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$\pi g\alpha$ Homeomorphisms In Intuitionistic Fuzzy Topological Spaces

Research Article

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Abstract: In this paper, we introduce the concepts of intuitionistic fuzzy πgα homeomorphisms and intuitionistic fuzzy i-πgα homeomorphisms. Further, we study some of their properties.

Keywords: Intuitionistic fuzzy topology, Intuitionistic fuzzy πgα closed set, Intuitionistic fuzzy πgα homeomorphism and Intuitionistic fuzzy i-πgα homeomorphism.

1. Introduction

The concept of fuzzy sets was introduced by Zadeh [22] and later Atanasov [1] generalized this idea to intuitionistic fuzzy sets using the notion of fuzzy sets. On the other hand Coker [3] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. The concept of semi homeomorphisms in topological spaces was introduced by Crossley and Hildebrand in 1972. The generalized homeomorphisms was introduced and investigated by Maki, Sundaram and Balachandran in 1991. Coker have introduced homeomorphisms in intuitionistic fuzzy topological spaces in 1997. In this paper we introduce intuitionistic fuzzy πgα homeomorphisms and intuitionistic fuzzy i-πgα homeomorphisms. The relations between intuitionistic fuzzy πgα homeomorphisms and other intuitionistic fuzzy homeomorphisms are given.

2. Preliminaries

Definition 2.1 ([1])**.** *Let X be a non empty set. An intuitionistic fuzzy set (IFS in short) A in X can be described in the form*

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$$

where the functions $\mu_A : X \to [0, 1]$ is called the membership function and $\mu_A(x)$ denotes the degree to which $x \in A$ and the function $\nu_A : X \to [0, 1]$ is called the non-membership function and $\nu_A(x)$ denotes the degree to which $x \notin A$ and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X. Throughout this paper, X denotes a non empty set.

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Definition 2.2 ([1]). *Let A and B be any two IFSs of the form* $A=\{\langle x,\ \mu_A(x),\ \nu_A(x)\rangle \mid x \in X\}$ *and* $B=\{\langle x,\ \mu_B(x),\ \nu_B(x)\rangle \mid x \in X\}$. *Then*

1. $A \subseteq B$ *if and only if* $\mu_A(x) \leq \mu_B(x)$ *and* $\nu_A(x) \geq \nu_B(x)$ *for all* $x \in X$,
2. $A = B$ *if and only if* $A \subseteq B$ *and* $B \subseteq A$,
3. $A^c = \{\langle x,\ \nu_A(x),\ \mu_A(x)\rangle \mid x \in X\}$,
4. $A \cap B = \{\langle x,\ \mu_A(x) \wedge \mu_B(x),\ \nu_A(x) \vee \nu_B(x)\rangle \mid x \in X\}$,
5. $A \cup B = \{\langle x,\ \mu_A(x) \vee \mu_B(x),\ \nu_A(x) \wedge \nu_B(x)\rangle \mid x \in X\}$.

Definition 2.3 ([1]). *The intuitionistic fuzzy sets* $0_\sim = \{\langle x,\ 0,\ 1\rangle \mid x \in X\}$ *and* $1_\sim = \{\langle x,\ 1,\ 0\rangle \mid x \in X\}$ *are the empty set and the whole set of X respectively.*

Definition 2.4 ([1]). *Let A and B be any two IFSs of the form* $A = \{\langle x,\ \mu_A(x),\ \nu_A(x)\rangle \mid x \in X\}$ *and* $B = \{\langle x,\ \mu_B(x),\ \nu_B(x)\rangle \mid x \in X\}$. *Then*

1. $A \subseteq B$ *and* $A \subseteq C \Rightarrow A \subseteq B \cap C$,
2. $A \subseteq C$ *and* $B \subseteq C \Rightarrow A \cup B \subseteq C$,
3. $A \subseteq B$ *and* $B \subseteq C \Rightarrow A \subseteq C$,
4. $(A \cup B)^c = A^c \cap B^c$ *and* $(A \cap B)^c = A^c \cup B^c$,
5. $((A)^c)^c = A$,
6. $(1_\sim)^c = 0_\sim$ *and* $(0_\sim)^c = 1_\sim$.

Definition 2.5 ([3]). *An intuitionistic fuzzy topology (IFT in short) on X is a family* τ *of IFSs in X satisfying the following axioms:*

1. $0_\sim,\ 1_\sim \in \tau$,
2. $G_1 \cap G_2 \in \tau$ *for any* $G_1,\ G_2 \in \tau$,
3. $\cup\ G_i \in \tau$ *for any family* $\{G_i \mid i \in J\} \subseteq \tau$.

In this case the pair (X, τ) *is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in* τ *is known as an intuitionistic fuzzy open set (IFOS in short) in X.*
The complement A^c *of an IFOS A in an IFTS* (X, τ) *is called an intuitionistic fuzzy closed set (IFCS in short) in X.*

Definition 2.6 ([3]). *Let* (X, τ) *be an IFTS and* $A = \{\langle x,\ \mu_A(x),\ \nu_A(x)\rangle \mid x \in X\}$ *be an IFS in X. Then the intuitionistic fuzzy interior and the intuitionistic fuzzy closure are defined as follows:*

$$int(A) = \cup\ \{G \mid G \text{ is an IFOS in } X \text{ and } G \subseteq A\},$$
$$cl(A) = \cap\ \{K \mid K \text{ is an IFCS in } X \text{ and } A \subseteq K\}.$$

Proposition 2.7 ([3]). *For any two IFSs A and B in* (X, τ), *we have*

1. $int(A) \subseteq A$,

2. $A \subseteq cl(A)$,

3. $A \subseteq B \Rightarrow int(A) \subseteq int(B)$ and $cl(A) \subseteq cl(B)$,

4. $int(int(A)) = int(A)$,

5. $cl(cl(A)) = cl(A)$,

6. $cl(A \cup B) = cl(A) \cup cl(B)$,

7. $int(A \cap B) = int(A) \cap int(B)$.

Proposition 2.8 ([3]). *For any IFS A in (X, τ), we have*

1. $int(0_\sim) = 0_\sim$ and $cl(0_\sim) = 0_\sim$,

2. $int(1_\sim) = 1_\sim$ and $cl(1_\sim) = 1_\sim$,

3. $(int(A))^c = cl(A^c)$,

4. $(cl(A))^c = int(A^c)$.

Proposition 2.9 ([3]). *If A is an IFCS in (X, τ) then $cl(A) = A$ and if A is an IFOS in (X, τ) then $int(A) = A$. The arbitrary union of IFCSs is an IFCS in (X, τ).*

Definition 2.10. *An IFS A in an IFTS (X, τ) is said to be an*

1. *intuitionistic fuzzy regular closed set (IFRCS in short) if $A = cl(int(A))$, [4]*

2. *intuitionistic fuzzy α-closed set (IFαCS in short) if $cl(int(cl(A))) \subseteq A$, [5]*

3. *intuitionistic fuzzy semi closed set (IFSCS in short) if $int(cl(A)) \subseteq A$, [4]*

4. *intuitionistic fuzzy pre closed set (IFPCS in short) if $cl(int(A)) \subseteq A$, [4]*

5. *intuitionistic fuzzy semi pre closed set (IFSPCS in short) if there exists an IFPCS B such that $int(B) \subseteq A \subseteq B$, [21]*

6. *intuitionistic fuzzy generalized closed set (IFGCS in short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ), [19]*

7. *intuitionistic fuzzy generalized semi closed set (IFGSCS in short) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ), [13]*

8. *intuitionistic fuzzy generalized semi pre closed set (IFGSPCS in short) if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ). [9]*

The complements of the above mentioned intuitionistic fuzzy closed sets are called their respective intuitionistic fuzzy open sets.

Definition 2.11 ([8]). *An IFS A in an IFTS (X, τ) is said to be an*

1. *intuitionistic fuzzy π open set (IFπOS in short) if A is a finite union of fuzzy regular open sets,*

2. *intuitionistic fuzzy π closed set (IFπCS in short) if A^c is an intuitionistic fuzzy π open set.*

Definition 2.12 ([11]). *Let A be an IFS in an IFTS (X, τ). Then the α-interior of A (αint(A) in short) and the α-closure of A (αcl(A) in short) are defined as*

$$\alpha int(A) = \cup \{G \mid G \text{ is an IF}\alpha OS \text{ in } (X, \tau) \text{ and } G \subseteq A\},$$
$$\alpha cl(A) = \cap \{K \mid K \text{ is an IF}\alpha CS \text{ in } (X, \tau) \text{ and } A \subseteq K\}.$$

sint(A), scl(A), pint(A), pcl(A), spint(A) and spcl(A) are similarly defined. For any IFS A in (X, τ), we have $\alpha cl(A^c) = (\alpha int(A))^c$ and $\alpha int(A^c) = (\alpha cl(A))^c$.

Remark 2.13 ([11]). *Let A be an IFS in an IFTS (X, τ). Then*

1. *$\alpha cl(A) = A \cup cl(int(cl(A)))$,*
2. *$\alpha int(A) = A \cap int(cl(int(A)))$.*

Remark 2.14.

1. *Every IFπOS in (X, τ) is an IFOS in (X, τ), [8]*
2. *Every IFOS in (X, τ) is an IFαOS in (X, τ), [4]*
3. *Every IFπOS in (X, τ) is an IFπGOS. [8]*

Definition 2.15 ([15]). *An IFS A in (X, τ) is said to be an intuituionistic fuzzy $\pi g\alpha$ closed set (IFπGαCS in short) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFπOS in (X, τ).*

Definition 2.16 ([15]). *An IFS A in (X, τ) is said to be an intuituionistic fuzzy $\pi g\alpha$ open set (IFπGαOS in short) if the complement A^c is an IFπGαCS in (X, τ).*

Remark 2.17 ([15]). *Every IFCS, IFαCS, IFRCS, IFGCS is an IFπGαCS, but the converses may not be true in general.*

Definition 2.18 ([3]). *Let X and Y are two nonempty sets. Let $f : X \to Y$ be a mapping. If $B = \{\langle y, \mu_B(y), \nu_B(y)\rangle \mid y \in Y\}$ is an IFS in Y, then the preimage of B under f, denoted by $f^{-1}(B)$, is the IFS in X defined by $f^{-1}(B) = \{\langle x, f^{-1}(\mu_B)(x), f^{-1}(\nu_B)(x)\rangle \mid x \in X\}$.*

Definition 2.19 ([3]). *Let X and Y are two nonempty sets. Let $f : X \to Y$ be a mapping. If $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ is an IFS in X, then the image of A under f, denoted by f(A), is the IFS in Y defined by $f(A) = \{\langle y, f(\mu_A)(y), f_-(\nu_A)(y)\rangle \mid y \in Y\}$, where $f_-(\nu_A)(y) = 1 - f(1 - \nu_A)$.*

Definition 2.20 ([4]). *Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an intuitionistic fuzzy closed mapping (IF closed map in short) if f(A) is an IFCS in (Y, σ) for every IFCS A of (X, τ).*

Definition 2.21 ([4]). *Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an intuitionistic fuzzy open mapping (IF open map in short) if f(A) is an IFOS in (Y, σ) for every IFOS A of (X, τ).*

Definition 2.22 ([17]). *A mapping $f : (X, \tau) \to (Y, \sigma)$ is said to be an intuituionistic fuzzy $\pi g\alpha$ closed mapping (IFπGα closed map in short) if f(A) is an IFπGαCS in (Y, σ) for every IFCS A in (X, τ).*

Definition 2.23 ([17]). *A mapping $f : (X, \tau) \to (Y, \sigma)$ is said to be an intuituionistic fuzzy $\pi g\alpha$ open mapping (IFπGα open map in short) if f(A) is an IFπGαOS in (Y, σ) for every IFOS A in (X, τ).*

Definition 2.24 ([4]). *Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an intuitionistic fuzzy continuous (IF continuous in short) if $f^{-1}(B)$ is an IFCS in (X, τ) for every IFCS B of (Y, σ).*

Definition 2.25. *Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an*

1. *intuitionistic fuzzy α-continuous (IFα continuous in short) if $f^{-1}(B)$ is an IFαCS in (X, τ) for every IFCS B of (Y, σ), [4]*
2. *intuitionistic fuzzy generalized continuous (IFG continuous in short) if $f^{-1}(B)$ is an IFGCS in (X, τ) for every IFCS B of (Y, σ), [18]*
3. *intuitionistic fuzzy generalized semi continuous (IFGS continuous in short) if $f^{-1}(B)$ is an IFGSCS in (X, τ) for every IFCS B of (Y, σ), [13]*
4. *intuitionistic fuzzy generalized semi pre continuous (IFGSP continuous in short) if $f^{-1}(B)$ is an IFGSPCS in (X, τ) for every IFCS B of (Y, σ), [14]*
5. *intuituionistic fuzzy $\pi g\alpha$ continuous (IF$\pi G\alpha$ continuous in short) if $f^{-1}(B)$ is an IF$\pi G\alpha$CS in (X, τ) for every IFCS B of (Y, σ). [16]*

Definition 2.26 ([16]). *A mapping f : (X, τ) $\to$ (Y, σ) is called an intuitionistic fuzzy $\pi g\alpha$ irresolute (IF$\pi G\alpha$ irresolute, in short) if $f^{-1}(A)$ is an IF$\pi G\alpha$CS in (X, τ) for every IF$\pi G\alpha$CS A in (Y, σ).*

Definition 2.27. *Let f be a bijective mapping from an IFTS (X, τ) into an IFTS (Y, σ).Then f is said to be an*

1. *intuitionistic fuzzy homeomorphism (IF homeomorphism in short) if f and f^{-1} are IF continuous mappings, [6]*
2. *intuitionistic fuzzy alpha homeomorphism (IFα homeomorphism in short) if f and f^{-1} are IFα continuous mappings, [12]*
3. *intuitionistic fuzzy alpha generalized homeomorphism (IFG homeomorphism in short) if f and f^{-1} are IFG continuous mappings, [12]*
4. *intuitionistic fuzzy alpha generalized semi homeomorphism (IFGS homeomorphism in short) if f and f^{-1} are IFGS continuous mappings, [10]*
5. *intuitionistic fuzzy alpha generalized semi pre homeomorphism (IFGSP homeomorphism in short) if f and f^{-1} are IFG continuous mappings, [20]*

Definition 2.28 ([15]). *An IFTS (X, τ) is said to be an intuitionistic fuzzy $_{\pi\alpha a}T_{1/2}$ (IF$_{\pi\alpha a}T_{1/2}$ in short) space if every IF$\pi G\alpha$CS in X is an IFCS in X.*

3. Intuitionistic Fuzzy $\pi g\alpha$ Homeomorphisms

In this section we introduce intuitionistic fuzzy $\pi g\alpha$ homeomorphisms and study some of its properties.

Definition 3.1. *A bijective mapping f : (X, τ) $\to$ (Y, σ) is called an intuitionistic fuzzy $\pi g\alpha$ homeomorphism (IF$\pi G\alpha$ homeomorphism in short) if f and f^{-1} are IF$\pi G\alpha$ continuous mappings.*

Example 3.2. *Let X = {a, b} and Y = {u, v}. Let T_1 = $\langle$x, (0.7, 0.6), (0.2, 0.3)$\rangle$ and T_2 = $\langle$y, (0.8, 0.7), (0.1, 0.2)$\rangle$. Then τ = {$0_\sim$, T_1, $1_\sim$} and σ = {$0_\sim$, T_2, $1_\sim$} are IFTs on X and Y respectively. Define a bijective mapping f : (X, τ) $\to$ (Y, σ) by f(a) = u and f(b) = v. Then f is an IF$\pi G\alpha$ continuous mapping and f^{-1} is also an IF$\pi G\alpha$ continuous mapping. Therefore the bijective mapping f is an IF$\pi G\alpha$ homeomorphism.*

Theorem 3.3. *Let f : (X, τ) $\to$ (Y, σ) be a bijective mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent:*

1. *f is an IF homeomorphism,*
2. *f is an IF continuous mapping and f is an IF open mapping,*
3. *f and f^{-1} are IF continuous mappings.*

Proof. (1) $\Rightarrow$ (2). Obvious.

(2) $\Rightarrow$ (3). Let f is an IF open mapping. That is f(A) is an IFOS in Y for each IFOS A in X. Now define a mapping f^{-1} : (Y, σ) $\to$ (X, τ). By hypothesis, for every IFOS A in X, we have f^{-1}(A) is an IFOS in Y. Hence f^{-1} is an IF continuous mapping. That is f and f^{-1} are IF continuous mappings.

(3) $\Rightarrow$ (1). Let f and f^{-1} are IF continuous mappings. Since f^{-1} : (Y, σ) $\to$ (X, τ) is an IF continuous mapping, f : (X, τ) $\to$ (Y, σ) is an IF open mapping. Hence f is an IF homeomorphism. □

Theorem 3.4. *Every IF homeomorphism is an IFπGα homeomorphism but not conversely.*

Proof. Let f : (X, τ) $\to$ (Y, σ) be an IF homeomorphism. Then f and f^{-1} are IF continuous mappings. This implies f and f^{-1} are IFπGα continuous mappings. That is f is an IFπGα homeomorphism. □

Example 3.5. *Let X = $\{a, b\}$ and Y = $\{u, v\}$. Let $T_1 = \langle x, (0.8, 0.7), (0.1, 0.2)\rangle$ and $T_2 = \langle y, (0.7, 0.6), (0.2, 0.3)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a bijective mapping f : (X, τ) $\to$ (Y, σ) by f(a) = u and f(b) = v. Consider an IFS A = $\langle x, (0.2, 0.3), (0.7, 0.6)\rangle$ is an IFCS in Y. Then f^{-1}(A) is not an IFCS in X. Therefore f is not an IF continuous mapping. Therefore the bijective mapping f is not an IF homeomorphism. But f is an IFπGα continuous mapping and f^{-1} is also an IFπGα continuous mapping. Therefore the bijective mapping f is an IFπGα homeomorphism.*

Theorem 3.6. *Every IFα homeomorphism is an IFπGα homeomorphism but not conversely.*

Proof. Let f : (X, τ) $\to$ (Y, σ) be an IFα homeomorphism. Then f and f^{-1} are IFα continuous mappings. This implies f and f^{-1} are IFπGα continuous mappings. That is f is an IFπGα homeomorphism. □

Example 3.7. *Let X = $\{a, b\}$ and Y = $\{u, v\}$. Let $T_1 = \langle x, (0.4, 0.3), (0.5, 0.6)\rangle$ and $T_2 = \langle y, (0.4, 0.5), (0.5, 0.4)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a bijective mapping f : (X, τ) $\to$ (Y, σ) by f(a) = u and f(b) = v. Consider an IFS A = $\langle x, (0.5, 0.4), (0.4, 0.5)\rangle$ is an IFCS in Y. Then f^{-1}(A) is not an IFαCS in X. Therefore f is not an IFα continuous mapping. Therefore the bijective mapping f is not an IFα homeomorphism. But f is an IFπGα continuous mapping and f^{-1} is also an IFπGα continuous mapping. Therefore the bijective mapping f is an IFπGα homeomorphism.*

Theorem 3.8. *Every IFG homeomorphism is an IFπGα homeomorphism but not conversely.*

Proof. Let f : (X, τ) $\to$ (Y, σ) be an IFG homeomorphism. Then f and f^{-1} are IFG continuous mappings. This implies f and f^{-1} are IFπGα continuous mappings. That is f is an IFπGα homeomorphism. □

Example 3.9. *Let X = $\{a, b\}$ and Y = $\{u, v\}$. Let $T_1 = \langle x, (0.6, 0.7), (0.3, 0.2)\rangle$ and $T_2 = \langle y, (0.5, 0.6), (0.4, 0.3)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a bijective mapping f : (X, τ) $\to$ (Y, σ) by f(a) = u and f(b) = v. Consider an IFS A = $\langle x, (0.4, 0.3), (0.5, 0.6)\rangle$ is an IFCS in Y. Then f^{-1}(A)* *is not an IFGCS in X. Therefore f is not an IFG continuous mapping. Therefore the bijective mapping f is not an IFG homeomorphism. But f is an IF$\pi G\alpha$ continuous mapping and f^{-1} is also an IF$\pi G\alpha$ continuous mapping. Therefore the bijective mapping f is an IF$\pi G\alpha$ homeomorphism.*

Remark 3.10. *An IF$\pi G\alpha$ homeomorphism is independent of an IFGS homeomorphism.*

Example 3.11. *Let X = {a, b} and Y = {u, v}. Let T_1 = ⟨x, (0.45, 0.5), (0.45, 0.4)⟩ and T_2 = ⟨y, (0.5, 0.6), (0.4, 0.3)⟩. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a bijective mapping f : (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Consider an IFS A = ⟨x, (0.45, 0.4), (0.45, 0.5)⟩ is an IFCS in X. Then f(A) is not an IFGSCS in Y. Therefore f^{-1} is not an IFGS continuous mapping. Therefore the bijective mapping f is not an IFGS homeomorphism. But f is an IF$\pi G\alpha$ continuous mapping and f^{-1} is also an IF$\pi G\alpha$ continuous mapping. Therefore the bijective mapping f is an IF$\pi G\alpha$ homeomorphism.*

Example 3.12. *Let X = {a, b} and Y = {u, v}. Let T_1 = ⟨x, (0.2, 0.3), (0.7, 0.6)⟩ and T_2 = ⟨y, (0.8, 0.7), (0.1, 0.2)⟩. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a bijective mapping f : (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Consider an A = ⟨x, (0.1, 0.2), (0.8, 0.7)⟩ is an IFCS in Y. Then f^{-1}(A) is not an IF$\pi G\alpha$CS in X. Therefore f is not an IF$\pi G\alpha$ continuous mapping. Therefore the bijective mapping f is not an IF$\pi G\alpha$ homeomorphism. But f is an IFGS continuous mapping and f^{-1} is also an IFGS continuous mapping. Therefore the bijective mapping f is an IFGS homeomorphism.*

Remark 3.13. *An IF$\pi G\alpha$ homeomorphism is independent of an IFGSP homeomorphism.*

Example 3.14. *Let X = {a, b} and Y = {u, v}. Let T_1 = ⟨x, (0.2, 0.6), (0.7, 0.3)⟩ and T_2 = ⟨y, (0.7, 0.3), (0.2, 0.6)⟩. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a bijective mapping f : (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Consider an A = ⟨x, (0.2, 0.6), (0.7, 0.3)⟩ is an IFCS in Y. Then f^{-1}(A) is not an IFGSPCS in X. Therefore f is not an IFGSP continuous mapping. Therefore the bijective mapping f is not an IFGSP homeomorphism. But f is an IF$\pi G\alpha$ continuous mapping and f^{-1} is also an IF$\pi G\alpha$ continuous mapping. Therefore the bijective mapping f is an IFGSP homeomorphism.*

Example 3.15. *Let X = {a, b} and Y = {u, v}. Let T_1 = ⟨x, (0.2, 0.3), (0.7, 0.6)⟩ and T_2 = ⟨y, (0.7, 0.6), (0.2, 0.3)⟩. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a bijective mapping f : (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Consider an A = ⟨x, (0.2, 0.3), (0.7, 0.6)⟩ is an IFCS in Y. Then f^{-1}(A) is not an IF$\pi G\alpha$CS in X. Therefore f is not an IF$\pi G\alpha$ continuous mapping. Therefore the bijective mapping f is not an IF$\pi G\alpha$ homeomorphism. But f is an IFGSP continuous mapping and f^{-1} is also an IFGSP continuous mapping. Therefore the bijective mapping f is an IFGSP homeomorphism.*

Theorem 3.16. *Let f : (X, τ) → (Y, σ) be an IF$\pi G\alpha$ homeomorphism. Then f is an IF homeomorphism if X and Y are IF$_{\pi\alpha a}T_{1/2}$ space.*

Proof. Let B be an IFCS in Y. Since f is an IFπGα homeomorphism, f is an IFπGα continuous mapping. Therefore f^{-1}(B) is an IFπGαCS in X. Since X is an IF$_{\pi\alpha a}$T$_{1/2}$ space, f^{-1}(B) is an IFCS in X. Hence f is an IF continuous mapping. Let A be an IFCS in X. Since f is an IFπGα homeomorphism, f^{-1} is an IFπGα continuous mapping. Therefore $(f^{-1})^{-1}$(A) = f(A) is an IFπGαCS in X. Since X is an IF$_{\pi\alpha a}$T$_{1/2}$ space, f(A) is an IFCS in X. Hence f^{-1} is an IF continuous mapping. Therefore f is an IF homeomorphism. □

Theorem 3.17. *Let f : (X, τ) → (Y, σ) be a bijective mapping. If f is an IF$\pi G\alpha$ continuous mapping, then the following are equivalent.*

1. *f is an IFπGα closed mapping,*
2. *f is an IFπGα open mapping,*
3. *f is an IFπGα homeomorphism.*

Proof. (1) $\Rightarrow$ (2): Let f : (X, τ) $\rightarrow$ (Y, σ) be a bijective mapping. Let f be an IFπGα closed mapping. This implies f^{-1} : (Y, σ) $\rightarrow$ (X, τ) is an IFπGα continuous mapping. Assume that A is an IFOS in X. Then by hypothesis, $(f^{-1})^{-1}(A)$ = f(A) is an IFπGαOS in Y. Hence f is an IFπGα open mapping.

(2) $\Rightarrow$ (3): Let f : (X, τ) $\rightarrow$ (Y, σ) be a bijective mapping. Let f is an IFπGα open mapping. This implies f^{-1} : (Y, σ) $\rightarrow$ (X, τ) is an IFπGα continuous mapping. That is f is an IFπGα homeomorphism.

(3) $\Rightarrow$ (1): Let f be an IFπGα homeomorphism. That is f and f^{-1} are IFπGα continuous mappings. Let A be an IFCS in X. Then by hypothesis, A is an IFπGαCS in Y. Hence f is an IFπGα closed mapping. □

Remark 3.18. *The composition of two IFπGα homeomorphisms need not be an IFπGα homeomorphism in general.*

Example 3.19. *Let X = {a, b}, Y = {c, d} and Z = {u, v}. Let T_1 = ⟨x, (0.2, 0.3), (0.7, 0.6)⟩, T_2 = ⟨y, (0.6, 0.5), (0.3, 0.4)⟩ and T_3 = ⟨z, (0.8, 0.7), (0.1, 0.2)⟩. Then τ = {$0_\sim$, T_1, $1_\sim$}, σ = {$0_\sim$, T_2, $1_\sim$} and δ = {$0_\sim$, T_3, $1_\sim$} are IFTs on X, Y and Z respectively. Define a bijective mapping f : (X, τ) $\rightarrow$ (Y, σ) by f(a) = c and f(b) = d, and define a bijective mapping g : (Y, σ) $\rightarrow$ (Z, δ) by f(c) = u and f(d) = v. Then f and g are IFπGα homeomorphisms. Consider an IFS A = ⟨z, (0.1, 0.2), (0.8, 0.7)⟩. Then A is an IFCS in Z. But $(g \circ f)^{-1}(A)$ is not an IFπGαCS in X. Therefore g ∘ f is not an IFπGα continuous mapping. Therefore g ∘ f is not an IFπGα homeomorphism. Hence the composition of two IFπGα homeomorphisms is need not be an IFπGα homeomorphism.*

4. Intuitionistic Fuzzy i-πgα Homeomorphisms

In this section, we introduce intuitionistic fuzzy i-πgα homeomorphisms and study some of their properties.

Definition 4.1. *A bijective mapping f : (X, τ) $\rightarrow$ (Y, σ) is called an intuitionistic fuzzy i-πgα homeomorphism (IFiπGα homeomorphism in short) if f and f^{-1} are IFπGα irresolute mappings.*

Example 4.2. *Let X = {a, b} and Y = {u, v}. Let T_1 = ⟨x, (0.8, 0.7), (0.1, 0.2)⟩ and T_2 = ⟨y, (0.7, 0.6), (0.2, 0.3)⟩. Then τ = {$0_\sim$, T_1, $1_\sim$} and σ = {$0_\sim$, T_2, $1_\sim$} are IFTs on X and Y respectively. Define a bijective mapping f : (X, τ) $\rightarrow$ (Y, σ) by f(a) = u and f(b) = v. Then f is an IFπGα irresolute mapping and f^{-1} is also an IFπGα irresolute mapping. Therefore the bijective mapping f is an IFiπGα homeomorphism.*

Theorem 4.3. *Every IFiπGα homeomorphism is an IFπGα homeomorphism but not conversely.*

Proof. Let f : (X, τ) $\rightarrow$ (Y, σ) be an IFiπGα homeomorphism. Let B be an IFCS in Y. This implies B is an IFπGαCS in Y. By hypothesis, $f^{-1}(B)$ is an IFπGαCS in X. Hence f is an IFπGα continuous mapping. Let A be an IFCS in Y. This implies A is an IFπGαCS in X. By hypothesis, $(f^{-1})^{-1}(A)$ = f(A) is an IFπGαCS in X. Hence f^{-1} is an IFπGα continuous mapping. This implies f is an IFπGα homeomorphism. □

Example 4.4. *Let X = {a, b} and Y = {u, v}. Let T_1 = ⟨x, (0.2, 0.3), (0.7, 0.6)⟩ and T_2 = ⟨y, (0.6, 0.5), (0.3, 0.4)⟩. Then τ = {$0_\sim$, T_1, $1_\sim$} and σ = {$0_\sim$, T_2, $1_\sim$} are IFTs on X and Y respectively. Define a bijective mapping f : (X, τ) $\rightarrow$ (Y, σ) by f(a) = u and f(b) = v. Consider an IFS A = ⟨x, (0.1, 0.2), (0.8, 0.7)⟩ is an IFπGαCS in Y. Then $f^{-1}(A)$ is not an IFπGαCS in X. Therefore f is not an IFπGα irresolute mapping. Therefore the bijective mapping f is not an IFiπGα*

homeomorphism. But f is an IF$\pi G\alpha$ continuous mapping and f^{-1} is also an IF$\pi G\alpha$ continuous mapping. Therefore the bijective mapping f is an IF$\pi G\alpha$ homeomorphism.

Definition 4.5. *Let A be an IFS in an IFTS (X, τ). Then $\pi g\alpha cl(A)$ is defined as*

$$\pi g\alpha cl(A) = \cap \{B \mid B \text{ is an IF}\pi G\alpha CS \text{ in } X \text{ and } A \subseteq B\}.$$

Theorem 4.6. *If the mapping $f : X \to Y$ is an IFi$\pi G\alpha$ homeomorphism, then $\pi g\alpha cl(f^{-1}(B)) \subseteq f^{-1}(\alpha cl(B))$ for every IFS B in Y.*

Proof. Let B be an IFS in Y. Then αcl(B) is an IFαCS in Y. This implies αcl(B) is an IFπGαCS in Y. Since f is an IFπGα irresolute mapping, $f^{-1}(\alpha cl(B))$ is an IFπGαCS in X. This implies $\pi g\alpha cl(f^{-1}(\alpha cl(B))) = f^{-1}(\alpha cl(B))$. Now $\pi g\alpha cl(f^{-1}(B)) \subseteq \pi g\alpha cl(f^{-1}(\alpha cl(B))) = f^{-1}(\alpha cl(B))$. Hence $\pi g\alpha cl(f^{-1}(B)) \subseteq f^{-1}(\alpha cl(B))$ for every IFS B in Y. □

Theorem 4.7. *If the mapping $f : X \to Y$ is an IFi$\pi G\alpha$ homeomorphism, then $\pi g\alpha cl(f^{-1}(B)) = f^{-1}(\pi g\alpha cl(B))$ for every IFS B in Y.*

Proof. Since f is an IFiπGα homeomorphism, $f : X \to Y$ is an IFπGα irresolute mapping. Consider an IFS B in Y. Clearly $\pi g\alpha cl(B)$ is an IFπGαCS in Y. By hypothesis, $f^{-1}(\pi g\alpha cl(B))$ is an IFπGαCS in X. Since $f^{-1}(B) \subseteq f^{-1}(\pi g\alpha cl(B))$, $\pi g\alpha cl(f^{-1}(B)) \subseteq \pi g\alpha cl(f^{-1}(\pi g\alpha cl(B))) = f^{-1}(\pi g\alpha cl(B))$. This implies $\pi g\alpha cl(f^{-1}(B)) \subseteq f^{-1}(\pi g\alpha cl(B))$.
Since f is an IFiπGα homeomorphism, $f^{-1} : Y \to X$ is an IFπGα irresolute mapping. Consider an IFS $f^{-1}(B)$ in X. Clearly $\pi g\alpha cl(f^{-1}(B))$ is an IFπGαCS in X. This implies $(f^{-1})^{-1}(\pi g\alpha cl(f^{-1}(B))) = f(\pi g\alpha cl(f^{-1}(B)))$ is an IFπGαCS in Y.
Clearly $B = (f^{-1})^{-1}(f^{-1}(B)) \subseteq (f^{-1})^{-1}(\pi g\alpha cl(f^{-1}(B))) = f(\pi g\alpha cl(f^{-1}(B)))$. Therefore $\pi g\alpha cl(B) \subseteq \pi g\alpha cl(f(\pi g\alpha cl(f^{-1}(B)))) = f(\pi g\alpha cl(f^{-1}(B)))$, since f^{-1} is an IFπGα irresolute mapping. Hence $f^{-1}(\pi g\alpha cl(B)) \subseteq f^{-1}(f(\pi g\alpha cl(f^{-1}(B)))) = \pi g\alpha cl(f^{-1}(B))$.
That is $f^{-1}(\pi g\alpha cl(B)) \subseteq \pi g\alpha cl(f^{-1}(B))$.
This implies $\pi g\alpha cl(f^{-1}(B)) = f^{-1}(\pi g\alpha cl(B))$. □

Theorem 4.8. *If the mapping $f : X \to Y$ is an IFi$\pi G\alpha$ homeomorphism, then $\pi g\alpha cl(f(B)) = f(\pi g\alpha cl(B))$ for every IFS B in X.*

Proof. Since f is an IFiπGα homeomorphism, f^{-1} is an IFiπGα homeomorphism. Let us consider an IFS B in X. By Theorem 4.7, $\pi g\alpha cl((f^{-1})^{-1})(B)) = (f^{-1})^{-1}(\pi g\alpha cl(B))$. Hence $\pi g\alpha cl(f(B)) = f(\pi g\alpha cl(B))$ for every IFS B in X. □

Corollary 4.9. *If the mapping $f : X \to Y$ is an IFi$\pi G\alpha$ homeomorphism where X and Y are IF$_{\pi\alpha a}T_{1/2}$ spaces, then cl(f(B)) = f(cl(B)) for every IFS B in X.*

Proof. Let us consider an IFS B in X. By Theorem 4.8, $\pi g\alpha cl(f(B)) = f(\pi g\alpha cl(B))$. Since X and Y are IF$_{\pi\alpha a}T_{1/2}$ spaces, we have cl(f(B)) = f(cl(B)). □

Theorem 4.10. *The composition of two IFi$\pi G\alpha$ homeomorphisms is an IFi$\pi G\alpha$ homeomorphism in general.*

Proof. Let $f : X \to Y$ and $g : Y \to Z$ are any two IFiπGα homeomorphisms. Let A be an IFπGαCS in Z. Then by hypothesis, $g^{-1}(A)$ is an IFπGαCS in Y. Again by hypothesis, $f^{-1}(g^{-1}(A))$ is an IFπGαCS in X. Hence $(g \circ f)^{-1}$ is an IFπGα irresolute mapping.
Now let B be an IFπGαCS in X. Then by hypothesis, f(B) is an IFπGαCS in Y. Again by hypothesis, g(f(B)) is an IFπGαCS in Z. This implies $g \circ f$ is an IFπGα irresolute mapping.
Hence $g \circ f$ is an IFiπGα homeomorphism. Therefore the composition of two IFiπGα homeomorphism is an IFiπGα homeomorphism in general. □

Theorem 4.11. *The set of all IFiπGα homeomorphisms in an IFTS (X, τ) is a group under the composition of maps.*

Proof. Let us denote the set of all IFiπGα homeomorphisms in an IFTS (X, τ) as IFiπGαHM(X). Define a binary operation $*$: IFiπGαHM(X) $\times$ IFiπGαHM(X) $\rightarrow$ IFiπGαHM(X) by f $*$ g = f $\circ$ g for every f, g $\in$ IFiπGαHM(X) and $\circ$ is the usual operation of composition of maps. If f and g $\in$ IFiπGαHM(X), then by Theorem 4.10, f $\circ$ g $\in$ IFiπGαHM(X). We know that the composition of maps is associative. The identity map I : (X, τ) $\rightarrow$ (X, τ) belonging to IFiπGαHM(X) is the identity element. If f $\in$ IFiπGαHM(X), then f^{-1} $\in$ IFiπGαHM(X). Therefore $f \circ f^{-1} = f^{-1} \circ f = I$ and so the inverse exists for each element of IFiπGαHM(X). Hence (IFiπGαHM(X), $\circ$) is a group. □

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