



Finding Rank Orders of the Elementary Concepts of Algebra Used in S.C.E. of Visva-Bharati

Research Article

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Abstract: In this paper, an attempt has been made to find the rank orders of identified top seventy two elementary concepts of algebra which were used to solve the algebraic problems and arithmetical problems which may be solved by algebraic method of two years question papers of mathematics of School Certificate Examination (S.C.E.) (1999 & 2000) of The Visva-Bharati Central University through task analysis technique. The teachers as well as learners will be benefited in several ways like to diagnose the weakness of the learners, to find the difficult order of these concepts, to emphasize on the learning of these concepts at appropriate grade levels etc. after knowing these elementary concepts. This study will help the entire curriculum development of school mathematics.

MSC: 97C99, 97H10.

Keywords: Identifying, major concept, elementary concept, algebra, task analysis technique, mathematics question papers, rank orders.

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1. Introduction

Algebra, is one and important branch of all renowned branches of mathematics, is framed logically by the nature of its abstraction. To make mathematics easier by algebra to the learners through the generalization of the content this is acquainted arithmetically instead of giving more mental stress. So, the learners can easily solve the real life arithmetical problems using the basic ideas of algebra. Again, it is a matter of pity that learners can't apparently perform better in mathematics at the 10th standard different Board examinations. On the other hand, the author is a mathematics teacher of secondary level under West Bengal Board of Secondary Education. So, he has emphasized to know which elementary concepts are more essential to solve the problems of mathematics of different Board examinations. Some works have been done by him to know these elementary concepts on algebra and arithmetic of mathematics of Madhyamik Pariksha (1999 & 2000) under West Bengal Board of Secondary Education and Indian Certificate of Secondary Education (I.C.S.E.) Examination (2005 & 2006) of The Council for the Indian School Certificate Examination (C.I.S.C.E.) through task analysis technique (1, 2, 6). Even, he has identified total 1253 elementary concepts of algebra and total 9007 elementary concepts of arithmetic through text book scanning process and task analysis technique(3, 4). In this paper, the author has made an attempt to find the rank order after identifying the most frequently used top seventy two algebraic elementary concepts to solve the algebra group problems of mathematics of School Certificate Examination (1999 & 2000) of Visva-Bharati Central University through task analysis technique whereas author had identified top thirty elementary concepts and presented in (5) .

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2. Objective of the Study

To find the rank orders of the top seventy two elementary concepts of algebra used in School Certificate Examination (1999 & 2000) of Visva-Bharati Central University.

3. Definitions

3.1. Major Concepts and Sub-concepts

A major concept is an idea which is complete in itself and is comprehended through a sequential process of step-by-step partial comprehension of its related concepts. These related concepts are called sub-concepts which are not complete in itself but are parts of the major concept.

3.2. Elementary Concepts

A sub-concept which can not be split further will be called an elementary concept.

4. Materials and Methods

It has been done in several stages

- (i). The two years question papers of mathematics of School Certificate Examination (1999 & 2000) of Visva-Bharati Central University have been collected for studying the major concepts of mathematics at secondary level through the task analysis technique.
- (ii). All the items of arithmetic (which may also be solved by algebraic method) and algebra which are relevant to our major concept analysis of the said question papers have been solved in such a way that a student may get full mark on each item. All the methods have been considered here where a sum has been solved more than one method.
- (iii). Then the answer scripts of the said question papers were given to some expert examiners to examine. They also have suggested some alternative methods or techniques to solve some items.
- (iv). The answer scripts have been re-written through incorporating the views of the expert examiners.
- (v). All the details of the solutions have been worked out (analysed in detail up to elementary level concepts) to find out all possible elementary concepts with their frequencies of occurrences which have been used in the solutions.
- (vi). In this phase of detail-analysis, again experts' opinions were sought and their opinions were incorporated. Then final form of the solutions for each items have been prepared.
- (vii). The elementary concepts which have been used in the solutions of each item have been identified and separated.
- (viii). Now the frequency of elementary concept of the item is the total number of elementary concept used to solve the sum in a single method. In case where a particular item has been solved in more than one method, the number of elementary concepts used and their frequency have been determined by using the following criteria: All the elementary concepts used in different methods have been selected and maximum frequency of each elementary concept within different methods has been taken as the frequency of that concept.
- (ix). Then elementary concepts which are used in School Certificate Examination (1999 & 2000) are counted.

- (x). Both algebraic and arithmetical elementary concepts have been considered in some cases when the algebraic method has been used to solve the arithmetical problem. For example, $3 + 2 = 5$ Algebraic elementary concept: addition of two positive integers. Arithmetical elementary concept: horizontal addition of two one-digit numbers except zero where the sum is one-digit number.
- (xi). Top seventy two frequently used elementary concepts determined by their frequency of use of algebra in School Certificate Examination (1999 & 2000) have been identified.
- (xii). The rank orders of these elementary concepts according to their frequency of use have been determined.

5. Major Findings

Using the above method, the rank orders of identified top seventy two elementary concepts according to frequency of use in algebra and arithmetic where algebraic method may be used in School Certificate Examination (1999 & 2000) of Visva-Bharati Central University have been determined. These concepts, their frequency of use and the rank order have been presented below in column 2, 3 and 4 of Table-1 respectively. For better understanding of these elementary concepts, Question (Q) is given in almost all concepts and Answer (A) is given for some questions in column 2 of Table-1.

Table 1. Rank orders of identified top seventy two elementary concepts of algebra used in S.C.E.

1	2	3	4
Sl. No.	Name of the elementary concept	Frequency of use	Rank
1.	Multiplication of monomials (when the monomials are two). Q. Multiply: i) $2 \times y$ ii) $2a^2 \times 3a$	203	1
2.	For all integers a and b, $a + b = b + a$. Q. Fill in the blanks. i) $2+5=5+\dots$ ii) $x+y=\dots+x$	60	2
3.	Multiplication of binomial by monomial (horizontal multiplication). Q. Multiply: i) $2x(3y+2z)$ ii) $(3m^2-2n)2m$	56	3
4.	Multiplication of positive integer by positive integer. Q. Multiply the first integer by the second. 7,12	51	4
5.	Division of integer: if both the dividend and divisor are of positive sign, then the quotient is positive. Q. Find: $18 \div 3$	50	5
6.	Addition of like monomials (when monomials are two). Q. Add: $2xy, -3xy$	49	6
7.	Solving the linear equation (polynomial form) without mentioning axiom (using process of transposition). Q. Solve the equation : $11x + 2 = -20$ A. : $11x + 2 = -20$ or $11x = -20 - 2$ or $11x = -22$ Therefore $x = -22/11 = -2$	36	7
8.	Addition of zero with any algebraic expression is equivalent to the same algebraic expression. Q. Add: $2x + 3y, 0$	34	8.5
9.	Subtraction of two positive integers (when minuend = subtrahend). Q. Subtract: $(+18) - (+18)$	34	8.5
10.	Division of a monomial by a monomial. Q. Divide p by p.	33	11
11.	If equal quantities are subtracted from equal quantities, the differences will also be equal.	33	11

In the table-1, first seven elementary concepts and concept no. 13, 14, 15, 16, 19, 28, 29, 30, 31 out of seventy two elementary

12.	Concept of cross multiplication. If $a/b = c/d$ then $ad = bc$	33	11																		
13.	Solving linear equation (in fractional form). Q. Solve: $2x/5 = 3/5$	32	13																		
14.	Subtraction of algebraic expression: subtraction of two like monomials. Q. Subtract: $a, 2a$	30	14																		
15.	If equal quantities are divided by equal quantities the quotients will also be equal.	25	15																		
16.	Finding the values of one unknown when the values of other unknown of simultaneous linear equations are given. Q. There is a simultaneous equation is given below. From the expression express x in terms of y and taking the value of y as 1, 2, 3, 4, -1, -2, -3, -4 find the corresponding values of x and complete the table given below. $x+y=6$ <table border="1" style="margin-left: 20px;"> <tbody> <tr> <td>x</td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td>y</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>-1</td> <td>-2</td> <td>-3</td> <td>-4</td> </tr> </tbody> </table>	x									y	1	2	3	4	-1	-2	-3	-4	23	16
x																					
y	1	2	3	4	-1	-2	-3	-4													
17.	Multiplying any integer by zero. Q. Find the product. -15×0	19	17.5																		
18.	Finding the value of an algebraic expression when the unknowns are other algebraic expressions. Q. Find the value of $(x+y)/z$ when $x=a+b, y=a-b, z=2a$	19	17.5																		
19.	Expression using multiplication: writing by using sign and symbols. Q. Write by using sign. 12 times of y	18	19																		
20.	Division of an algebraic expression by 1. Q. Divide: i) $2x + 3y$ by 1 ii) x by 1	17	20.5																		
21.	Multiplying positive integer by negative integer or negative integer by positive integer. Q. Multiply the first integer by the second integer. i) $3, (-3)$ ii) $(-6), 3$	17	20.5																		
22.	If equal quantities are added to equal quantities, the sums will also be equal.	16	23																		
23.	Expression of polynomial as rational expression. Q. Express $11-x$ as a rational expression. [A: $(x-1)/1, (22-2x)/2$ etc]	16	23																		
24.	Factorization of binomial using the commutative law and distributive law of multiplication (when common factor is monomial positive). Q. Factorize: $8 + 4x$	16	23																		
25.	Expression using addition: writing by using sign and symbols. Q. Write by using sign. Sum of x and y	15	26																		
26.	The quantities which are equal to a particular quantity are themselves equal. If $a = b$ and $b = c$, then $a = b = c$.	15	26																		
27.	Integers as rational numbers. Q. Write the following integers as rational numbers with denominator 1. $9, -13$ (A : $9=9/1, -13=-13/1$)	15	26																		
28.	If x, m and n be any algebraic symbols representing number, then $x^m \times x^n = x^{m+n}$ Q. Multiply: $x \times x^2$	14	28																		
29.	Division of a monomial by a monomial (when not exactly divisible). Q. Divide: $3y^3$ by y^2x	13	29																		
30.	Rearrangement of term for factorization. Q. Find the rearrangement of term for factorization. $ab + ka + kb + kc$ (A: $ab+bc+ka+kc$)	12	30																		
31.	Addition of two positive integers. Q. Add: $+5, +7$	11	31																		
32.	Addition of integer and zero. Q. Add: $+5, 0$	10	32.5																		
33.	Subtraction of zero from any algebraic expression is equivalent to the same algebraic expression. Q. Subtract: $2x-0$	10	32.5																		
34.	Subtraction of integer and zero. Q. Find: $0-5$	9	36																		
35.	Concept of opposite numbers. Q. What is the opposite number of $+5$?	9	36																		

36.	Subtraction of two positive integers when minuend > subtrahend. Q. Subtract: $(+12) - (+5)$	9	36
37.	Algebraic expression when all the terms are of positive sign: expression using subtraction: writing by using sign and symbols. Q. Rahim is 20 years old. What were his age y years before?	9	36
38.	Expression of one unknown in terms of others unknown of a linear equation in two unknowns. Q. Express x in terms of y : $2x + 3y = 5$	9	36
39.	Factorization using formula of differences of two squares: simple factorization (direct form). Q. Factorize: $25a^2 - 49b^2$	8	39.5
40.	Division of a polynomial by a polynomial: long division method. Q. Divide: $a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$ by $a + b + c$	8	39.5
41.	Repeated factorization after necessary simplification: Involving any two of the following forms: i. taking out common factor. ii. $a^2 + 2ab + b^2 = (a + b)^2$ iii. $a^2 - 2ab + b^2 = (a - b)^2$ iv. $a^2 - b^2 = (a + b)(a - b)$ Q. Factorize: $ax^2 - a^3$	7	41.5
42.	Equations of x -axis and y -axis. Q. What is the equation of x -axis?	7	41.5
43.	Factorization of binomial using the commutative law and distributive law of multiplication: when common factor binomial positive. Q. Factorize: $4(2+x) + x(2+x)$	6	45.5
44.	Subtraction of two positive integers when minuend < subtrahend. Q. Subtract: $(+5) - (+12)$	6	45.5
45.	If the dividend and divisor are of opposite sign then the quotient is negative. Q. Find: $-16 \div 4$	6	45.5
46.	The rational expression 0 (that is, the zero polynomial) is such that $r(x) + 0 = r(x) = 0 + r(x)$ for any rational expression.	6	45.5
47.	Every rational expression $\frac{p(x)}{q(x)}$ has an additive inverse $-\frac{p(x)}{q(x)}$ such that $\frac{p(x)}{q(x)} + \frac{-p(x)}{q(x)} = 0 = \frac{-p(x)}{q(x)} + \frac{p(x)}{q(x)}$.	6	45.5
48.	All the rational expressions whose expressions in lowest term are same are called equivalent rational expression. Q. Write two equivalent rational expressions of $-\frac{a}{a+1}$.	6	45.5
49.	Factorization of binomial using the commutative law and distributive law of multiplication: when common factor monomial negative. Q. Factorize: $-8 + 4x$	5	51.5
50.	Factorization using $a^2 + 2ab + b^2 = (a + b)^2$. Q. Factorize: $m^2 + 14m + 49$	5	51.5
51.	Multiplying negative integer by negative integer. Q. Multiply the first integer by the second. i) $(-3), (-6)$	5	51.5
52.	Solving simultaneous linear equations in two unknowns: method of substitution: simple sum (having numeral co-efficient of the simultaneous equations). Q. Solve using method of substitution: $5x - 3y = 9$ $2x + 5y = 16$	5	51.5
53.	L.C.M. of more than one algebraic expression each having single term: when number of algebraic expression is two: without involving numeral co-efficient of the expression. Q. Find L.C.M: x^2y, y^2z	5	51.5
54.	L.C.M. of more than one algebraic expression having no common factor other than one. Q. Find L.C.M: $x - a, x - b$	5	51.5
55.	Standard form of rational number. A rational number $\frac{p}{q}$ is said to be in standard form if q is positive, and the integers p and q have no common divisor other than 1. Q. Express the rational number in standard form.	4	63.5

	$\frac{-26}{39}$		
56.	If both the dividend and divisor are of negative sign then the quotient is of the positive sign. Q. Find: $(-18) \div (-3)$	4	63.5
57.	If a is a non-zero integer, then $0 \div a = 0$ but $a \div 0$ is not meaningful. Q. Find: $0 \div (-3)$	4	63.5
58.	Finding $a + (-b)$ or $(-b) + a$ if $a = -b $: addition without number line. Q. Add: +3, -3	4	63.5
59.	Solving simultaneous linear equations in two unknowns: method of elimination: simple sum (having numeral co-efficient of the simultaneous equations). Q. Solve using method of elimination: $5x - 3y = 9$ $2x + 5y = 16$	4	63.5
60.	If equal quantities are multiplied by equal quantities, the products will also be equal.	4	63.5
61.	Solving simultaneous linear equations in two unknowns: method of comparison: simple sum (having numeral co-efficient of the simultaneous equations). Q. Solve using method of comparison: $x + y = 6$, $x - y = 2$	4	63.5
62.	Solving simultaneous linear equations in two unknowns: method of cross multiplication: simple sum (having numeral co-efficient of the simultaneous equations). Q. Solve using cross multiplication: $5x - 3y = 9$, $2x + 5y = 16$	4	63.5
63.	Formation of linear equation in two unknowns. Q. Express the following statement in the form of equation. The cost of two tables and three chairs together is Rs. 900. Express the statement in a linear equation of two unknowns.	4	63.5
64.	Graph of linear equation in two unknowns: solution and graph: finding solution of a linear equation in two unknowns. Q. Find four different solution of the equation $x + 2y = 3$.	4	63.5
65.	Graph of linear equation in two unknowns: solution and graph: writing the different solution in tabular form. Q. Writing the different solutions in tabular form. $x = 1, y = 1$; $x = 3, y = 0$; $x = 5, y = 1.5$; $x = 0, y = 1.5$	4	63.5
66.	Plotting the point when the abscissa and ordinate of the point are given. Q. Plot the point on a graph paper. (2, -5)	4	63.5
67.	Plotting two points and their joining. Q. Plot the following points on a graph paper and join them by a straight line. (3,4) (10,12)	4	63.5
68.	Plotting more than two points and their joining. Q. Plot the three points (1, 2), (-1, 1), (11, 7) on a graph paper and verify that lie on the same line.	4	63.5
69.	Finding the product of two rational expressions when at least one of the numerator or denominator of the two rational expressions is not monomial. Q. Find the product of the pairs of rational expressions. $\frac{5x + 2}{5x - 3}$, $\frac{x + 6}{x + 2}$	4	63.5
70.	Drawing a graph of a linear equation in two unknowns. Q. Draw a graph of the following linear equation. $2x - 5y = 10$	4	63.5
71.	Division of a binomial by a monomial: dividing each term by separation method. Q. Divide: $3y^3 + 15y^2$ by $3y^2x$	4	63.5
72.	If x, m and n be any algebraic symbols representing number then $(x^m)^n = x^{m \times n} = x^{mn}$	4	63.5

concepts which have different frequency of use and rank order. The elementary concept sl. nos. 8 & 9, each of their frequency is 34 and rank order 8.5, elementary concept nos. from 10 to 12, each of their frequency is 33 and rank order is 11, concept nos. 17 & 18, each of their frequency of use is 19 and rank order is 17.5, concept nos. 20 & 21, each of their frequency of use is 17 and rank order is 20.5, concept nos. 22-24, each of their frequency is 16 and rank order is 23, concept nos. 25-27, each of their frequency is 15 and rank order is 26, concept nos. 32 & 33, each of their frequency is 10 and rank order is 32.5, concept nos. 34-38, each of their frequency is 9 and rank order is 36, concept nos. 39 & 40, each of their frequency is 8 and rank order is 39.5, concept nos. 41 & 42, each of their frequency is 7 and rank order is 41.5, concept nos. 43-48, each of their frequency is 6 and rank order is 45.5, concept nos. 49-54, each of their frequency is 5 and rank order is 51.5, concept nos. 55-72, each of their frequency is 4 and rank order is 63.5.

6. Conclusion

- (i). This methodology helps to identify the elementary concepts of algebra and their rank orders which are used to solve the mathematical problems of different Board examinations.
- (ii). Table-1 reveals the rank orders of identified top seventy two elementary concepts of algebra which were used to solve the algebraic group problems and arithmetical problems may be solved by algebraic method in two years mathematics question papers of S.C.E. Examination (1999 & 2000) have been determined and presented. It may be said that a large number of elementary concepts that they have same importance according to their rank orders.
- (iii). This study will help to identify the weakness of the learners after taking an investigation, those learners who are going to appear the S.C.E. Examination.
- (iv). The teachers as well as learners will also be benefited in several ways like to diagnose the weakness of the learners, to find the difficult order of these concepts, to emphasize on the learning of these concepts at appropriate grade levels etc. after knowing these elementary concepts.
- (v). Learners should be properly acquainted with these elementary concepts at the appropriate level which will help to perform better at the 10th standard final examination of Visva-Bharati.

7. Further Study

- (i). To identify the order of difficulty of the top seventy two elementary concepts of algebra among the 10th standard learners.
- (ii). To diagnose the weakness of the learners in the top seventy two elementary concepts of algebra.

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