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Propagation of Surface Wave in Fluid Saturated Porous Medium Sandwiched Between Magneto-Elastic Self-Reinforced Layer and Heterogeneous Isotropic half-space

Seminar Paper*

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Abstract: The present study deals with the propagation of surface waves in the fluid saturated porous medium sandwiched between magneto-elastic self-reinforced medium and heterogeneous isotropic half-space. Frequency equation of surface wave has been obtained. Numerical results and particular cases have also been discussed. In the isotropic case, when heterogeneity, magnetic field, self-reinforcement and porosity are absent, the frequency equation reduces to classical Love wave equation. Effects of reinforcement, magneto-elastic coupling parameter and heterogeneity on phase velocity have been depicted by means of graphs.

Keywords: Surface wave, Heterogeneity, Magneto-elastic, Self-reinforcement, Porosity.

1. Introduction

Effects of earthquakes on artificial structures are of prime importance to engineers and architects. During an earthquake and similar disturbances, a structure is excited into a more or less vibrant with oscillatory stress that depends upon both the ground motion and physical properties of the structure. Most concrete construction includes nominal steel reinforcing as given in the book of Richter [13].

This is potentially the strongest and most earthquake resistant type of construction. So, wave propagation in a reinforced medium plays a very important role in the civil engineering and geophysics. The characteristic property of a self-reinforced material is that its component act together as a single anisotropic unit as long as they remain in elastic condition, i.e. the two components are bound together so that there is no relative displacement between them. There is sufficient evidence in the literature that the Earth crust may contain some hard/soft rocks or material that may exhibit self-reinforcement properties. The study of mineral explorations and geophysics also proves the existence of magnetic field and inhomogeneity characteristic of the earth.

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The study of porous medium in recent time has acquired prime interest. The layer of the earth usually of such materials and the medium is generally dealt under the name of poro-elastic medium. Investigation on propagation of waves in liquid saturated porous solids are relevant to geophysical prospecting methods, survey techniques are very useful in oil industry.

The role of pore water in seismology has been emphasized in many studies. Biot [2] has established the theory of the propagation of elastic waves in a porous elastic solid saturated by a viscous fluid. Biot [3] has developed the mathematical theory for the propagation of elastic waves in a fluid saturated porous medium. The problem of magneto-elastic transverse surface waves in self-reinforced elastic solids was studied by Verma et al. [12]. Chattopadhyay and Chaudhury [4] studied the propagation of magnetoelastic shear waves in an infinite self-reinforced plate. The magnetic and thermal effect on shear wave propagation is highlighted by Sethi and Gupta [11].

A detailed investigation has been made by Kumar and Hundal [10] to notice the propagation pattern of surface waves in uniform liquid layer overlying a fluid saturated porous half space. The existence and asymptotic behavior of the surface waves at a free interface of a saturated porous medium are investigated in the low frequency range by Edelman [7]. Gupta et al. [9] have shown the effect of initial stress on propagation of Love waves in anisotropic porous layer. Chattopadhyay and De [5] investigated the propagation of love waves in an isotropic fluid saturated porous layer with irregular interface. Chattopadhyay et al. [6] have discussed the dispersion of G-type seismic wave in magnetoelastic self-reinforced layer.

The idea of our present paper is taken from this. In the present paper we have discussed the propagation of surface waves in a fluid saturated porous medium sandwiched between magneto-elastic self-reinforced layer and a heterogeneous half-space.

2. Formulation and Solution of the Problem

We have considered an anisotropic fluid saturated porous layer of thickness H_1, sandwiched between magneto-elastic self-reinforced medium of finite width and heterogeneous isotropic half-spaces. The heterogeneity of the half-space has been taken as

$$\begin{aligned} \mu_3 &= \mu_0 e^{az} \\ \rho_3 &= \rho_0 e^{az} \end{aligned} \tag{1}$$

where μ_3 and ρ_3 are rigidity and density of the half-space respectively. The x-axis is taken horizontally in the direction of wave propagation and z-axis is taken vertically downwards as shown in the Fig.1.

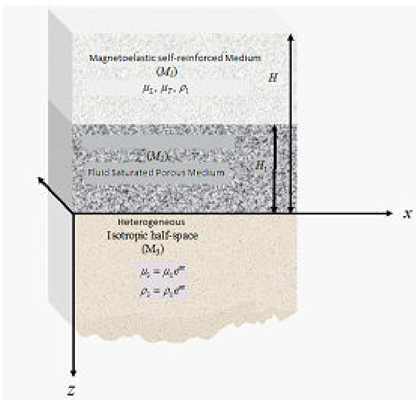

Figure 1. **Geometry of the Problem**

2.1. Formulation and Solution for Upper Layer M1

We deduce the equation of motion for the propagation of surface wave in Magneto-elastic self-reinforced media and then find its appropriate solution. The constitutive equations used in a self-reinforced linearly elastic model are [1]:

$$\tau_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu_T e_{ij} + \alpha(a_k a_m e_{km}\delta_{ij} + e_{kk}a_i a_j) + 2(\mu_L - \mu_T)(a_i a_k e_{kj} + a_j a_k e_{ki}) + \beta a_k a_m e_{km} a_i a_j,\ i,j,k,m = 1,2,3. \quad (2)$$

Where τ_{ij} are components of stress, $e_{ij}$are components of infinitesimal strain, δ_{ij} Kronecker delta, a_i components of $\vec{a}$, all referred to rectangular Cartesian co-ordinates x_i . $\vec{a} = (a_1, a_2, a_3)$ is the preferred direction of reinforcement such that $a_1^2 + a_2^2 + a_3^2 = 1$. The vector $\vec{a}$may be a function of position. Indices take the values 1, 2, 3 and summation convention is employed. The coefficients λ, μ_L, μ_T, α and β are elastic constants with dimension of stress. μ_T can be identified as the shear modulus in transverse shear across the preferred direction, and μ_L as the shear modulus in longitudinal shear in the preferred direction. α and β are specific stress components to take into account different layers for the concrete part of the composite material. Equations governing the propagation of small elastic disturbances in a perfectly conducting self-reinforced elastic medium having electromagnetic force$\vec{J} \times \vec{B}$ (the Lorentz force, $\vec{J}$ being the electric current density and $\vec{B}$ being the magnetic induction vector) as the only body forces are

$$\tau_{ij,j} + \left(\vec{J} \times \vec{B}\right)_i = \rho \frac{\partial^2 u_i}{\partial t^2} \quad (3)$$

Where $\left(\vec{J} \times \vec{B}\right)_i$ is the x_i-component of the force $\left(\vec{J} \times \vec{B}\right)$. Here interaction of mechanical and electromagnetic fields are considered. Let $u_i = (u_1, v_1, w_1)$ and taking $x_1 = x, x_2 = y, x_3 = z$, then equation (3) becomes

$$\begin{aligned} &\tfrac{\partial \tau_{11}}{\partial x} + \tfrac{\partial \tau_{12}}{\partial y} + \tfrac{\partial \tau_{13}}{\partial z} + \left(\vec{J} \times \vec{B}\right)_x = \rho \tfrac{\partial^2 u_1}{\partial t^2} \\ &\tfrac{\partial \tau_{12}}{\partial x} + \tfrac{\partial \tau_{22}}{\partial y} + \tfrac{\partial \tau_{23}}{\partial z} + \left(\vec{J} \times \vec{B}\right)_y = \rho \tfrac{\partial^2 v_1}{\partial t^2} \\ &\tfrac{\partial \tau_{13}}{\partial x} + \tfrac{\partial \tau_{23}}{\partial y} + \tfrac{\partial \tau_{33}}{\partial z} + \left(\vec{J} \times \vec{B}\right)_z = \rho \tfrac{\partial^2 w_1}{\partial t^2} \end{aligned} \quad (4)$$

For SH wave propagating in the x- direction and causing displacement in the y- direction only, we have

$$u_1 = w_1 = 0,\ v_1 = v_1(x,z,t)\ \text{ and }\ \frac{\partial}{\partial y} \equiv 0 \quad (5)$$

Using equation (5) in equation (4), we have the only non vanishing equation as

$$\frac{\partial \tau_{12}}{\partial x} + \frac{\partial \tau_{23}}{\partial z} + \left(\vec{J} \times \vec{B}\right)_y = \rho \frac{\partial^2 v_1}{\partial t^2} \quad (6)$$

Where

$$\begin{aligned} \tau_{12} &= \mu_T \tfrac{\partial v_1}{\partial x} + (\mu_L - \mu_T)\, a_1 \left(a_1 \tfrac{\partial v_1}{\partial x} + a_3 \tfrac{\partial v_1}{\partial z}\right) \\ \tau_{23} &= \mu_T \tfrac{\partial v_1}{\partial z} + (\mu_L - \mu_T)\, a_3 \left(a_1 \tfrac{\partial v_1}{\partial x} + a_3 \tfrac{\partial v_1}{\partial z}\right) \end{aligned}$$

The electromagnetic fields are governed by the following well known Maxwell's equations

$$\vec{\nabla}.\vec{B} = 0,\ \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t},\ \vec{\nabla} \times \vec{H} = \vec{J} \text{ with } \vec{B} = \mu_e \vec{H} \text{ and } \vec{J} = \sigma\left(\vec{E} + \frac{\partial u_i}{\partial t} \times \vec{B}\right) \quad (7)$$

Where $\vec{E}$ is the induced electric field, $\vec{J}$ is the current density vector and magnetic field $\vec{H}$ includes both primary and induced magnetic fields. μ_e and σ are the induced permeability and conduction coefficient respectively. The linearized Maxwell's stress tensor $\left(\tau_{ij}^0\right)^{M_x}$ due to the magnetic field is

$$\left(\tau_{ij}^0\right)^{M_x} = \mu_e\left(H_i h_j + H_j h_i - H_k h_k \delta_{ij}\right)$$

Let $\vec{H} = (H_x, H_y, H_z)$ and $\vec{h} = (h_1, h_2, h_3)$. Where $\vec{h}_i$ is the change in the magnetic field. In writing the above equations, we have neglected the displacement current. From equation (7), we get

$$\nabla^2 \vec{H} = \mu_e \sigma \left\{ \frac{\partial \vec{H}}{\partial t} + \vec{\nabla} \times \left(\frac{\partial u_i}{\partial t} \times \vec{H} \right) \right\} \tag{8}$$

Equation (8) can be written in component form as

$$\begin{aligned} \frac{\partial H_x}{\partial t} &= \frac{1}{\mu_e \sigma} \nabla^2 H_x \\ \frac{\partial H_z}{\partial t} &= \frac{1}{\mu_e \sigma} \nabla^2 H_z \\ \frac{\partial H_y}{\partial t} &= \frac{1}{\mu_e \sigma} \nabla^2 H_y + \frac{\partial \left(H_x \frac{\partial v_1}{\partial t} \right)}{\partial x} + \frac{\partial \left(H_z \frac{\partial v_1}{\partial t} \right)}{\partial z} \end{aligned} \tag{9}$$

For perfectly conducting medium (i.e.$\sigma \to \infty$), we have from equation (9)

$$\frac{\partial H_x}{\partial t} = \frac{\partial H_z}{\partial t} = 0 \tag{10}$$

And

$$\frac{\partial H_y}{\partial t} = \frac{\partial \left(H_x \frac{\partial v_1}{\partial t} \right)}{\partial x} + \frac{\partial \left(H_z \frac{\partial v_1}{\partial t} \right)}{\partial z} \tag{11}$$

Assuming that primary magnetic field is uniform throughout space. It is clear from Equation (10) that there is no perturbation in H_x and H_z, however from Equation (11) there may be perturbation in H_y. Therefore, taking small perturbation, say $h_2$in H_y , we have $H_x = H_{01}$, $H_y = H_{02} + h_2$ and $H_z = H_{03}$. Where (H_{01}, H_{02}, H_{03}) are components of the initial magnetic field $\vec{H}_0$. We can write $\vec{H}_0 = (H_0 \cos\phi, 0, H_0 \sin\phi)$, where $H_0 = \left| \vec{H}_0 \right|$ and ϕ is the angle at which the wave crosses the magnetic field. Thus we have

$$\vec{H} = (H_0 \cos\phi, h_2, H_0 \sin\phi) \tag{12}$$

We shall consider initial value of h_2 to be zero. Using equation (12) in equation (11), we get

$$\frac{\partial h_2}{\partial t} = \frac{\partial \left(H_0 \cos\phi \frac{\partial v_1}{\partial t} \right)}{\partial x} + \frac{\partial \left(H_0 \sin\phi \frac{\partial v_1}{\partial t} \right)}{\partial z} \tag{13}$$

Integrating with respect to 't', we get

$$h_2 = H_0 \cos\phi \frac{\partial v_1}{\partial x} + H_0 \sin\phi \frac{\partial v_1}{\partial z} \tag{14}$$

Considering $\vec{\nabla} \left(\frac{H^2}{2} \right) = -\left(\vec{\nabla} \times \vec{H} \right) \times \vec{H} + \left(\vec{H} . \vec{\nabla} \right) \vec{H}$ and the equation (7), we get

$$\vec{J} \times \vec{B} = \mu_e \left\{ -\vec{\nabla} \left(\frac{H^2}{2} \right) + \left(\vec{H} . \vec{\nabla} \right) \vec{H} \right\} \tag{15}$$

Using equations (2) and (15), we obtain the only non vanishing equation of the motion for layer as

$$P \frac{\partial^2 v_1}{\partial z^2} + Q \frac{\partial^2 v_1}{\partial x^2} + R \frac{\partial^2 v_1}{\partial x \partial z} = \rho \frac{\partial^2 v_1}{\partial t^2} \tag{16}$$

Where,

$$\begin{aligned} P &= \mu_T + a_3^2 (\mu_L - \mu_T) + \mu_e H_0^2 \sin^2\phi \\ Q &= \mu_T + a_1^2 (\mu_L - \mu_T) + \mu_e H_0^2 \cos^2\phi \\ R &= 2a_1 a_3 (\mu_L - \mu_T) + \mu_e H_0^2 \sin 2\phi \end{aligned} \tag{17}$$

Now let we consider

$$v_1\left(x,z,t\right)=V_1\left(z\right)e^{i(kx-\omega t)} \tag{18}$$

Where k is wave number and ω is the angular frequency. Substituting Equation (18) in Equation (16), we get

$$P\frac{\partial^2 V_1}{\partial z^2}+ikR\frac{\partial V_1}{\partial z}+k^2\left(\rho_1 c^2-Q\right)V_1=0 \tag{19}$$

Solution of Equation (19) is

$$V_1(z)=e^{-\frac{\eta z}{2}}\left(A\cos Tz+B\sin Tz\right) \tag{20}$$

Where $\eta=\frac{ikR}{P}$, $T=\left(\psi-\frac{\eta^2}{4}\right)^{\frac{1}{2}}$, $\psi=\frac{1}{P}\left(\rho_1\omega^2-Qk^2\right)$ and A, B are constants. Using Equation (20) in Equation (18), we get the solution for upper layer M1 as

$$v_1\left(x,z,t\right)=e^{-\frac{\eta z}{2}}\left(A\cos Tz+B\sin Tz\right)e^{ik(x-ct)} \tag{21}$$

2.2. Solution for Fluid Saturated Porous Medium M2

Equation of motion for fluid saturated porous layer in the absence of body forces are taken as

$$\sigma_{ij,j}=\frac{\partial^2}{\partial t^2}\left(\rho_{11}u_i+\rho_{12}U_i\right)-b_{ij}\frac{\partial}{\partial t}\left(U_j-u_j\right) \tag{22}$$

and

$$\sigma_{ij,j}=\frac{\partial^2}{\partial t^2}\left(\rho_{12}u_i+\rho_{22}U_i\right)+b_{ij}\frac{\partial}{\partial t}\left(U_j-u_j\right) \tag{23}$$

Where σ_{ij} are the components of the stress tensor and u_i are the components of the displacement vector of the solid and U_i are the components of the displacement vector of the fluid and

$$\sigma=-pf \tag{24}$$

where p is the pressure in the fluid and f is the porosity of the medium. Mass coefficients $\rho_{11},\rho_{12},\rho_{22}$ are related to the densities ρ',ρ_s,ρ_f of the layer, solid and fluid, respectively

$$\rho_{11}+\rho_{12}=(1-f)\rho_s \tag{25}$$

And

$$\rho_{12}+\rho_{22}=f\rho_f \tag{26}$$

so the mass density of the aggregate is

$$\rho'=\rho_{11}+2\times\rho_{12}+\rho_{22}=\rho_s+f(\rho_f-\rho_s) \tag{27}$$

the mass coefficients must follow the following inequalities

$$\rho_{11}>0,\rho_{12}<0,\rho_{22}>0,\rho_{11}\rho_{22}-\rho_{22}^2>0 \tag{28}$$

The constitutive equations for anisotropic fluid-saturated porous medium is

$$\sigma_{11} = (A_p \in +ME) + 2Nu_{1,1} + (F - A_p)u_{3,3} \tag{29}$$

$$\sigma_{22} = (A_p \in +ME) + 2Nu_{2,2} + (F - A_p)u_{3,3} \tag{30}$$

$$\sigma_{33} = (F \in +QE) + (2C - F)u_{3,3} \tag{31}$$

$$\sigma_{23} = G(u_{2,3} + u_{3,2}) \tag{32}$$

$$\sigma_{31} = G(u_{1,3} + u_{3,1}) \tag{33}$$

$$\sigma_{12} = G(u_{2,1} + u_{1,2}) \tag{34}$$

$$\sigma = (M \in +RE) + (Q - M)u_{3,3} \tag{35}$$

where A_p, F, C, G, M, Q, N, R are the material constants and

$$\in_{ij}= \frac{1}{2}\left(\frac{\partial u_i}{\partial u_j} + \frac{\partial u_j}{\partial u_i}\right) \tag{36}$$

With the help of Equation (29) and Equation (36), the Equation (22) and Equation (23) reduces to

$$N\frac{\partial^2 u}{\partial x^2} + G\frac{\partial^2 u}{\partial z^2} = \frac{\partial^2}{\partial t^2}(\rho_{11}u + \rho_{12}U) - b_{11}\frac{\partial}{\partial t}(U - u) \tag{37}$$

and

$$\frac{\partial^2}{\partial t^2}(\rho_{12}u + \rho_{22}U) + b_{11}\frac{\partial}{\partial t}(U - u) = 0 \tag{38}$$

the solution of Equation (37) and Equation (38) may be taken as

$$u(x, z, t) = u(z)e^{i(kx-\omega t)} \tag{39}$$

And

$$U(x, z, t) = U(z)e^{i(kx-\omega t)} \tag{40}$$

with the help of Equation (39) and (40), the Equation (37) and Equation (38) becomes

$$\left(\frac{\partial^2}{\partial z^2} + L_2^2\right)\binom{u}{U} = 0 \tag{41}$$

where

$$L_2^2 = \xi^2 - \frac{N}{G}k^2 \tag{42}$$

and

$$\xi^2 = (F + iR)\frac{\omega^2}{\beta} \tag{43}$$

$$F = \left(\frac{b_{11}^2 + \gamma_{22}d\rho'^2\omega^2}{b_{11}^2 + (\rho'\gamma_{22}\omega)^2}\right)\frac{\gamma_{22}}{d} \tag{44}$$

$$\beta = \sqrt{\frac{G}{d'}},\ d' = \rho_{11} - \frac{\rho_{12}^2}{\rho_{22}},\ d = \gamma_{11} - \frac{\gamma_{12}^2}{\gamma_{22}},\ \gamma = \frac{N}{G},\ \beta_a = \sqrt{\frac{N}{\rho'}},\ \rho' = \rho_{11} + 2\rho_{12} + \rho_{22} \tag{45}$$

the solution of Equation (41) are

$$v_2(x, z, t) = u(x, z, t) = (A_1 \cos L_2 z + A_2 \sin L_2 z)e^{i(kx-\omega t)} \tag{46}$$

And

$$U(x, z, t) = \left(\overline{A_1} \cos L_2 z + \overline{A_2} \sin L_2 z\right)e^{i(kx-\omega t)} \tag{47}$$

2.3. Solution for the half-space M3

The equation of motion for the isotropic medium, without body forces is taken as

$$\tau_{ij,j} = \rho \frac{\partial^2 v_i}{\partial t^2} \qquad i,j = 1,2,3 \tag{48}$$

where v_i are the components of the displacement and τ_{ij} is the component of stress tensor and ρ is the density of medium.

$$\tau_{ij} = \lambda \delta_{ij} e_{kk} + 2\mu e_{ij} \tag{49}$$

where λ and μ are Lame's constants, δ_{ij} is kronnecker delta and

$$e_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) \text{ and } e_{kk} = \frac{\partial v_i}{\partial x_j} \tag{50}$$

with the help of Equation (49) and (50), Equation (48) becomes

$$\mu_3\left(\frac{\partial^2 v_3}{\partial x^2} + \frac{\partial^2 v_3}{\partial z^2}\right) + \left(\frac{\partial \mu_3}{\partial z}\right)\left(\frac{\partial v_3}{\partial z}\right) = \rho \frac{\partial^2 v_3}{\partial t^2} \tag{51}$$

Now let we consider

$$v_3(x,z,t) = V_3(z)\, e^{i(kx-\omega t)} \tag{52}$$

using Equation (1) and (52) in Equation (51), we get

$$V_3{}''(z) + aV_3{}'(z) + \left(\frac{\rho_3}{\mu_3}\omega^2 - k^2\right)V_3(z) = 0 \tag{53}$$

Let we consider

$$V_3(z) = \varphi(z) e^{-\frac{az}{2}} \tag{54}$$

Using Equation (54) in Equation (53), we get

$$\varphi''(z) + N_1^2 \varphi(z) = 0 \tag{55}$$

where $N_1 = \sqrt{k^2 - \frac{\omega^2}{\beta_3^2} + \frac{a^2}{4}}$ and $\beta_3 = \sqrt{\frac{\mu_3}{\rho_3}}$ solution of the Equation (55) is

$$\varphi(z) = De^{-N_1 z} \tag{56}$$

using Equation(56) and (54) in Equation (52), we get

$$v_3(x,z,t) = De^{-\left(N_1+\frac{a}{2}\right)z} e^{i(kx-\omega t)} \tag{57}$$

3. Boundary Condition

The boundary conditions are:

(i). $P\frac{\partial v_1}{\partial z} + \frac{R}{2}\frac{\partial v_1}{\partial x} = 0$, at $z = -H$

(ii). $v_1 = v_2$ and $P\frac{\partial v_1}{\partial z} + \frac{R}{2}\frac{\partial v_1}{\partial x} = \left(-GA_1L_2 \sin L_2 z + GA_2 L_2 \cos L_2 z\right) e^{i(kx-\omega t)}$, at $z = -H_1$

(iii). $v_2 = v_3$ and $(-GA_1L_2 \sin L_2z + GA_2L_2 \cos L_2z)\, e^{i(kx-\omega t)} = \mu_3 \frac{\partial v_3}{\partial z}$ at $z = 0$

With the help of boundary condition (i), we get

$$APT \sin TH + BPT \cos TH = 0 \tag{58}$$

By boundary condition (ii), we have

$$Ae^{\frac{\eta H_1}{2}} \cos TH_1 - Be^{\frac{\eta H_1}{2}} \sin TH_1 - A_1 \cos L_2H_1 + A_2 \sin L_2H_1 = 0 \tag{59}$$

and

$$APT \sin TH_1 + BPT \cos TH_1 - A_1GL_2 \sin L_2H_1 - A_2GL_2 \cos L_2H_1 = 0 \tag{60}$$

By boundary condition (iii), we get

$$A_1 = D \tag{61}$$

and

$$A_1 = -\frac{GL_2A_2}{\mu_0\left(N_1 + \frac{a}{2}\right)} \tag{62}$$

using Equation (61) and (62) in Equations (59) and (60), we get

$$Ae^{\frac{\eta H_1}{2}} \cos TH_1 - Be^{\frac{\eta H_1}{2}} \sin TH_1 + A_2\left(\frac{GL_2}{\mu_0\left(N_1 + \frac{a}{2}\right)} \cos L_2H_1 + \sin L_2H_1\right) = 0 \tag{63}$$

And

$$APT \sin TH_1 + BPT \cos TH_1 + A_2\left(\frac{(GL_2)^2}{\mu_0\left(N_1 + \frac{a}{2}\right)} \sin L_2H_1 - GL_2 \cos L_2H_1\right) = 0 \tag{64}$$

Now eliminating the constants from Equations (58), (63) and (64), we get

$$\begin{vmatrix} PT \sin TH & PT \cos TH & 0 \\ e^{\frac{\eta H_1}{2}} \cos TH_1 & -e^{\frac{\eta H_1}{2}} \sin TH_1 & \left(\frac{GL_2}{\mu_0\left(N_1+\frac{a}{2}\right)} \cos L_2H_1 + \sin L_2H_1\right) \\ PT \sin TH_1 & PT \cos TH_1 & \left(\frac{(GL_2)^2}{\mu_0\left(N_1+\frac{a}{2}\right)} \sin L_2H_1 - GL_2 \cos L_2H_1\right) \end{vmatrix} = 0$$

after solving this determinant, we get

$$\tan L_2H_1 = \frac{GL_2\left\{\frac{PT}{\mu_0\left(N_1+\frac{a}{2}\right)} \sin T\left(H_1 - H\right) + e^{\frac{\eta H_1}{2}} \cos T\left(H_1 - H\right)\right\}}{\frac{(GL_2)^2 e^{\frac{\eta H_1}{2}}}{\mu_0\left(N_1+\frac{a}{2}\right)} \cos T\left(H - H_1\right) + PT \sin T\left(H - H_1\right)} \tag{65}$$

This is the frequency equation of surface wave in fluid saturated porous medium sandwiched between magneto-elastic self-reinforced medium and heterogeneous isotropic half-space.

4. Particular Cases

Case 1: When $P = Q = \mu_1$, $R = 0$, $\varepsilon_H = 0$, $a \neq 0$, then we get from Equation (65)

$$\tan L_2 H_1 = \frac{GL_2\left\{\frac{\mu_1 T_1}{\mu_0\left(N_1+\frac{a}{2}\right)}\sin T_1\left(H_1 - H\right) + \cos T_1\left(H_1 - H\right)\right\}}{\frac{(GL_2)^2}{\mu_0\left(N_1+\frac{a}{2}\right)}\cos T_1\left(H - H_1\right) + \mu_1 T_1 \sin T_1\left(H - H_1\right)} \tag{66}$$

where

$$T_1 = k\sqrt{\frac{c^2}{\beta_1^2} - 1}\ , \qquad \beta_1 = \sqrt{\frac{\mu_1}{\rho_1}}\ , \qquad \varepsilon_H = \frac{\mu_e H_0^2}{\mu_T}$$

This is the frequency equation of surface wave in fluid saturated porous medium sandwiched between isotropic layer and heterogeneous isotropic half-space.

Case 2: when $P = Q = \mu_1$, $R = 0$, $a = 0$, then from Equation (65), we get

$$\tan L_2 H_1 = \frac{GL_2\left\{\frac{\mu_1 T_1}{\mu_0 N_2}\sin T_1\left(H_1 - H\right) + \cos T_1\left(H_1 - H\right)\right\}}{\frac{(GL_2)^2}{\mu_0 N_2}\cos T_1\left(H - H_1\right) + \mu_1 T_1 \sin T_1\left(H - H_1\right)} \tag{67}$$

where $N_2 = k\sqrt{1 - \frac{c^2}{\beta_3^2}}$ and $\beta_3 = \sqrt{\frac{\mu_3}{\rho_3}}$. This is the frequency equation of surface wave in fluid saturated porous medium sandwiched between isotropic layer and homogeneous isotropic half-space.

Case 3: When $P = Q = \mu_1$, $R = 0$, $a = 0$ and $H \rightarrow H_1$, then Equation (65) reduces to

$$\tan\left(kH_1\sqrt{\frac{c^2}{\beta^2} - 1}\right) = \frac{\mu_0\sqrt{1 - \frac{c^2}{\beta_3^2}}}{G\sqrt{\frac{c^2}{\beta^2} - 1}} \tag{68}$$

where $\beta = \sqrt{\frac{G}{d'}}$. The Equation (68) gives the dispersion equation of surface wave in isotropic layer lying over a homogeneous isotropic half-space. This is the classical Love-wave equation.

5. Numerical Examples and Discussion

For computation of dimensionless phase velocity of surface wave propagation in a fluid saturated porous medium sandwiched between a magneto-elastic self-reinforced layer and a heterogeneous isotropic half-space, we have considered the following data (Gubbins [8]):

1. For Magneto-elastic self-reinforced medium $\mu_L = 4.4 \times 10^9$ N/m^2, $\mu_T = 1.89 \times 10^9$ N/m^2, $\rho_1 = 5600\,Kg/m^3$, $a_1 = 0.00316227$, $\phi = 10°$.

2. For heterogeneous isotropic half-space $\mu_3 = 6.77 \times 10^{10}$ N/m^2, $\rho_3 = 3323 Kg/m^3$.

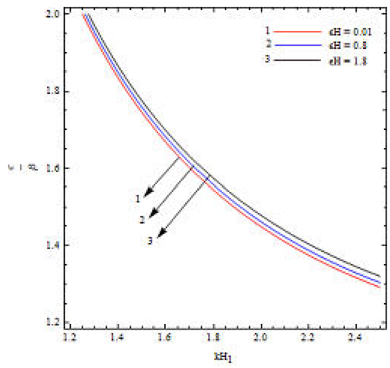

Figure 2. Variation in dimensionless phase velocity against dimensionless wave number for different values of magneto-elastic coupling parameter

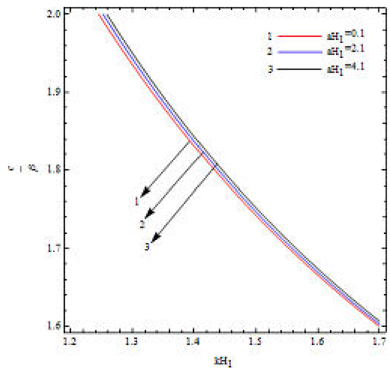

Figure 3. Variation in dimensionless phase velocity against dimensionless wave number for different values of inhomogeneity parameter

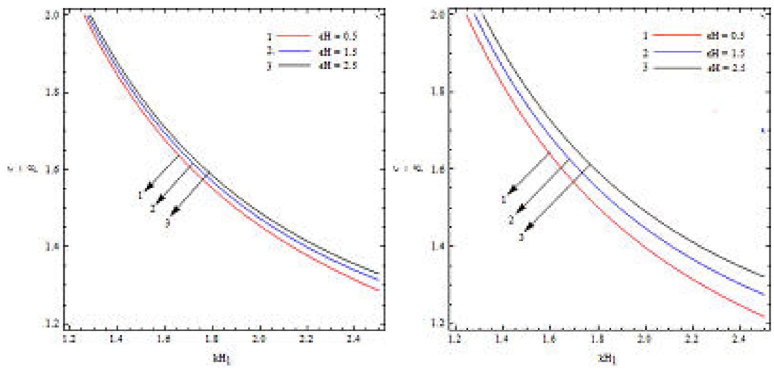

Figure 4. Variation in dimensionless phase velocity against dimensionless wave number for different values of magneto-elastic coupling parameter for different heights of layers

6. Conclusion

Propagation of surface wave in fluid saturated porous medium sandwiched between magneto-elastic self-reinforced layer and heterogeneous isotropic half-space has been studied. Frequency equation of surface wave has been obtained. Numerical results and particular cases have also been discussed. In the isotropic case, when heterogeneity, magnetic field, self-reinforcement and porosity are absent, the frequency equation reduces to classical Love wave equation. Graphs have been plotted between phase velocity and wave number. It is observed that the heterogeneity of the medium increases the phase velocity of surface waves significantly. Also, anisotropy (magneto-elasticity, self-reinforcement and porosity) of the medium has been found in favour of the phase velocity of considered wave.

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