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Fuzzy Basis

Research Article

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Abstract: We introduce fuzzy basis and strong fuzzy basis.

Keywords: Fuzzy basis, Strong fuzzy basis.

1. Introduction

In metric space and topological space the concept of base(basis) plays an important role. In the year 1965 Lotfi A.Zadeh [2] introduced the concept of fuzzy sets and fuzzy logic.In the year 1968 C.L.Chang [1] introduced Fuzzy topological spaces. In the year 2014 We [3] introduced fuzzy sequences in a metric space.In the same year we [6] introduced fuzzy subsequences and limit points. In the same year we [4, 5] defined fuzzy net and fuzzy filter.

In this paper we introduce fuzzy basis and strong fuzzy basis.

2. Fuzzy Basis

Now we recall the definition base for a topology in a topological space. Let X be a non empty set. $\mathbf{B} \subset P(X)$ is called a base if (1). $\cup\{B/B \in \mathbf{B}\} = X$. (2). $U, V \in \mathbf{B}$ and $x \in U \cap V$ implies there exists $W \in \mathbf{B}$ such that $x \in W \subset U \cap V$.

Let T be the collection of all union of finite number of elements of **B**. Then T is a topology and **B** is a base for the topology.

Now we fuzzify the concept.

Definition 2.1 (Fuzzy Basis)**.** *Let X be a non empty set. A function $f : P(X) \to [0, 1]$ is called a fuzzy base if*

(1). $\cup\{B/f(B) = 1\} = X$.

(2). For each $\alpha \in (0, 1]$. $f(U) \geq \alpha$, $f(V) \geq \alpha$ and $x \in U \cap V$ implies there exists W with $f(W) \geq \alpha$ and $x \in W \subset U \cap V$.

Example 2.2. *$X = \{a, b, c, d\}$, $f : P(X) \to [0, 1]$ as $f(X) = 1, f\{a, b, c\} = 1, f\{b, c, d\} = 1, f\{b\} = 1, f\{c\} = 1, f\{a, b, d\} = 0.6, f\{a, b, c\} = 0.6, f\{a\} = 0.6, f\{d\} = 0.6$ and $f(A) = 0$ for all other $A \subset X$, f is a fuzzy basis. Take $\alpha = 0.8$. Take $U = \{a, b, c\}, V = \{b, c, d\}$, $f(U) \geq \alpha$, $f(V) \geq \alpha$, $b \in U \cap V$.*

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Take $W = \{b\}$*.* $b \in W \subset U \cap V$ *and* $f(W) = f\{b\} = 1 \geq \alpha$*. Take* $\alpha = 0.6$*. Take* $U = \{a, b, d\}, V = \{a, b, c\}$*.* $f(U) \geq \alpha$*,* $f(V) \geq \alpha$ *and* $a \in U \cap V$*. Take* $W = \{a\}$*. Then* $f(W) \geq \alpha$ *and* $a \in W \subset U \cap V$*.* $f(U) \geq \alpha$*,* $f(V) \geq \alpha$ *and* $b \in U \cap V$*. Take* $= \{b\}$*. Then* $f(W) \geq \alpha$ *and* $b \in W \subset U \cap V$*.*

We can verify all other possibilities. Hence f is a fuzzy basis.

Example 2.3. $X = \{a, b, c\}$*. Define* $f : P(X) \to [0, 1]$ *as* $f\{a, b, c\} = 1, f\{a, b\} = 0.6, f\{b, c\} = 0.6$ *and* $f(A) = 0$ *for all other cases. Take* $\alpha = 0.6$*. Take* $U = \{a, b\}, V = \{b, c\}$*,* $f(U) \geq \alpha$*,* $f(V) \geq \alpha$*,* $b \in U \cap V$*. The only* W *with* $b \in W \subset U \cap V$ *is* $\{b\}$ *and* $f\{b\} = 0$*. Hence there exists no* W *with* $f(W) \geq \alpha$ *and* $b \in W \subset U \cap V$*. Hence* f *is not a fuzzy basis.*

Example 2.4. $X = \{a, b, c\}$*. Define* $f : P(X) \to [0, 1]$ *as* $f\{a, b\} = 1, f\{b\} = 1$ *and* $f(A) = 0.5$ *for all other* A*. Now* $\cup\{B/f(B) = 1\} = \{a, b\} \cup \{b\} = \{a, b\} \neq X$*. Hence* f *is not a fuzzy basis.*

We can give another definition of fuzzy basis. The second condition is modified.

Definition 2.5 (Strong fuzzy basis)**.** *Let* X *be a non empty set. A function* $f : P(X) \to [0, 1]$ *is called a strong fuzzy basis if*

(1). $\cup\{B/f(B) = 1\} = X$*.*

(2). $f(U \cap V) \geq \min\{f(U), f(V)\}$ *for* $U, V \subset X$ *with* $U \cap V \neq \phi$*.*

Example 2.6. $X = \{a, b, c, d\}$*. Define* $f : P(X) \to [0, 1]$ *as* $f(X) = 1$*,* $f\{a, b, c\} = 0.5$*,* $f\{a, b, d\} = 0.6$*,* $f\{a, b\} = 0.5$*,* $f(A) = 0$ *for all other* A*.*

(1). $\cup\{B/f(B) = 1\} = X$*.*

(2). Take $U = \{a, b, c\}$*,* $V = \{a, b, d\}$*,* $U \cap V = \{a, b\}$*,* $f(U \cap V) = 0.5$*,* $\min\{f(U), f(V)\} = \min\{0.5, 0.6\} = 0.5$*. Now* $f(U \cap V) \geq \min\{f(U), f(V)\}$*.* f *is a strong fuzzy basis.*

Theorem 2.7. *Every strong fuzzy basis is a fuzzy basis.*

Proof. Let X be a non empty set. Let $f : P(X) \to (0, 1]$ be a strong fuzzy basis. Then (1). $\cup\{B/f(B) = 1\} = X$. (2). Take U,V with $U \cap V \neq \phi$. Then $f(U \cap V) \geq \min\{f(U), f(V)\}$.
Claim : f is a fuzzy basis. (1). $\cup\{B/f(B) = 1\} = X$. (2). Let $\alpha \in (0, 1]$. Let $f(U) \geq \alpha$, $f(V) \geq \alpha$ and $x \in U \cap V$. Now take $W = U \cap V$. Now $x \in W \subset U \cap V$. Also $f(W) = f(U \cap V) \geq \min\{f(U), f(V)\} \geq \alpha$. So $f(W) \geq \alpha$. Hence there exists W such that $x \in W \subset U \cap V$ and $f(W) \geq \alpha$. Hence f is a fuzzy basis. □

Result 2.8. *Converse is not true. A fuzzy basis need not be a strong fuzzy basis.*

Example 2.9. $X = \{a, b, c, d\}$*. Define:* $f : P(X) \to [0, 1]$ *as* $f\{a, b, c, d\} = 1$*,* $f\{a, b, c\} = 0.6$*,* $f\{b, c, d\} = 0.6$*,* $f\{b\} = 0.6$*,* $f\{c\} = 0.6$*,* $f(A) = 0$ *for all other* A*. Now condition 1 is satisfied. Condition 2 is satisfied. Hence* f *is a fuzzy basis.*
$U = \{a,b,c\}$ *,* $V = \{b,c,d\}$*,* $U \cap V = \{b, c\} f(U \cap V) = f\{b, c\} = 0$*.* $\min\{f(U), f(V)\} = min\{0.6, 0.6\} = 0.6 f(U \cap V)$ *is not greater than or equal to min* $\{f(U), f(V)\}$*. Hence* f *is not a strong fuzzy basis.*

Theorem 2.10. *Every crisp basis induces a fuzzy basis.*

Proof. Let X be a non empty set. Let **B** be a crisp basis. Define $f : P(X) \to [0, 1]$ as $f(A) = 1$ if $A \in \mathbf{B}$ and $f(A) = 0$ if $A \notin \mathbf{B}$

(1). $\cup\{B/f(B) = 1\} = \cup\{B/B \in \mathbf{B}\} = X$, by definition of crisp basis.

(2). Take $\alpha \in (0,1]$. Let $f(U) \geq \alpha$, $f(V) \geq \alpha$ and $x \in U \cap V$. $f(U) \geq \alpha$ and $f(V) \geq \alpha$ implies $f(U) = 1$ and $f(V) = 1$. This implies that $U, V \in \mathbf{B}$. Since $\mathbf{B}$ is a crisp basis $\exists\, W \in \mathbf{B}$ $x \in W \subset U \cap V$. Since $W \in \mathbf{B}$, $f(W) = 1$ and hence $f(W) \geq \alpha$.

Hence $\exists\, W$ such that $x \in W \subset U \cap V$ and $f(W) \geq \alpha$. Hence f is a fuzzy basis. Hence every crisp basis induces a fuzzy basis. □

Result 2.11. *The fuzzy basis induced by a crisp basis need not a strong fuzzy basis.*

Example 2.12. *Consider R^2 with usual metric topology. Let $\boldsymbol{B}$ = Set of all open circles. The induced fuzzy basis is $f : P(X) \to [0,1]$ defined as $f(A) = 1$ if A is a circle and $f(A) = 0$ if A is not a circle. Clearly f is a fuzzy basis. Now we claim that f is not strong. Let U and V be two circles where $U \cap V \neq \phi$. Let U and V be not concentric. Then $U \cap V$ is not a circle. Hence $f(U \cap V) = 0$. Since U and V are circles $f(U) = 1$ and $f(V) = 1$. Hence $\min\{f(U), f(V)\} = 1$. Now $f(U \cap V)$ is not greater than or equal to $\min\{f(U), f(V)\}$. Hence f is not strong. Therefore the fuzzy basis induced by a crisp basis need not be strong.*
Now we try to identify crisp basis giving strong fuzzy basis. For this we improve the crisp basis.

Definition 2.13 (Strong crisp basis)**.** *Let X be a non empty set. $\boldsymbol{B} \subset P(X)$ is called a strong crisp basis if*

(1). $\cup\{B/B \in \boldsymbol{B}\} = X$.

(2). $U, V \in \boldsymbol{B}$, $U \cap V \neq \phi \Rightarrow U \cap V \in \boldsymbol{B}$.

Example 2.14.

(1). $X = \{a, b, c\}$, $\boldsymbol{B} = \{\{a, b\}, \{b, c\}, \{b\}\}$

(2). $X = \{a, b, c, d\}$, $\boldsymbol{B} = \{\{a, b, c\}, \{b, c, d\}, \{b, c\}\}$.

Theorem 2.15. *The fuzzy basis induced by a strong crisp basis is a strong fuzzy basis.*

Proof. Let X be a non empty set. Let $\mathbf{B}$ be a strong crisp basis. Let f be the induced fuzzy basis. We claim that f is a strong fuzzy basis. Take $U, V \in P(X)$ where $U \cap V \neq \phi$. If $f(U) = 0$ or $f(V) = 0$ then whatever be the value of $f(U \cap V)$, we have $f(U \cap V) \geq \min\{f(U), f(V)\}$. If $f(U) = 1$ and $f(V) = 1$ then $U, V \in B$. Since B is a strong basis, $U \cap V \in \mathbf{B}$. Hence $f(U \cap V) = 1$. Hence $f(U \cap V) \geq \min\{f(U), f(V)\}$. Hence f is a strong fuzzy basis. □

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