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Intuitionistic Fuzzy α-Generalized Semi Homeomorphisms

Research Article

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Abstract: Using the notion of Intuitionistic fuzzy α-generalized semi continuous mapping, the concept of Intuitionistic fuzzy α-generalized semi homeomorphism and Intuitionistic fuzzy iα-generalized semi homeomorphism are introduced. Also we have provided some characterizations of intuitionistic fuzzy α-generalized semi homeomorphisms.

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Keywords: Intuitionistic fuzzy topological space, Intuitionistic fuzzy α-generalized semi closed set,Intuitionistic fuzzy α-generalized semi continuous mapping, Intuitionistic fuzzy α-generalized semi homeomorphism, Intuitionistic fuzzy iα-generalized semi homeomorphism.

1. Introduction

The concept of intuitionistic fuzzy sets was introduced by Atanassov[1] as a generalization of fuzzy sets. In 1997 Coker[3] introduced the concept of intuitionistic fuzzy topological spaces. In the last 30 years the various concepts of fuzzy mathematics have been extended for intuitionistic fuzzy sets. In this paper, we study the concepts of intuitionistic fuzzy α-generalized semi homeomorphism and Intuitionistic fuzzy iα-generalized semi homeomorphism as an extension of our work done in paper[5]. We studied some characterizations of intuitionistic fuzzy α-generalized semi homeomorphism and Intuitionistic fuzzy iα-generalized semi homeomorphism.

2. Preliminaries

Definition 2.1 ([1]). *Let X be a non empty fixed set. An intuitionistic fuzzy set(IFS in short) A in X is an object having the form*

$$A = \{\langle\ x,\ \mu_A(x),\ \nu_A(x)\ \rangle\ /\ x \in X\}$$

where the function $\mu_A(x)$:$X \to$ [0,1] denotes the degree of membership(namely $\mu_A(x)$) and the function $\nu_A(x)$:$X \to$ [0,1] denotes the degree of non-membership(namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

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IFS(X) denote the set of all intuitionistic fuzzy sets in X.

Definition 2.2 ([1]). *Let A and B be IFSs of the form A=$\{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$ and B=$\{\langle x, \mu_B(x), \nu_B(x) \rangle / x \in X\}$. Then*

1. *$A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,*
2. *$A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,*
3. *$A^c = \{\langle x, \nu_A(x), \mu_A(x) \rangle / x \in X\}$,*
4. *$A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle / x \in X\}$,*
5. *$A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle / x \in X\}$.*

For the sake of simplicity, we shall use the notation A=$\langle x, \mu_A, \nu_A \rangle$ instead of
$A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$.
The intuitionistic fuzzy sets $0_\sim = \{\langle x, 0, 1 \rangle / x \in X\}$ and $1_\sim = \{\langle x, 1, 0 \rangle / x \in X\}$ are the empty set and the whole set of X respectively.

Definition 2.3 ([3]). *An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms.*

1. *$0_\sim, 1_\sim \in \tau$,*
2. *$G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,*
3. *$\cup G_i \in \tau$ for any family $\{G_i / i \in J\} \subseteq \tau$.*

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space(IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set(IFOS in short) in X. The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set(IFCS in short) in X.

Definition 2.4 ([3]). *Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X. Then the intuitionistic fuzzy interior and the intuitionistic fuzzy closure are defined as follows:*

1. *$int(A) = \cup\{G / G$ is an IFOS in X and $G \subseteq A\}$,*
2. *$cl(A) = \cap\{K / K$ is an IFCS in X and $A \subseteq K\}$.*

Note that for any IFS A in (X, τ), we have $cl(A^c) = (int(A))^c$ and $int(A^c) = (cl(A))^c$.

Definition 2.5. *An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be an*

1. *intuitionistic fuzzy regular closed set(IFRCS in short) if $A = cl(int(A))$ [3],*
2. *intuitionistic fuzzy α-closed set(IFαCS in short) if $cl(int(cl(A))) \subseteq A$ [4],*
3. *intuitionistic fuzzy semiclosed set(IFSCS in short) if $int(cl(A)) \subseteq A$ [3].*

   The complements of the above mentioned intuitionistic fuzzy closed sets are called their respective intuitionistic fuzzy open sets.

Definition 2.6. *[5] Let A be an IFS of an IFTS (X, τ). Then*

1. *$\alpha cl(A) = \cap\{K$ / K is an IFαCS in X and $A \subseteq K\}$,*

2. *$\alpha int(A) = \cup\{K$ / K is an IFαOS in X and $K \subseteq A\}$.*

Definition 2.7. *[5] An IFS A of an IFTS (X, τ) is an*

1. *intuitionistic fuzzy generalized closed set(IFGCS in short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X,*

2. *intuitionitic fuzzy alpha generalized semi-closed set(IFαGSCS in short) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFSOS in X. Every IFCS and IFαCS are IFαGSCS but the converse may not be true in general.*

The complements of the above mentioned intuitionistic fuzzy closed sets are called their respective intuitionistic fuzzy open sets.

Definition 2.8. *Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an*

1. *intuitionistic fuzzy closed mapping (IF closed map in short) if f(A) is an IFCS in (Y, σ) for every IFCS A of (X, τ)[7],*

2. *intuitionistic fuzzy α generalized semi closed mapping (IFαGS closed map in short) if f(A) is an IFαGSCS in (Y, σ) for every IFCS A of (X, τ)[6].*

Definition 2.9 ([5]). *An IFTS (X, τ) is said to be an*

1. *intuitionistic fuzzy $\alpha gaT_{1/2}$(in short $IF_{\alpha ga}T_{1/2}$)space if every IFαGSCS in X is an IFCS in X,*

2. *intuitionistic fuzzy $\alpha gbT_{1/2}$(in short $IF_{\alpha gb}T_{1/2}$)space if every IFαGSCS in X is an IFGCS in X.*

Definition 2.10. *Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ).Then f is said to be an*

1. *intuitionistic fuzzy continuous (IF continuous in short) if $f^{-1}(B) \in IFO(X)$ for every $B \in \sigma$[7],*

2. *intuitionistic fuzzy α continuous (IFα continuous in short) if $f^{-1}(B) \in IF\alpha O(X)$ for every $B \in \sigma$[7].*

Every IF continuous mapping is an IFα-continuous mapping but the converse may not be true in general.

Definition 2.11 ([5]). *Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ).Then f is said to be*

1. *intuitionistic fuzzy generalized continuous (IFG continuous in short) if $f^{-1}(B) \in IFGC(X)$ for every IFCS B in Y,*

2. *intuitionistic fuzzy α-generalized semi continuous (IFαGS continuous in short) if $f^{-1}(B) \in IF\alpha GSC(X)$ for every IFCS B in Y.*

Every IF continuous mapping, IFα continuous mapping is an IFαGS continuous mapping but the converse may not be true in general.

Definition 2.12 ([6]). *Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ).Then f is said to be an intuitionistic fuzzy α-generalized semi open mapping(IFαGS open mapping) if $f(A) \in IF\alpha GSO(X)$ for every IFOS A in X,*

Definition 2.13 ([5]). *Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ).Then f is said to be an intuitionistic fuzzy α-generalized semi irresolute (IFαGS irresolute) if $f^{-1}(B) \in IF\alpha GSCS(X)$ for every IFαGSCS B in Y.*

Definition 2.14. *Let f be a bijection from an IFTS (X, τ) into an IFTS (Y, σ).Then f is said to be*

1. *intuitionistic fuzzy homeomorphism (IF homeomorphism in short) if f and f^{-1} are IF continuous mappings [8],*

2. *intuitionistic fuzzy alpha homeomorphism (IFα homeomorphism in short) if f and f^{-1} are IFα continuous mappings [9],*

3. *intuitionistic fuzzy alpha generalized homeomorphism (IFG homeomorphism in short) if f and f^{-1} are IFG continuous mappings [9].*

3. Intuitionistic Fuzzy α-generalized Semi Homeomorphism

In this section we introduce intuitionistic fuzzy alpha generalized semi homeomorphism and study some of its properties.

Definition 3.1. *A bijection mapping $f : (X, \tau) \to (Y, \sigma)$ is called an intuitionistic fuzzy alpha generalized semi homeomorphism (IFαGS homeomorphism in short) if f and f^{-1} are IFαGS continuous mappings.*

Example 3.2. *Let $X = \{a, b\}$, $Y = \{u, v\}$, $G_1 = \langle x, (0.1, 0.2), (0.2, 0.3)\rangle$ and $G_2 = \langle y, (0.3, 0.4), (0.4, 0.5) \rangle$. Then $\tau = \{ 0_\sim, G_1, 1_\sim \}$ and $\sigma = \{ 0_\sim, G_2, 1_\sim \}$ are IFTs on X and Y respectively. Define a bijective mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IFαGS continuous and f^{-1} is also an IFαGS continuous mapping. Therefore the bijective mapping f is an IFαGS homeomorphism.*

Theorem 3.3. *Let $f:(X, \tau) \to (Y, \sigma)$ be a bijective mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent:*

1. *f is an IF homeomorphism*

2. *f is an IF continuous mapping and f is an IF open mapping*

3. *f and f^{-1} IF continuous mappings*

Proof. (i) $\Rightarrow$ (ii): It is obviously true.
(ii) $\Rightarrow$ (iii): Let f is an IF open mapping. That is f(A) is IFOS in Y for each IFOS A in X. Now define a mapping f^{-1}: $(Y, \sigma) \to (X, \tau)$. By hypothesis, for every IFOS A in X, we have f−1(A) is an IFOS in Y. Hence f^{-1} is an IF continuous mapping. That is f and f^{-1} are IF continuous mappings.
(ii) $\Rightarrow$ (iii): Let f and f^{-1} are IF continuous mappings. Since f^{-1}: $(Y, \sigma) \to (X, \tau)$ is an IF continuous mapping, $f:(X, \tau) \to (Y, \sigma)$ is an IF open mapping. Hence f is an IF homeomorphism. □

Theorem 3.4. *Every IF homeomorphism is an IFαGS homeomorphism but not conversely.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an IF homeomorphism. Then f and f^{-1} are IF continuous mappings. Since every IF continuous mappings is an IFαGS continuous mapping, f and f^{-1} are IFαGS continuous mappings. Therefore f is an IFαGS homeomorphism. □

Example 3.5. *Let $X = \{a, b\}$, $Y = \{u, v\}$, $G_1 = \langle x, (0.3, 0.1), (0.4, 0.2)\rangle$ and $G_2 = \langle y, (0.1, 0.2), (0.2, 0.4) \rangle$. Then $\tau = \{ 0_\sim, G_1, 1_\sim \}$ and $\sigma = \{ 0_\sim, G_2, 1_\sim \}$ are IFTs on X and Y respectively. Define a bijective $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Since every IFCS in (Y, σ) is an IFαGSCS in (X, τ), f is an IFαGS continuous mapping and every IFCS in (X, τ) is an IFαGSCS in (Y, σ), f^{-1} is an IFαGS continuous mapping. Hence f is an IFαGS homeomorphism. But f is not an IF homeomorphism since f and f^{-1} are not an IF continuous mappings.*

Theorem 3.6. *Every IFα homeomorphism is an IFαGS homeomorphism but not conversely.*

Proof. Let f:(X, τ) $\rightarrow$ (Y, σ) be an IFα homeomorphism. Then f and f^{-1} are IFα continuous mappings. Since every IFα continuous is an IFαGS continuous mapping, f and f^{-1} are IFαGS continuous mappings. Therefore f is an IFαGS homeomorphism. □

Example 3.7. *Let $X = \{a, b\}$, $Y = \{u, v\}$, $G_1 = \langle\ x, (0.3, 0.4), (0.4, 0.5)\rangle$ and $G_2 = \langle\ y, (0.4, 0.2), (0.5, 0.3)\ \rangle$. Then $\tau = \{\ 0_\sim, G_1, 1_\sim\ \}$ and $\sigma = \{\ 0_\sim, G_2, 1_\sim\ \}$ are IFTs on X and Y respectively. Define a bijective $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Consider an IFCS $G_2{}' = \langle y, (0.5, 0.3), (0.4, 0.2)\rangle$ in Y. Then f^{-1} $(G_2{}') = \langle y, (0.5, 0.3), (0.4, 0.2)\rangle$ is not an IFαCS in X. This implies f is not an IFα continuous mapping. Hence f is not an IFα homeomorphism.*

Theorem 3.8. *Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an IFαGS homeomorphism. Then f is an IF homeomorphism if X and Y are $IF_{\alpha ga}T_{1/2}$ space.*

Proof. Let B be an IFCS in Y. By hypothesis, $f^{-1}(B)$ is an IFαGSCS in X. Since X is an $IF_{\alpha ga}T_{1/2}$ space, $f^{-1}(B)$ is an IFCS in X. Hence f is an IF continuous mapping. By hypothesis f^{-1}:(Y, σ) $\rightarrow$ (X, τ) is an IFαGS continuous mapping. Let A be an IFCS in X. Then $(f^{-1})^{-1}(A) = f(A)$ is an IFαGSCS in Y. Since Y is an $IF_{\alpha ga}T_{1/2}$ space, f(A) is an IFCS in Y. Hence f^{-1} is an IF continuous mapping. Therefore f is an IF homeomorphism. □

Theorem 3.9. *Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an IFαGS homeomorphism. Then f is an IFG homeomorphism if X and Y are $IF_{\alpha gb}T_{1/2}$ space.*

Proof. Let B be an IFCS in Y. By hypothesis, $f^{-1}(B)$ is an IFαGSCS in X. Since X is an $IF_{\alpha gb}T_{1/2}$ space, $f^{-1}(B)$ is an IFGCS in X. Hence f is an IFG continuous mapping. By hypothesis f^{-1}:(Y, σ) $\rightarrow$ (X, τ) is an IFαGS continuous mapping. Let A be an IFCS in X. Then $(f^{-1})^{-1}(A) = f(A)$ is an IFαGSCS in Y. Since Y is an $IF_{\alpha gb}T_{1/2}$ space, f(A) is an IFGCS in X. Hence f^{-1} is an IFG continuous mapping. Therefore f is an IFG homeomorphism. □

Theorem 3.10. *Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a bijective mapping. If f is an IFαGS continuous mapping, then the following are equivalent:*

1. *f is an IFαGS closed mapping*
2. *f is an IFαGS open mapping*
3. *f is an IFαGS homeomorphism.*

Proof. (i) $\Rightarrow$ (ii): Let f : (X, τ) $\rightarrow$ (Y, σ) be a bijective mapping and let f be an IFαGS closed mapping. This implies f^{-1}:(Y, σ) $\rightarrow$ (X, τ) is an IFαGS continuous mapping. Assume that A is an IFOS in X. Then by hypothesis, $(f^{-1})^{-1}(A)$ is an IFαGSOS in Y. Hence f is an IFαGS open mapping.
(ii) $\Rightarrow$ (iii): Let f:(X, τ) $\rightarrow$ (Y, σ) be a bijective mapping and let f is an IFαGS open mapping. This implies f^{-1}:(Y, σ) $\rightarrow$ (X, τ) is an IFαGS continuous mapping. Hence f and f^{-1} are IFαGS continuous mappings. Therefore f is an IFαGS homeomorphism.
(iii) $\Rightarrow$ (i): Let f be an IFαGS homeomorphism. That is f and f^{-1} are IFαGS continuous mappings. Assume that A is an IFCS in X. Then by hypothesis, A is an IFαGSCS in Y. Hence f is an IFαGS closed mapping. □

Remark 3.11. *The composition of two IFαGS homeomorphisms need not be an IFαGS homeomorphism in general.*

Example 3.12. *Let $X = \{a, b\}$, $Y = \{c, d\}$ and $Z = \{u, v\}$. Let $G_1 = \langle\ x, (0.2, 0.3), (0.4, 0.3)\rangle$, $G_2 = \langle\ y, (0.4, 0.5), (0.5, 0.5)\ \rangle$ and $G_3 = \langle\ z, (0.1, 0.2), (0.4, 0.4)\rangle$. Then $\tau = \{\ 0_\sim, G_1, 1_\sim\ \}$, $\sigma = \{\ 0_\sim, G_2, 1_\sim\ \}$ and $\eta = \{\ 0_\sim, G_3, 1_\sim\ \}$ are IFTs on X,Y and Z respectively. Here $\mu_{G1}(a) = 0.2$, $\mu_{G1}(b) = 0.3$, $\nu_{G1}(a) = 0.4$, $\nu_{G1}(b) = 0.3$, $\mu_{G2}(c) = 0.4$, $\mu_{G2}(d)$*

= 0.5, $\nu_{G2}(c)$ = 0.5, $\nu_{G2}(d)$ = 0.5, $\mu_{G3}(u)$ = 0.1, $\mu_{G3}(v)$ = 0.2, $\nu_{G3}(u)$ = 0.4, $\nu_{G3}(v)$ = 0.4. Define a bijective mapping f:(X, τ) $\to$ (Y, σ) by f(a) = c and f(b) = d and g:(Y, σ) $\to$ (Z, η) by g(c) = u and g(d) = v. Then f and f^{-1} are IFαGS continuous mappings. Also g and g^{-1} are IFαGS continuous mappings. Hence f and g are IFαGS homeomorphisms. But the composition g∘f: X $\to$ Z is not an IFαGS homeomorphism since g∘f is not an IFαGS continuous mapping.

Definition 3.13. *A bijective mapping f:(X, τ) $\to$ (Y, σ) is called an intuitionistic fuzzy iα-generalized semi homeomorphism (IFiαGS homeomorphism in short) if f and f^{-1} are IFαGS irresolute mappings.*

Theorem 3.14. *Every IFiαGS homeomorphism is an IFαGS homeomorphism but not conversely.*

Proof. Let f:(X, τ) $\to$ (Y, σ) be an IFiαGS homeomorphism. Let B be IFCS in Y. Since every IFCS is an IFαGSCS, B is an IFαGSCS in Y. By hypothesis f^{-1}(B) is an IFαGSCS in X. Hence f is an IFαGS continuous mapping. Similarly we can prove f^{-1} is an IFαGS continuous mapping. Hence f and f^{-1} are IFαGS continuous mappings. Therefore the mapping f is an IFαGS homeomorphism. □

Example 3.15. *Let X = {a, b}, Y = {u, v}, G_1 = ⟨ x, (0.3, 0.1), (0.4, 0.2)⟩ and G_2 = ⟨ y, (0.1, 0.2), (0.2, 0.4) ⟩. Then τ = { $0_\sim$, G_1, $1_\sim$ } and σ = { $0_\sim$, G_2, $1_\sim$ } are IFTs on X and Y respectively. Define a mapping f:(X, τ) $\to$ (Y, σ) by f(a) = u and f(b) = v. Then f is IFαGS homeomorphism. Let us consider an IFS A = ⟨ x, (0.2, 0.3), (0.1, 0.3)⟩ in X. Clearly A is an IFαGSCS in X. But f(A) is not an IFαGSCS in Y. That is f^{-1} is not an IFαGS irresolute mapping. Hence f is not an IFiαGS homeomorphism.*

Theorem 3.16. *Let f : (X, τ) $\to$ (Y, σ) be an IFiαGS homeomorphism. Then f is an IF homeomorphism if X and Y are IF$_{\alpha ga}T_{1/2}$ space.*

Proof. Let B be an IFCS in Y. Since every IFCS is an IFαGSCS, B is an IFαGSCS in Y. Since f is an IFαGS irresolute mapping, f^{-1}(B) is an IFαGSCS in X. Since X is an IF$_{\alpha ga}T_{1/2}$ space, f^{-1}(B) is an IFCS in X. Hence f is an IF continuous mapping. By hypothesis f^{-1}:(Y, σ) $\to$ (X, τ) is an IFαGS irresolute mapping. Let A be an IFCS in X. Since every IFCS is an IFαGSCS, A is an IFαGSCS in X. Then $(f^{-1})^{-1}$(A) = f(A) is an IFαGSCS in Y. Since Y is an IF$_{\alpha ga}T_{1/2}$ space, f(A) is an IFCS in Y. Hence f^{-1} is an IF continuous mapping. Therefore f is an IF homeomorphism. □

Definition 3.17. *Let A be an IFS in an IFTS (X, τ). Then αgscl(A) is defined as αgscl(A) = ∩ {B / B is an IFαGSCS in X and A ⊆ B}.*

Theorem 3.18. *If f : (X, τ) $\to$ (Y, σ) is an IFiαGS homeomorphism, then αgscl(f^{-1}(B)) ⊆ f^{-1}(αcl(B)) for every IFS B in Y.*

Proof. Let B be an IFS in Y. Then αcl(B) is an IFαCS in Y. This implies αcl(B) is an IFαGSCS in Y. Since f is an IFαGS irresolute mapping, f^{-1}(αcl(B)) is an IFαGSCS in X. This implies αgscl(f^{-1}(αcl(B))) = f^{-1}(αcl(B)). Now αgscl(f^{-1}(B)) ⊆ αgscl(αgscl(f^{-1}(αcl(B))) = f^{-1}(αcl(B)). Hence αgscl(f^{-1}(*B*)) ⊆ f^{-1}(αcl(B)) for every IFS B in Y. □

Theorem 3.19. *If f:(X, τ) $\to$ (Y, σ) is an IFiαGS homeomorphism, then αgscl(f^{-1}(B)) = f^{-1}(αgscl(B)) for every IFS B in Y.*

Proof. Since f is an IFiαGS homeomorphism, f is an IFαGS irresolute mapping. Consider an IFS B in Y. Clearly αgscl(B) is an IFαGSCS in Y. By hypothesis f^{-1}(αgscl(B)) is an IFαGSCS in X. Since f^{-1}(B) ⊆ f^{-1}(αgscl(B)), αgscl(f^{-1}(B)) ⊆ αgscl(f^{-1}(αgscl(B))) = f^{-1}(αgscl(B)). This implies αgscl(f^{-1}(B)) ⊂ f^{-1}(αgscl(B)).

Since f is an IFiαGS homeomorphism, f^{-1}:Y $\to$ X is an IFαGS irresolute mapping. Consider an IFS f^{-1}(B) in X.

Clearly αgscl(f^{-1}(B)) is an IFαGSCS in X. This implies $(\mathrm{f}^{-1})^{-1}$(αgscl(f^{-1}(B))) = f(αgscl(f^{-1}(B))) is an IFαGSCS in Y. Clearly B = $(\mathrm{f}^{-1})^{-1}(\mathrm{f}^{-1}$(B)) $\subseteq$ $(\mathrm{f}^{-1})^{-1}$(αgscl(f^{-1}(B))) = f(αgscl(f^{-1}(B))). Therefore αgscl(B) $\subseteq$ αgscl(f(αgscl(f^{-1}(B)))) = f(αgscl(f^{-1}(B)), Since f^{-1} is an IFαGS irresolute mapping. Hence f^{-1}(αgscl(B)) $\subseteq$ f^{-1}(f(αgscl(f^{-1}(B)) = αgscl(f^{-1}(B)).That is f^{-1}(αgscl(B)) $\subseteq$ αgscl(f^{-1}(B)). This implies αgscl(f^{-1}(B)) = f^{-1}(αgscl(B)). □

Theorem 3.20. *If f:(X, τ) $\rightarrow$ (Y, σ) is an IFiαGS homeomorphism, then αgscl(f(B)) = f(αgscl(B)) for every IFS B in X.*

Proof. Since f is an IFiαGS homeomorphism, f^{-1} is an IFiαGS homeomorphism. Let us consider an IFS B in X. By theorem : 3.18, αgscl($(\mathrm{f}^{-1})^{-1}$(B)) = $(\mathrm{f}^{-1})^{-1}$(αgscl(B)). Hence αgscl(f(B)) = f(αgscl(B)) for every IFS B in X. □

Proposition 3.21. *The composition of two IFiαGS homeomorphisms is an IFiαGS homeomorphism in general.*

Proof. Let f : (X, τ) $\rightarrow$ (Y, σ) and g : (Y, σ) $\rightarrow$ (Z, η) be two IFiαGS homeomorphisms. Let A be an IFαGSCS in Z. Then by hypothesis, $\mathrm{g}^{-1}(A)$ is an IFαGSCS in Y. Then by hypothesis,$\mathrm{f}^{-1}(g^{-1}(A))$ is an IFαGSCS in X. Hence $(\mathrm{g}\circ\mathrm{f})^{-1}$ is an IFαGS irresolute mapping. Let B be an IFαGSCS in X. Then by hypothesis, f(B) is an IFαGSCS in Y. Then by hypothesis g(f(B)) is an IFαGSCS in Z. This implies g$\circ$f is an IFαGS irresolute mapping. Hence g$\circ$f is an IFiαGS homeomorphism. Therefore the composition of two IFiαGS homeomorphism in general. □

We denote the family of all IFiαGS homeomorphisms of an IFTS (X, τ) onto itself by IFiαGS-h(X, τ).

Theorem 3.22. *The set IFiαGS-h(X, τ) is a group under the composition of maps.*

Proof. Define a binary operation $*$: IFiαGS-h((X, τ) $\times$ IFiαGS-h(X, τ) $\rightarrow$ IFiαGS-h(X, τ) by f$*$g = g$\circ$f for all f, g $\in$ IFiαGS-h(X, τ) and $\circ$ is the usual operation of composition of maps. Then by Proposition : 3.20, g$\circ$f $\in$ IFiαGS-h(X, τ). We know that, the composition of maps is associative and the identity map I : (X, τ) $\rightarrow$ (X, τ) belonging to IFiαGS-h(X, τ) serves as the identity element. If f $\in$ IFiαGS-h(X, τ), then f^{-1} $\in$ IFiαGS-h(X, τ) such that f$\circ\mathrm{f}^{-1}$ = $\mathrm{f}^{-1}\circ$f = I and so inverse exists for each element of IFiαGS-h(X, τ). Therefore, (IFiαGS-h(X, τ), $\circ$) is a group under the operation of composition of maps. □

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