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An Analytical Study of Brownian Motion

Seminar Paper*

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Abstract: In the present paper it has been emphasized the analytical study of Brownian motion. This phenomenon was discovered by an English botanist Broun in 1827 while examining the suspension of fine inanimate spores in water microscopically. It has been found that the spores dancing about in a state of random motion. It has also been observed that if a colloidal solution be examined particles appear moving to and fro, rapidly and continuously in an entirely random way. Further Einsteins theory of translational Brownian motion explains the physical nature of the phenomenon. Clearly a development in the experimental and theoretical investigations of Brownian motion is presented. Einstein who did not like Gods game of playing disc for electrons in an atom himself put forward a theory of Brownian movement allowing God to play the disc. The vital role played by his random walk model in the evolution of non-equilibrium statistical mechanics multitude of its applications is highlighted. Also induced are the basics of Langevins theory for Brownian motion.

Keywords: Brownian Motion, Microscopically, colloidal Solution, random walk.

1. Introduction

Brownian motion is the temperature - dependent perpetual, irregular motion of the particles immersed in a fluid, caused by their continuous bombardment by the surrounding molecules of much smaller size. Brownian motion can be easily observed in the colloidal Solutions under powerful microscope. It can be readily observed in smoke in a directed beam of light. Smaller the size of the particle more strongly the Brownian motion is observed. On the basis of the Kinetic theory of matter the Brownian motion can be easily understood. The size of the colloid particles are smoke particles is of the order of 10^{-4} cm which is about to 10^4 times the size of molecules. The Brownian motion of the colloid particles in a direct proof of the existence of molecular agitation in the substance. Brownian motion was first discovered by the Jan Ingenhousz (1785), when he found that fine powder of charcoal floating on alcohol surface exhibited a highly random motion. However, it got the name Brownian motion after Scottish botanist Robert Brown, who in 1828-29 published the results of his extensive studies on the incessant random movement of tiny particles like pollen grains, dust and soot suspended in a fluid. In 1863, Wiener attributed Brownian motion to molecular movement of the liquid and this viewpoint was supported by Delsaux (1877-80) and Gouy (1888-95). On the basis of a series of experiments Gouy convincingly ruled out the exterior factors as causes of Brownian motion and argued in favour of contribution of the surrounding fluid. He also discussed the connection between Brownian motion and Carnot's principle and thereby brought out the statistical nature of the laws of thermodynamics. Such ideas put Brownian motion at the great of the then prevalent controversial views about philosophy of science. Then,

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in 1900 Bachelier obtained a diffusion equation for random processes and thus, a theory of Brownian motion in his Ph.D. thesis on stock market fluctuation. Unfortunately, this work was not recognized by the scientific community, including his supervisor Poincare, because it was in the field of economics and it did not involve any of the relevant physical aspects. Eventually, Einstein in a number of research publications beginning in 1905 put forward an acceptable model for Brownian motion. His work was followed by an almost similar expression for the time dependence of displacement of the Brownian particle by Smoluch owski in 1906. In 1908, Langevin gave a phenomenological theory of Brownian motion and obtained essentially the same formulae for displacement of the particles. Their results were experimentally verified by perrin 1908 using precise measurements on sedimentation in colloidal suspensions to determine the Avogadro's number. Later on, more accurate values of this constant K were found out by many workers by performing similar experiments.

The correctness of the random walk model and Langevin's theory made a very strong case in favour of molecular kinetic model of matter and unleashed a wave of activity for a systematic development of dynamical theory of Brownian motion by Fokker, Planck, Unlenback, Ornstein and several other scientists. As such, during the last 100 years, concerted efforts by a galaxy of physicists, mathematicians, chemists etc. have not only provided a proper mathematical foundation to the physical theory but have also led to extensive diverse applications of the techniques developed.

The two basic ingredients of Einstein's theory were: (i) the movement of the Brownian particle is a consequence of continuous impacts of the randomly moving surrounding molecules of the fluid; (ii) These impacts can be described only probabilistically so that time evolution of the particle under observation is also probabilistic in nature. However, science the trajectory of a Brownian particle is random, it grows only as square root of time and therefore one cannot define its derivative at a point. To handle this problem, N. wiener (1923) put forward a measure theory which formed the basis of the so-called stable distributions or levy distribution. As a follow-up of these and to give a firm footing to the theory of Brownian motion, its (1944), developed stochastic calculus and an alternative to Brownian motion - the Geometrical Brownian motion. These aspects have enlarged the scope of applicability of theoretical methods for beyond Einstein's random walks. These concepts have been fruitfully exploited in a multitude of phenomena in not only physics, chemistry and biology but also physiology, economics, sociology and politics.

2. Randomly Walk Model of Brownian Motion

Brown found the spores dancing about in a state of random motion. It has also been observed that if a colloidal solution be examined under an ultra microscope, the suspended particles appear moving to and fro, rapidly and continuously in an entirely random way. This irregular motion is called the Brownian motion of the particle, Brownian motion is so irregular that nothing can be predicated about the next step. All one can talk about is the probability of his covering a specific distance in given time. Such a problem was originally solved by Markov and therefore, such process are known as Markov processes. Einstein adopted this approach to obtain the probability of the Brownian particle covering a particular distance in this time t. The randomness of the drunkard's walk has given this treatment the name random walk model and in the case of a Brownian particle, the steps of the walk are caused by molecular Collisions.

To simplify the derivation, we get consider the problem in one-dimension along X-axis, taking the position of the particle to be 0 at t = 0 and x(t) at time t and make the following assumptions,

1. Each molecular impact on the particle under observation takes place after the same interval τ_0 (10^{-8}sec) so that the number of collisions in time t is $n = t/\tau_0$
2. Each collision makes the Brownian particle jump by the same distance δ which turns out, we will set to be about 1nm, for a particle of radius 1μm in a fluid with the viscosity of water, at room temperature along either positive or negative

direction with equal probability and δ is smaller than the displacement x(t) which we resolve through the microscope only on the scale of a μm of the particle.

3. Successive jumps of the particle are independent of each other.

4. At time t, the particle has not positive displacement x(t).

3. Langevin's Theory of Brownian Motion

The intimate relationship between irreversibility and randomness of Collisions of the fluid molecules with the Brownian particle prompted Langevin (1908) to put forward a more logical theory of Brownian movement. According to Langevin's theory, any particle of the colloidal solution suffers collisions of molecules of solvent from all possible directions, causing its path to change continuously. It may be assumed that each particle of the colloid in addition to viscous force is also acted upon by a variable force which is responsible for its Brownian motion. If a particle of mass m and radius a is moving with velocity v in the solution of coefficient of viscosity η, the viscous force acting on it is (Stock's law).

$$F_v = 6\ \pi\eta\ a\ v = 6\ \pi\eta\ a\ \left(\frac{dx}{dt}\right) = C\ \left(\frac{dx}{dt}\right)$$

where, C = 6 $\pi\eta\ a$ if the component of the variable force F, in the X - direction is F_x, then the net force acting on the particle in the X - direction is = F_x - $C\ \left(\frac{dx}{dt}\right)$

Therefore, in the X - direction the equation of motion of the particle will be,

$$m\ \frac{d^2x}{dt^2} = -C\ \left(\frac{dx}{dt}\right) + F_x \tag{1}$$

Since the frequency of collision is very high F_x changes very rapidly and can have positive or negative value, therefore, the motion of the particle can be either in the positive or in the negative X -direction. As the time of observation is much larger than the collision intervals, we get a finite displacement s of the particle. The value of x will be different for different particles. The average value of x for a large number of particles will be zero as the probability of x being positive or negative are equal. Therefore Equation (1) can not provide any information. Since x^2 does not change in sign, we try to find the average value of x^2 as follows : Multiplying both sides of Equation (1) by x, we have,

$$m\ x\frac{d^2x}{dt^2} = -C\ x\ \frac{dx}{dt} + xF_x \tag{2}$$

but

$$m\ x\frac{d^2x}{dt^2} = mx\frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{1}{2}m\frac{d^2}{dt^2}(x^2) - m\left(\frac{dx}{dt}\right)^2$$

and $C\ x\ \frac{dx}{dt} = \frac{1}{2}\ C\ \frac{dx^2}{dt}$

$$\frac{1}{2}m\frac{d^2}{dt^2}\left(x^2\right) - m\left(\frac{dx}{dt}\right)^2 = -\frac{1}{2}C\frac{dx^2}{dt} + xF_x$$

Integrate over a time interval t,

$$\frac{1}{2}m\int_0^t \frac{d^2}{dt^2}\left(x^2\right)dt - m\int_0^t \left(\frac{dx}{dt}\right)^2 dt = -\frac{1}{2}C\int_0^t \frac{d}{dt}\left(x^2\right)dt + \int_0^t F_x x dt$$

$$\frac{1}{2}m\left[\frac{d}{dt}\left(x^2\right)\right]_0^t - m\int_0^t \left(\frac{dx}{dt}\right)^2 dt = -\frac{1}{2}C\ \left[x^2\right]_0^t + \int_0^t F_x x dt$$

$$\frac{1}{2}m\left[2x\frac{dx}{dt}\right]_0^t - m\int_0^t \left(\frac{dx}{dt}\right)^2 dt = -\frac{1}{2}C\ \left[x^2\right]_0^t + \int_0^t F_x\ .\ xdt$$

Writing $\frac{dx}{dt}= \dot{x}$, we have

$$\frac{1}{2}m[2x\ \dot{x}]_0^t - m\int_0^t \dot{x}^2 dt = -\frac{1}{2}C\ \left[x^2\right]_0^t + \int_0^t F_x\ xdt \tag{3}$$

Since x changes randomly with time in taking averages over a large number of particles in Equation (3) the first term can be neglected in comparison to the second term, therefore,

$$-m\int_0^t \dot{x}^2 dt = -\frac{1}{2}C\ \left[x^2\right]_0^t + \int_0^t x\ F_x\ dx \tag{4}$$

As F_x is positive for same particles and negative for the other , the second term of the right sight of Equation (4), becomes zero, thus,

$$\begin{aligned}\int_0^t m\dot{x}^2 dt &= \frac{1}{2}C\left[x^2\right]_0^t \\ m\dot{x}^2 dt &= \frac{1}{2}Cx^2\end{aligned} \tag{5}$$

Since the system is in the thermal equilibrium the average kinetic energy of the particles can be obtained from the law of equipartition of energy, or, $\frac{1}{2}\ m\ \dot{x}^2 = \frac{1}{2}KT$. Therefore,

$$\begin{aligned}KTt &= \frac{1}{2}Cx^2 \\ x^2 &= \frac{2KTt}{c} = \frac{2KTt}{6\phi\eta a} = \frac{N_a KTt}{3\phi\eta a N_a} = \frac{RT}{3\phi\eta a N}\ t\end{aligned} \tag{6}$$

Where $R=N_A K$, and N_A is the Avogadro's number. Perrin, studied the motion of a single colloid particle for a long time interval and using graphs of the field of view determined the value of x^2 experimentally. With the help of this value of x^2 he calculated the value of Avogadro number. N_A as 6.88×10^{23} per mole which is very close to the actual value (6.02×10^{23} per mole) of the Avogadro number proving the validity of the Equation (6) and hence the Langevin theory.

4. Einstein's Theory of Brownian Motion

Langevin's theory of Brownian motion does not say anything about its physical nature. Einstein's proposed a theory for the Brownian motion according to which, the Brownian particles get diffused into the medium due to their Brownian motion. Therefore, Brownian motion should be related to the diffusion coefficient.

Einstein's deduced the value of the diffusion coefficient by two methods, viz,(i) from random motion of the Brownian or colloid particles and (ii) from the osmotic pressure due to difference of concentration of suspended particles in different regions of the colloidal solution. Equation these tow values of diffusion coefficients Einstein studied the Brownian motion.

5. Summary and Conclusion

An early realization of the consequences of the studies on Brownian motion was that it imposes limit on the precision of measurements by small instruments as there are under the influence of random impacts of the surrounding air. In 1883, wiener attributed Brownian motion to molecular movement of the liquid. The correctness of the random walk model and Langevin's theory made a very strong case in favour of molecular kinetic model of matter. The techniques developed for the theory of Brownian motion form cornerstones for investigating a variety of phenomena. The main mathematical aspect of the early theories of Brownian motion was the "Central limit theorem", which states that the distribution function for random walk is quite close to a Gaussian. The proportionality of root mean square displacement of the Brownian particle to the square root of time is a typical consequence of the random nature of the process. The Einstein relation gives a relation between the diffusion coefficient and mobility of the particle and hence the viscosity of the fluid. Thus, viscosity of a medium is a consequences of fluctuating forces arising from their continuous and random motion and is an irreversible phenomenon.

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