continuous Functions

Research Article

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Abstract: In this paper, we introduce the concepts of intuitionistic fuzzy $\ddot{g}$-continuous functions and intuitionistic fuzzy $\ddot{g}$-irresolute functions. Further, we study some of their properties.

MSC: 54A40, 03F55.

Keywords: Intuitionistic fuzzy topology, Intuitionistic fuzzy $\ddot{g}$-closed set, Intuitionistic fuzzy $\ddot{g}$-open set, Intuitionistic fuzzy $\ddot{g}$-continuous function, Intuitionistic fuzzy $\ddot{g}$-irresolute function.

1. Introduction

As a generalization of fuzzy sets, the concepts of intuitionistic fuzzy sets was introduced by Atanassov [1]. Recently, Coker [3] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets and studied its properties. Since the study of generalized continuous functions in intuitionistic fuzzy topological spaces is recent trend, such an attempt is taken in this paper. In this paper we introduce Intuitionistic fuzzy $\ddot{g}$-continuous functions and Intuitionistic fuzzy $\ddot{g}$-irresolute functions. Also the interconnections between the intuitionistic fuzzy continuous functions and the intuitionistic fuzzy irresolute functions are investigated. Some examples are given to illustrate the results.

2. Preliminaries

Definition 2.1 ([1]). *Let X be a non empty set. An intuitionistic fuzzy set (IFS in short) A in X can be described in the form*

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$$

where the function $\mu_A : X \to [0, 1]$ *is called the membership function and* $\mu_A(x)$ *denotes the degree to which* $x \in A$ *and the function* $\nu_A : X \to [0, 1]$ *is called the non-membership function and* $\nu_A(x)$ *denotes the degree to which* $x \notin A$ *and* $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ *for each* $x \in X$. *Denote IFS(X), the set of all intuitionistic fuzzy sets in X. Throughout the paper, X denotes a non empty set.*

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Definition 2.2 ([1]). *Let A and B are any two IFSs of the form* $A=\{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ *and* $B=\{\langle x, \mu_B(x), \nu_B(x)\rangle \mid x \in X\}$. *Then*

(1). $A \subseteq B$ *if and only if* $\mu_A(x) \leq \mu_B(x)$ *and* $\nu_A(x) \geq \nu_B(x)$ *for all* $x \in X$,

(2). $A = B$ *if and only if* $A \subseteq B$ *and* $B \subseteq A$,

(3). $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle \mid x \in X\}$,

(4). $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle \mid x \in X\}$,

(5). $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle \mid x \in X\}$.

Definition 2.3 ([1]). *The intuitionistic fuzzy sets* $0_\sim = \{\langle x, 0, 1\rangle \mid x \in X\}$ *and* $1_\sim = \{\langle x, 1, 0\rangle \mid x \in X\}$ *are called the empty set and the whole set of X respectively.*

Definition 2.4 ([1]). *Let A and B be any two IFSs of the form* $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ *and* $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle \mid x \in X\}$. *Then*

(1). $A \subseteq B$ *and* $A \subseteq C \Rightarrow A \subseteq B \cap C$,

(2). $A \subseteq C$ *and* $B \subseteq C \Rightarrow A \cup B \subseteq C$,

(3). $A \subseteq B$ *and* $B \subseteq C \Rightarrow A \subseteq C$,

(4). $(A \cup B)^c = A^c \cap B^c$ *and* $(A \cap B)^c = A^c \cup B^c$,

(5). $((A)^c)^c = A$,

(6). $(1_\sim)^c = 0_\sim$ *and* $(0_\sim)^c = 1_\sim$.

Definition 2.5 ([3]). *An intuitionistic fuzzy topology (IFT in short) on X is a family* τ *of IFSs in X satisfying the following axioms :*

(1). $0_\sim, 1_\sim \in \tau$,

(2). $G_1 \cap G_2 \in \tau$ *for any* $G_1, G_2 \in \tau$,

(3). $\cup G_i \in \tau$ *for any family* $\{G_i \mid i \in J\} \subseteq \tau$.

In this case the pair (X, τ) *is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in* τ *is known as an intuitionistic fuzzy open set(IFOS in short) in X.*

The complement A^c *of an IFOS A in an IFTS* (X, τ) *is called an intuitionistic fuzzy closed set (IFCS in short) in X.*

Definition 2.6 ([3]). *Let* (X, τ) *be an IFTS and* $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ *be an IFS in X. Then the intuitionistic fuzzy interior and the intuitionistic fuzzy closure are defined as follows:*

$$int(A) = \cup \{G \mid G \text{ is an IFOS in } X \text{ and } G \subseteq A\},$$
$$cl(A) = \cap \{K \mid K \text{ is an IFCS in } X \text{ and } A \subseteq K\}.$$

Proposition 2.7 ([3]). *For any IFSs A and B in* (X, τ), *we have*

(1). $int(A) \subseteq A$,

(2). $A \subseteq cl(A)$,

(3). A is an IFCS in X $\Leftrightarrow$ $cl(A) = A$,

(4). A is an IFOS in X $\Leftrightarrow$ $int(A) = A$,

(5). $A \subseteq B \Rightarrow int(A) \subseteq int(B)$ and $cl(A) \subseteq cl(B)$,

(6). $int(int(A)) = int(A)$,

(7). $cl(cl(A)) = cl(A)$,

(8). $cl(A\cup B) = cl(A) \cup cl(B)$,

(9). $int(A\cap B) = int(A) \cap int(B)$.

Proposition 2.8 ([3]). *For any IFS A in (X, τ), we have*

(1). $int(0_\sim) = 0_\sim$ and $cl(0_\sim) = 0_\sim$,

(2). $int(1_\sim) = 1_\sim$ and $cl(1_\sim) = 1_\sim$,

(3). $(int(A))^c = cl(A^c)$,

(4). $(cl(A))^c = int(A^c)$.

Proposition 2.9 ([3]). *If A is an IFCS in (X, τ) then $cl(A) = A$ and if A is an IFOS in (X, τ) then $int(A) = A$. The arbitrary union of IFCSs is an IFCS in (X, τ).*

Definition 2.10 ([3]). *Let X and Y be two nonempty sets and $f : X \to Y$ be a function. If B=$\{ \langle y, \mu_B(y), \nu_B(y) \rangle \mid y \in Y \}$ is an IFS in Y, then the preimage of B under f, denoted by $f^{-1}(B)$, is an IFS in X defined by $f^{-1}(B) = \{ \langle x, f^{-1}(\mu_B)(x), f^{-1}(\nu_B)(x) \rangle \mid x \in X \}$ where $f^{-1}(\mu_B)(x) = \mu_B(f(x))$ for every $x \in X$.*

Definition 2.11 ([3]). *Let X and Y be two nonempty sets and $f : X \to Y$ be a function. If $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X \}$ is an IFS in X, then the image of A under f denoted by f(A), is an IFS in Y defined by $f(A) = \{ \langle y, f(\mu_A)(y), f(\nu_A)(y) \rangle \mid y \in Y \}$, where*

$$f(\mu_A)(y) = \begin{cases} sup\ \mu_A(x) & if\ f^{-1}(y) \neq \emptyset \ \ and \ \ x \in f^{-1}(y) \\ 0 & otherwise \end{cases}$$

and

$$f(\nu_A)(y) = \begin{cases} inf\ \nu_A(x) & if\ f^{-1}(y) \neq \emptyset \ \ and \ \ x \in f^{-1}(y) \\ 1 & otherwise \end{cases}$$

for each $y \in Y$.

Result 2.12 ([3]). *Let A, A_1, A_2 be IFS in X and B, B_1, B_2 be IFS in Y. Let $f : X \to Y$ be a function. Then*

(1). $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$,

(2). $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$,

(3). $A \subseteq f^{-1}(f(A))$,

(4). $f(f^{-1}(B)) \subseteq B$,

(5). *If f is injective, then* $f^{-1}(f(A)) = A$,

(6). *If f is surjective, then* $f(f^{-1}(B)) = B$,

(7). $f^{-1}(0_\sim) = 0_\sim$,

(8). $f^{-1}(1_\sim) = 1_\sim$,

(9). $f(0_\sim) = 0_\sim$,

(10). *If f is surjective, then* $f(1_\sim) = 1_\sim$,

(11). $f^{-1}(B^c) = (f^{-1}(B))^c$,

(12). *If f is bijective, then* $(f(A))^c = f(A^c)$.

Definition 2.13. *An IFS A in an IFTS* (X, τ) *is said to be an*

(1). intuitionistic fuzzy α*-closed set (IF*α*CS in short) if* $cl(int(cl(A))) \subseteq A$, [5]

(2). intuitionistic fuzzy semi closed set (IFSCS in short) if $int(cl(A)) \subseteq A$, [2]

(3). intuitionistic fuzzy semi pre closed set (IFSPCS in short) if $int(cl(int(A))) \subseteq A$. [19]

(4). intuitionistic fuzzy α*-open set (IF*α*OS in short) if* $A \subseteq int(cl(int(A)))$, [5]

(5). intuitionistic fuzzy semi open set (IFSOS in short) if $A \subseteq cl(int(A))$, [4]

(6). intuitionistic fuzzy semi pre open set (IFSPOS in short) if $A \subseteq cl(int(cl(A)))$. [19]

Definition 2.14 ([14])**.** *Let A be an IFS in an IFTS* (X, τ)*. Then the* α*-interior of A (*$\alpha int(A)$ *in short) and the* α*-closure of A (*$\alpha cl(A)$ *in short) are defined as*

$$\alpha int(A) = \cup \{G \mid G \text{ is an IF}\alpha\text{OS in } (X, \tau) \text{ and } G \subseteq A\},$$
$$\alpha cl(A) = \cap \{K \mid K \text{ is an IF}\alpha\text{CS in } (X, \tau) \text{ and } A \subseteq K\}.$$

$sint(A)$, $scl(A)$, $spint(A)$ *and* $spcl(A)$ *are similarly defined. For any IFS A in* (X, τ)*, we have* $\alpha cl(A^c) = (\alpha int(A))^c$ *and* $\alpha int(A^c) = (\alpha cl(A))^c$.

Definition 2.15. *An IFS A in* (X, τ) *is said to be an*

(1). intuitionistic fuzzy generalized closed set (IFGCS in short) if $cl(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is an IFOS in* (X, τ), [18]

(2). intuitionistic fuzzy generalized semi closed set (IFGSCS in short) if $scl(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is an IFOS in* (X, τ), [13]

(3). intuitionistic fuzzy semi generalized closed set (IFSGCS in short) if $scl(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is an IFSOS in* (X, τ), [16]

(4). intuitionistic fuzzy α *generalized closed set (IF*α*GCS in short) if* $\alpha cl(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is an IFOS in* (X, τ), [14]

(5). intuitionistic fuzzy α generalized semi closed set (IFαGSCS in short) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFSOS in (X, τ), [7]

(6). intuitionistic fuzzy ω closed set (IFωCS in short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFSOS in (X, τ), [16]

(7). intuitionistic fuzzy ψ-closed set (IFψCS in short) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFSGOS in (X, τ), [15]

(8). intuitionistic fuzzy generalized semi pre closed set (IFGSPCS in short) if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ). [11]

The complements of the above mentioned intuitionistic fuzzy closed sets are called their respective intuitionistic fuzzy open sets.

Definition 2.16 ([15]). *An IFS A in (X, τ) is said to be an*

(1). intuitionistic fuzzy $\ddot{g}$-closed set (IF$\ddot{G}$CS in short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFSGOS in (X, τ),

(2). intuitionistic fuzzy $\ddot{g}_\alpha$-closed set (IF$\ddot{G}_\alpha$CS in short) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFSGOS in (X, τ).

The complements of the above mentioned intuitionistic fuzzy closed sets are called their respective intuitionistic fuzzy open sets.

Definition 2.17. *Let f be a function from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an*

(1). intuitionistic fuzzy continuous (IF continuous in short) if $f^{-1}(B)$ is IFOS in X for every $B \in \sigma$, [4]

(2). intuitionistic fuzzy α-continuous (IFα continuous in short) if $f^{-1}(B)$ is IFαOS in X for every $B \in \sigma$, [4]

(3). intuitionistic fuzzy semi continuous (IFS continuous in short) if $f^{-1}(B)$ is IFSOS in X for every $B \in \sigma$, [4]

(4). intuitionistic fuzzy ω continuous (IFω continuous in short) if $f^{-1}(B)$ is IFωOS in X for every $B \in \sigma$, [16]

(5). intuitionistic fuzzy generalized continuous (IFG continuous in short) if $f^{-1}(B)$ is an IFGCS in X for every IFCS B of (Y, σ), [17]

(6). intuitionistic fuzzy generalized semi continuous (IFGS continuous in short) if $f^{-1}(B)$ is an IFGSCS in X for every IFCS B of (Y, σ), [13]

(7). intuitionistic fuzzy generalized semi pre continuous (IFGSP continuous in short) if $f^{-1}(B)$ is an IFGSPCS in X for every IFCS B of (Y, σ), [6]

(8). intuitionistic fuzzy α-generalized continuous (IFαG continuous in short) if $f^{-1}(B)$ is an IFαGCS in X for every IFCS B of (Y, σ), [14]

(9). intuitionistic fuzzy α-generalized semi continuous (IFαGS continuous in short) if $f^{-1}(B)$ is an IFαGSCS in X for every IFCS B of (Y, σ). [7]

3. Intuitionistic Fuzzy $\ddot{g}$-continuous Functions

In this section we introduce intuitionistic fuzzy $\ddot{g}$-continuous functions and study some of their properties.

Definition 3.1. *A function $f : (X, \tau) \to (Y, \sigma)$ is called an intuituionistic fuzzy $\ddot{g}$-continuous (IF$\ddot{G}$ continuous in short) if $f^{-1}(B)$ is an IF$\ddot{G}$CS in (X, τ) for every IFCS B of (Y, σ).*

Example 3.2. *Let $X = \{a,\ b\}$ and $Y = \{u,\ v\}$. Let $T_1 = \langle x,\ (0.3,\ 0.4),\ (0.6,\ 0.5)\rangle$ and $T_2 = \langle y,\ (0.3,\ 0.4),\ (0.6,\ 0.5)\rangle$. Then $\tau = \{0_\sim,\ T_1,\ 1_\sim\}$ and $\sigma = \{0_\sim,\ T_2,\ 1_\sim\}$ are IFTs on X and Y respectively. Define a function $f : (X,\ \tau) \to (Y,\ \sigma)$ by $f(a) = u$ and $f(b) = v$. Then $f^{-1}(B)$ is an IF$\ddot{G}$CS in $(X,\ \tau)$ for every IFCS B of $(Y,\ \sigma)$. Therefore f is an IF$\ddot{G}$ continuous function.*

Example 3.3. *Let $X = \{a,\ b\}$ and $Y = \{u,\ v\}$. Let $T_1 = \langle x,\ (0.3,\ 0.4),\ (0.6,\ 0.5)\rangle$ and $T_2 = \langle y,\ (0.6,\ 0.5),\ (0.3,\ 0.4)\rangle$. Then $\tau = \{0_\sim,\ T_1,\ 1_\sim\}$ and $\sigma = \{0_\sim,\ T_2,\ 1_\sim\}$ are IFTs on X and Y respectively. Define a function $f : (X,\ \tau) \to (Y,\ \sigma)$ by $f(a) = u$ and $f(b) = v$. Then for IFCS $B = \langle y,\ (0.3,\ 0.4),\ (0.6,\ 0.5)\rangle$ of $(Y,\ \sigma)$, $f^{-1}(B)$ is not an IF$\ddot{G}$CS in $(X,\ \tau)$. Therefore f is not an IF$\ddot{G}$ continuous function.*

Theorem 3.4. *Every IF continuous function is an IF$\ddot{G}$ continuous function, but not conversely.*

Proof. Let $f : (X,\ \tau) \to (Y,\ \sigma)$ be an IF continuous function. Let B be an IFCS in Y. Since f is an IF continuous function, $f^{-1}(B)$ is an IFCS in X. Since every IFCS is an IF$\ddot{G}$CS, $f^{-1}(B)$ is an IF$\ddot{G}$CS in X. Hence f is an IF$\ddot{G}$ continuous function. □

Example 3.5. *Let $X = \{a,\ b\}$ and $Y = \{u,\ v\}$. Let $T_1 = \langle x,\ (0.6,\ 0.7),\ (0.3,\ 0.2)\rangle$ and $T_2 = \langle y,\ (0.7,\ 0.8),\ (0.2,\ 0.1)\rangle$. Then $\tau = \{0_\sim,\ T_1,\ 1_\sim\}$ and $\sigma = \{0_\sim,\ T_2,\ 1_\sim\}$ are IFTs on X and Y respectively. Define a function $f : (X,\ \tau) \to (Y,\ \sigma)$ by $f(a) = u$ and $f(b) = v$. Then for IFCS $B = \langle y,\ (0.2,\ 0.1),\ (0.7,\ 0.8)\rangle$ of $(Y,\ \sigma)$, $f^{-1}(B)$ is not an IFCS in $(X,\ \tau)$. Therefore f is not an IF continuous function. But f is an IF$\ddot{G}$ continuous function.*

Theorem 3.6. *Every IF$\ddot{G}$ continuous function is an IFGSP continuous function, but not conversely.*

Proof. Let $f : (X,\ \tau) \to (Y,\ \sigma)$ be an IF$\ddot{G}$ continuous function. Let B be an IFCS in Y. Since f is an IF$\ddot{G}$ continuous function, $f^{-1}(B)$ is an IF$\ddot{G}$CS in X. Since every IF$\ddot{G}$CS is an IFGSPCS [15], $f^{-1}(B)$ is an IFGSPCS in X. Hence f is an IFGSP continuous function. □

Example 3.7. *Let $X = \{a,\ b\}$ and $Y = \{u,\ v\}$. Let $T_1 = \langle x,\ (0.3,\ 0.4),\ (0.6,\ 0.5)\rangle$ and $T_2 = \langle y,\ (0.5,\ 0.4),\ (0.4,\ 0.5)\rangle$. Then $\tau = \{0_\sim,\ T_1,\ 1_\sim\}$ and $\sigma = \{0_\sim,\ T_2,\ 1_\sim\}$ are IFTs on X and Y respectively. Define a function $f : (X,\ \tau) \to (Y,\ \sigma)$ by $f(a) = u$ and $f(b) = v$. Then for IFCS $B = \langle y,\ (0.4,\ 0.5),\ (0.5,\ 0.4)\rangle$ of $(Y,\ \sigma)$, $f^{-1}(B)$ is not an IF$\ddot{G}$CS in $(X,\ \tau)$. Therefore f is not an IF$\ddot{G}$ continuous function. But f is an IFGSP continuous function.*

Theorem 3.8. *Every IF$\ddot{G}$ continuous function is an IFω continuous function, but not conversely.*

Proof. Let $f : (X,\ \tau) \to (Y,\ \sigma)$ be an IF$\ddot{G}$ continuous function. Let B^c be an IFOS and B is an IFCS in Y. Since f is an IF$\ddot{G}$ continuous function, $f^{-1}(B)$ is an IF$\ddot{G}$CS in X. Since every IF$\ddot{G}$CS is an IFωCS [15], $f^{-1}(B)$ is an IFωCS in X and $X \setminus f^{-1}(B) = f^{-1}(B^c)$ is IFωOS in X. Hence f is an IFω continuous function. □

Example 3.9. *Let $X = \{a,\ b\}$ and $Y = \{u,\ v\}$. Let $T_1 = \langle x,\ (0.4,\ 0.3),\ (0.5,\ 0.6)\rangle$ and $T_2 = \langle y,\ (0.3,\ 0.2),\ (0.6,\ 0.7)\rangle$. Then $\tau = \{0_\sim,\ T_1,\ 1_\sim\}$ and $\sigma = \{0_\sim,\ T_2,\ 1_\sim\}$ are IFTs on X and Y respectively. Define a function $f : (X,\ \tau) \to (Y,\ \sigma)$ by $f(a) = u$ and $f(b) = v$. Then for IFCS $B = \langle y,\ (0.6,\ 0.7),\ (0.3,\ 0.2)\rangle$ of $(Y,\ \sigma)$, $f^{-1}(B)$ is not an IF$\ddot{G}$CS in $(X,\ \tau)$. Therefore f is not an IF$\ddot{G}$ continuous function. But f is an IFω continuous function.*

Theorem 3.10. *Every IF$\ddot{G}$ continuous function is an IFG continuous function, but not conversely.*

Proof. Let $f : (X,\ \tau) \to (Y,\ \sigma)$ be an IF$\ddot{G}$ continuous function. Let B be an IFCS in Y. Since f is an IF$\ddot{G}$ continuous function, $f^{-1}(B)$ is an IF$\ddot{G}$CS in X. Since every IF$\ddot{G}$CS is an IFGCS [15], $f^{-1}(B)$ is an IFGCS in X. Hence f is an IFG continuous function. □

Example 3.11. *Let $X = \{a, b\}$ and $Y = \{u, v\}$. Let $T_1 = \langle x, (0.6, 0.5), (0.3, 0.4)\rangle$ and $T_2 = \langle y, (0.2, 0.3), (0.7, 0.6)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then for IFCS $B = \langle y, (0.7, 0.6), (0.2, 0.3)\rangle$ of (Y, σ), $f^{-1}(B)$ is not an IF$\ddot{G}$CS in (X, τ). Therefore f is not an IF$\ddot{G}$ continuous function. But f is an IFG continuous function.*

Theorem 3.12. *Every IF$\ddot{G}$ continuous function is an IFαG continuous function, but not conversely.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an IF$\ddot{G}$ continuous function. Let B be an IFCS in Y. Since f is an IF$\ddot{G}$ continuous function, $f^{-1}(B)$ is an IF$\ddot{G}$CS in X. Since every IF$\ddot{G}$CS is an IFαGCS [15], $f^{-1}(B)$ is an IFαGCS in X. Hence f is an IFαG continuous function. □

Example 3.13. *Let $X = \{a, b\}$ and $Y = \{u, v\}$. Let $T_1 = \langle x, (0.5, 0.6), (0.4, 0.3)\rangle$ and $T_2 = \langle y, (0.3, 0.2), (0.6, 0.7)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then for IFCS $B = \langle y, (0.6, 0.7), (0.3, 0.2)\rangle$ of (Y, σ), $f^{-1}(B)$ is not an IF$\ddot{G}$CS in (X, τ). Therefore f is not an IF$\ddot{G}$ continuous function. But f is an IFαG continuous function.*

Theorem 3.14. *Every IF$\ddot{G}$ continuous function is an IFGS continuous function, but not conversely.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an IF$\ddot{G}$ continuous function. Let B be an IFCS in Y. Since f is an IF$\ddot{G}$ continuous function, $f^{-1}(B)$ is an IF$\ddot{G}$CS in X. Since every IF$\ddot{G}$CS is an IFGSCS [15], $f^{-1}(B)$ is an IFGSCS in X. Hence f is an IFGS continuous function. □

Example 3.15. *Let $X = \{a, b\}$ and $Y = \{u, v\}$. Let $T_1 = \langle x, (0.2, 0.4), (0.7, 0.5)\rangle$ and $T_2 = \langle y, (0.3, 0.2), (0.6, 0.7)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then for IFCS $B = \langle y, (0.6, 0.7), (0.3, 0.2)\rangle$ of (Y, σ), $f^{-1}(B)$ is not an IF$\ddot{G}$CS in (X, τ). Therefore f is not an IF$\ddot{G}$ continuous function. But f is an IFGS continuous function.*

Definition 3.16. *A function $f : (X, \tau) \to (Y, \sigma)$ is called an intuituionistic fuzzy ψ-continuous (IFψ continuous in short) if $f^{-1}(B)$ is an IFψCS in (X, τ) for every IFCS B of (Y, σ).*

Theorem 3.17. *Every IF$\ddot{G}$ continuous function is an IFψ continuous function, but not conversely.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an IF$\ddot{G}$ continuous function. Let B be an IFCS in Y. Since f is an IF$\ddot{G}$ continuous function, $f^{-1}(B)$ is an IF$\ddot{G}$CS in X. Since every IF$\ddot{G}$CS is an IFψCS [15], $f^{-1}(B)$ is an IFψCS in X. Hence f is an IFψ continuous function. □

Example 3.18. *Let $X = \{a, b\}$ and $Y = \{u, v\}$. Let $T_1 = \langle x, (0.3, 0.2), (0.6, 0.7)\rangle$ and $T_2 = \langle y, (0.5, 0.6), (0.4, 0.3)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then for IFCS $B = \langle y, (0.4, 0.3), (0.5, 0.6)\rangle$ of (Y, σ), $f^{-1}(B)$ is not an IF$\ddot{G}$CS in (X, τ). Therefore f is not an IF$\ddot{G}$ continuous function. But f is an IFψ continuous function.*

Theorem 3.19. *Every IF$\ddot{G}$ continuous function is an IFαGS continuous function, but not conversely.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an IF$\ddot{G}$ continuous function. Let B be an IFCS in Y. Since f is an IF$\ddot{G}$ continuous function, $f^{-1}(B)$ is an IF$\ddot{G}$CS in X. Since every IF$\ddot{G}$CS is an IFαGSCS [15], $f^{-1}(B)$ is an IFαGSCS in X. Hence f is an IFαGS continuous function. □

Example 3.20. *Let $X = \{a, b\}$ and $Y = \{u, v\}$. Let $T_1 = \langle x, (0.3, 0.4), (0.6, 0.5)\rangle$ and $T_2 = \langle y, (0.2, 0.3), (0.7, 0.6)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then for IFCS $B = \langle y, (0.7, 0.6), (0.2, 0.3)\rangle$ of (Y, σ), $f^{-1}(B)$ is not an IF$\ddot{G}$CS in (X, τ). Therefore f is not an IF$\ddot{G}$ continuous function. But f is an IFαGS continuous function.*

Definition 3.21. *A function $f : (X, \tau) \to (Y, \sigma)$ is called an intuituionistic fuzzy $\ddot{g}_\alpha$-continuous (IF$\ddot{G}_\alpha$ continuous in short) if $f^{-1}(B)$ is an IF$\ddot{G}_\alpha$CS in (X, τ) for every IFCS B of (Y, σ).*

Theorem 3.22. *Every IF$\ddot{G}$ continuous function is an IF$\ddot{G}_\alpha$ continuous function, but not conversely.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an IF$\ddot{G}$ continuous function. Let B be an IFCS in Y. Since f is an IF$\ddot{G}$ continuous function, $f^{-1}(B)$ is an IF$\ddot{G}$CS in X. Since every IF$\ddot{G}$CS is an IF$\ddot{G}_\alpha$CS [15], $f^{-1}(B)$ is an IF$\ddot{G}_\alpha$CS in X. Hence f is an IF$\ddot{G}_\alpha$ continuous function. □

Example 3.23. *Let $X = \{a, b\}$ and $Y = \{u, v\}$. Let $T_1 = \langle x, (0.3, 0.7), (0.6, 0.2)\rangle$ and $T_2 = \langle y, (0.6, 0.7), (0.3, 0.2)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then for IFCS $B = \langle y, (0.3, 0.2), (0.6, 0.7)\rangle$ of (Y, σ), $f^{-1}(B)$ is not an IF$\ddot{G}$CS in (X, τ). Therefore f is not an IF$\ddot{G}$ continuous function. But f is an IF$\ddot{G}_\alpha$ continuous function.*

Remark 3.24. *IFα continuous function and IF$\ddot{G}$ continuous function are independent.*

Example 3.25. *Let $X = \{a, b\}$ and $Y = \{u, v\}$. Let $T_1 = \langle x, (0.3, 0.2), (0.7, 0.8)\rangle$ and $T_2 = \langle y, (0.8, 0.6), (0.2, 0.4)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then for IFCS $B = \langle y, (0.3, 0.4), (0.6, 0.5)\rangle$ of (Y, σ), $f^{-1}(B)$ is not an IF$\ddot{G}$CS in (X, τ). Therefore f is not an IF$\ddot{G}$ continuous function. But f is an IFα continuous function.*

Example 3.26. *Let $X = \{a, b\}$ and $Y = \{u, v\}$. Let $T_1 = \langle x, (0.2, 0.3), (0.7, 0.6)\rangle$, $T_2 = \langle x, (0.3, 0.4), (0.6, 0.5)\rangle$ and $T_3 = \langle y, (0.15, 0.25), (0.75, 0.65)\rangle$. Then $\tau = \{0_\sim, T_1, T_2, 1_\sim\}$ and $\sigma = \{0_\sim, T_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then for IFCS $B = \langle y, (0.75, 0.65), (0.15, 0.25)\rangle$ of (Y, σ), $f^{-1}(B)$ is not an IFαCS in (X, τ). Therefore f is not an IFα continuous function. But f is an IF$\ddot{G}$ continuous function.*

Remark 3.27. *IFS continuous function and IF$\ddot{G}$ continuous function are independent.*

Example 3.28. *Let $X = \{a, b\}$ and $Y = \{u, v\}$. Let $T_1 = \langle x, (0.2, 0.3), (0.7, 0.6)\rangle$ and $T_2 = \langle y, (0.6, 0.5), (0.3, 0.4)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then for IFCS $B = \langle y, (0.3, 0.4), (0.6, 0.5)\rangle$ of (Y, σ), $f^{-1}(B)$ is not an IF$\ddot{G}$CS in (X, τ). Therefore f is not an IF$\ddot{G}$ continuous function. But f is an IFS continuous function.*

Example 3.29. *Let $X = \{a, b\}$ and $Y = \{u, v\}$. Let $T_1 = \langle x, (0.2, 0.3), (0.7, 0.6)\rangle$, $T_2 = \langle x, (0.3, 0.4), (0.6, 0.5)\rangle$ and $T_3 = \langle y, (0.1, 0.2), (0.8, 0.7)\rangle$. Then $\tau = \{0_\sim, T_1, T_2, 1_\sim\}$ and $\sigma = \{0_\sim, T_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then for IFCS $B = \langle y, (0.8, 0.7), (0.1, 0.2)\rangle$ of (Y, σ), $f^{-1}(B)$ is not an IFSCS in (X, τ). Therefore f is not an IFS continuous function. But f is an IF$\ddot{G}$ continuous function.*

Theorem 3.30. *A function $f : X \to Y$ is an IF$\ddot{G}$ continuous if and only if the inverse image of every IFOS in Y is an IF$\ddot{G}$OS in X.*

Proof. Necessary Part: Let A be an IFOS in Y. This implies A^c is an IFCS in Y. Since f is an IF$\ddot{G}$ continuous, $f^{-1}(A^c)$ is an IF$\ddot{G}$CS in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IF$\ddot{G}$OS in X. Hence the inverse image of an IFOS is an IF$\ddot{G}$OS.

Sufficient Part: Let A be an IFCS in Y. Then A^c is an IFOS on Y. By hypothesis, $f^{-1}(A^c)$ is an IF$\ddot{G}$OS in X. Since $f^{-1}(A^c)$ = $(f^{-1}(A))^c$, $f^{-1}(A)$ is an IF$\ddot{G}$CS in X. Hence f is an IF$\ddot{G}$ continuous function. □

Let us introduce IF$_{\ddot{g}}T_{1/2}$ and IF$_{\ddot{g}a}T_{1/2}$ spaces.

Definition 3.31. *An IFTS (X, τ) is said to be an intuitionistic fuzzy $_{\ddot{g}}T_{1/2}$ (IF$_{\ddot{g}}T_{1/2}$ in short) space if every IF$\ddot{G}$CS in (X, τ) is an IFCS in (X, τ).*

Definition 3.32. *An IFTS (X, τ) is said to be an intuitionistic fuzzy $_{\ddot{g}a}T_{1/2}$ (IF$_{\ddot{g}a}T_{1/2}$ in short) space if every IFGCS in (X, τ) is an IF$\ddot{G}$CS in (X, τ).*

Theorem 3.33. *Let f : (X, τ) → (Y, σ) be an IF$\ddot{G}$ continuous function. Then f is an IF continuous function if X is an IF$_{\ddot{g}}T_{1/2}$ space.*

Proof. Let A be an IFCS in Y. By hypothesis, $f^{-1}(A)$ is an IF$\ddot{G}$CS in X. Since X is an IF$_{\ddot{g}}T_{1/2}$ space, $f^{-1}(A)$ is an IFCS in X. Hence f is an IF continuous function. □

Theorem 3.34. *Let f : (X, τ) → (Y, σ) be an IFG continuous function. Then f is an IF$\ddot{G}$ continuous function if X is an IF$_{\ddot{g}a}T_{1/2}$ space.*

Proof. Let A be an IFCS in Y. By hypothesis, $f^{-1}(A)$ is an IFGCS in X. Since X is an IF$_{\ddot{g}a}T_{1/2}$ space, $f^{-1}(A)$ is an IF$\ddot{G}$CS in X. Hence f is an IF$\ddot{G}$ continuous function. □

Remark 3.35. *The composition of two IF$\ddot{G}$ continuous functions need not be an IF$\ddot{G}$ continuous function.*

Example 3.36. *Let X = {a, b}, Y = {c, d} and Z = {u, v}. Let T_1 = ⟨x, (0.3, 0.4), (0.6, 0.5)⟩ and T_2 = ⟨z, (0.7, 0.6), (0.2, 0.3)⟩. Then τ = {$0_\sim$, T_1, $1_\sim$}, σ = {$0_\sim$, $1_\sim$} and δ = {$0_\sim$, T_2, $1_\sim$} are IFTs on X, Y and Z respectively. Define a function f : (X, τ) → (Y, σ) by f(a) = c and f(b) = d, and define a function g : (Y, σ) → (Z, δ) by g(c) = u and g(d) = v. Then f and g are IF$\ddot{G}$ continuous functions. Consider an IFS A = ⟨z, (0.2, 0.3), (0.7, 0.6)⟩. Then A is an IFCS in Z. But $(g \circ f)^{-1}(A)$ is not an IF$\ddot{G}$CS in X. Hence the composition of two IF$\ddot{G}$ continuous functions need not be an IF$\ddot{G}$ continuous function.*

Theorem 3.37. *Let f : (X, τ) → (Y, σ) be an IF$\ddot{G}$ continuous function. Let g : (Y, σ) → (Z, δ) be an IF continuous. Then g ∘ f : (X, τ) → (Z, δ) is an IF$\ddot{G}$ continuous function.*

Proof. Let A be an IFCS in Z. By hypothesis, $g^{-1}(A)$ is an IFCS in Y. Since f is an IF$\ddot{G}$ continuous function, $f^{-1}(g^{-1}(A))$ is an IF$\ddot{G}$CS in X. Hence g ∘ f is an IF$\ddot{G}$ continuous function. □

4. Intuitionistic Fuzzy $\ddot{g}$-irresolute Functions

In this section, we introduce intuitionistic fuzzy $\ddot{g}$-irresolute functions and study some of their properties.

Definition 4.1. *A function f : (X, τ) → (Y, σ) is called an intuitionistic fuzzy $\ddot{g}$-irresolute (IF$\ddot{G}$ irresolute, in short) if $f^{-1}(A)$ is an IF$\ddot{G}$CS in (X, τ) for every IF$\ddot{G}$CS A in (Y, σ).*

Example 4.2. *Let X = {a, b} and Y = {u, v}. Let T_1 = ⟨x, (0.5, 0.6), (0.4, 0.3)⟩ and T_2 = ⟨y, (0.6, 0.7), (0.3, 0.2)⟩. Then τ = {$0_\sim$, T_1, $1_\sim$} and σ = {$0_\sim$, T_2, $1_\sim$} are IFTs on X and Y respectively. Define a function f : (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is not an IF$\ddot{G}$ irresolute function.*

Theorem 4.3. *Let f : (X, τ) → (Y, σ) be an IF$\ddot{G}$ irresolute. Then f is an IF$\ddot{G}$ continuous function, but not conversely.*

Proof. Let f be an IF$\ddot{G}$ irresolute function. Let A be any IFCS in Y. Since every IFCS is an IF$\ddot{G}$CS [15], A is an IF$\ddot{G}$CS in Y. By hypothesis, $f^{-1}(A)$ is an IF$\ddot{G}$CS in X. Hence f is an IF$\ddot{G}$ continuous function. □

Example 4.4. *Let X = {a, b} and Y = {u, v}. Let $T_1 = \langle x, (0.3, 0.4), (0.7, 0.6)\rangle$ and $T_2 = \langle y, (0.7, 0.6), (0.3, 0.4)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a function f : (X, τ) → (Y, σ) by f(a) = 1−u and f(b) = 1−v. Then f is an IF$\ddot{G}$ continuous function. But f is not an IF$\ddot{G}$ irresolute function. We have the IFS $A = \langle y, (0.2, 0.3), (0.8, 0.7)\rangle$ is an IF$\ddot{G}$CS in Y, but $f^{-1}(A) = \langle x, (0.8, 0.7), (0.2, 0.3)\rangle$ is not an IF$\ddot{G}$CS in X. Hence f is an IF$\ddot{G}$ continuous function, but not an IF$\ddot{G}$ irresolute function.*

Theorem 4.5. *Let f : (X, τ) → (Y, σ) be an IF$\ddot{G}$ irresolute. Then f is an IF continuous function if X is an $IF_{\ddot{g}}T_{1/2}$ space.*

Proof. Let A be an IFCS in Y. Then A is an IF$\ddot{G}$CS in Y. By hypothesis, $f^{-1}(A)$ is an IF$\ddot{G}$CS in X. Since X is an $IF_{\ddot{g}}T_{1/2}$ space, $f^{-1}(A)$ is an IFCS in X. Hence f is an IF continuous function. □

Theorem 4.6. *Let f : (X, τ) → (Y, σ) and g : (Y, σ) → (Z, δ) be IF$\ddot{G}$ irresolute functions. Then g ∘ f : (X, τ) → (Z, δ) is an IF$\ddot{G}$ irresolute function.*

Proof. Let A be an IF$\ddot{G}$CS in Z. Then $g^{-1}(A)$ is an IF$\ddot{G}$CS in Y. Since f is an IF$\ddot{G}$ irresolute function, $f^{-1}(g^{-1}(A))$ is an IF$\ddot{G}$CS in X. Hence g ∘ f is an IF$\ddot{G}$ irresolute function. □

Theorem 4.7. *Let f : (X, τ) → (Y, σ) be an IF$\ddot{G}$ irresolute function and g : (Y, σ) → (Z, δ) be an IF$\ddot{G}$ continuous function. Then g ∘ f : (X, τ) → (Z, δ) is an IF$\ddot{G}$ continuous function.*

Proof. Let A be an IFCS in Z. Since g is an IF$\ddot{G}$ continuous function, $g^{-1}(A)$ is an IF$\ddot{G}$CS in Y. Since f is an IF$\ddot{G}$ irresolute function, $f^{-1}(g^{-1}(A))$ is an IF$\ddot{G}$CS in X. Hence g ∘ f is an IF$\ddot{G}$ continuous function. □

Theorem 4.8. *Let f : (X, τ) → (Y, σ) be an IF$\ddot{G}$ irresolute function. Then f is an IFG continuous function if X is an $IF_{\ddot{g}a}T_{1/2}$ space.*

Proof. Let A be an IFCS in Y. Then A is an IF$\ddot{G}$CS in Y. By hypothesis, $f^{-1}(A)$ is an IF$\ddot{G}$CS in X and hence IFGCS in X. Since X is an $IF_{\ddot{g}a}T_{1/2}$ space, $f^{-1}(A)$ is an IF$\ddot{G}$CS in X and hence IFGCS in X. Hence f is an IFG continuous function. □

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