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Decompositions of $\breve{g}$-Continuity in Topological Spaces

Research Article

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Abstract: The aim of this paper is to give decompositions of a weaker form of continuity, namely $\breve{g}$-continuity, by providing the concepts of $\breve{g}_t$-sets, $\breve{g}_{\alpha^\star}$-sets, $\breve{g}_t$-continuity and $\breve{g}_{\alpha^\star}$-continuity.

MSC: 54C10, 54C08, 54C05.

Keywords: $\breve{g}$-closed set, $\breve{g}_\alpha$-closed set, $\breve{g}_p$-closed set, $\breve{g}_t$-set, $\breve{g}_{\alpha^\star}$-set, $\breve{g}_t$-continuity and $\breve{g}_{\alpha^\star}$-continuity.

1. Introduction

Various interesting problems arise when one considers continuity and generalized continuity. In recent years, the decomposition of continuity is one of the main interest for general topologists. In 1961, Levine [6] obtained a decomposition of continuity which was later improved by Rose [20]. Tong [21] decomposed continuity into α-continuity and A-continuity and showed that his decomposition is independent of Levine's. Ganster and Reilly [4] have improved Tong's decomposition result and provided a decomposition of A-continuity. Przemski [13] obtained some decomposition of continuity. Hatir et al. [5] also obtained a decomposition of continuity. Recently, Dontchev and Przemski [3] and Noiri et al. [12] obtained some more decompositions of continuity. In this paper, we obtain decompositions of $\breve{g}$-continuity in topological spaces using $\breve{g}_p$-continuity [19], $\breve{g}_\alpha$-continuity [16], $\breve{g}_t$-continuity and $\breve{g}_{\alpha^\star}$-continuity.

2. Preliminaries

Throughout this paper (X, τ) and (Y, σ) (or X and Y) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), cl(A) and int(A) denote the closure of A and the interior of A respectively. We recall the following definitions which are useful in the sequel.

Definition 2.1. *A subset A of a space (X, τ) is called:*

(1). semi-open set [7] if $A \subseteq cl(int(A))$;

(2). preopen set [8] if $A \subseteq int(cl(A))$;

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(3). α-open set [10] if A$\subseteq$ int(cl(int(A))).

The complements of the above mentioned open sets are called their respective closed sets. The preclosure [11] (resp. semi-closure [2], α-closure [10]) of a subset A of X, pcl(A) (resp. scl(A), αcl(A)), is defined to be the intersection of all preclosed (resp. semi-closed, α-closed) sets of (X, τ) containing A. It is known that pcl(A) (resp. scl(A), αcl(A)) is a preclosed (resp. semi-closed, α-closed) set. For any subset A of an arbitrarily chosen topological space, the preinterior [11] (resp. semi-interior [2], α-interior [10]) of A, denoted by pint(A) (resp. sint(A), αint(A)), is defined to be the union of all preopen (semi-open, α-open) sets of (X, τ) contained in A.

Definition 2.2. *A subset A of a space (X, τ) is called:*

(1). a semi-generalized closed (briefly, sg-closed) set [1] if scl(A)$\subseteq$U whenever A$\subseteq$U and U is semi-open in X. The complement of sg-closed set is called sg-open set;

(2). a $\ddot{g}$-closed set [14] if cl(A)$\subseteq$U whenever A$\subseteq$U and U is sg-open in X. The complement of $\ddot{g}$-closed set is called $\ddot{g}$-open set;

(3). a A-closed set [15] if cl(A)$\subseteq$U whenever A$\subseteq$U and U is $\ddot{g}$-open in X. The complement of A-closed set is called A-open set;

(4). a B-closed set [15] if scl(A)$\subseteq$U whenever A$\subseteq$U and U is A-open in X. The complement of B-closed set is called B-open set;

(5). a $\breve{g}$-closed set [15] if cl(A)$\subseteq$U whenever A$\subseteq$U and U is B-open in X. The complement of $\breve{g}$-closed set is called g-open set;

(6). an $\breve{g}_\alpha$-closed set [17] if αcl(A)$\subseteq$U whenever A$\subseteq$U and U is B-open in X. The complement of $\breve{g}_\alpha$-closed set is called $\breve{g}_\alpha$-open set;

(7). a $\breve{g}_p$-closed set [18] if pcl(A)$\subseteq$U whenever A$\subseteq$U and U is B-open in X. The complement of $\breve{g}_p$-closed set is called $\breve{g}_p$-open set.

Definition 2.3. *A subset A of a space (X, τ) is called:*

(1). t-set [21] if int(A) = int(cl(A));

(2). $\alpha^\star$-set [5] if int(A)= int(cl(int(A))).

Remark 2.4.

(1). Every closed set is $\breve{g}$-closed but not conversely [15].

(2). Every $\breve{g}$-closed set is $\breve{g}_\alpha$-closed but not conversely [17].

(3). Every α-closed set is $\breve{g}_\alpha$-closed but not conversely [17].

(4). Every $\breve{g}_\alpha$-closed set is $\breve{g}_p$-closed but not conversely [18].

(5). The concepts of α-closed sets and $\breve{g}$-closed sets are independent [15].

Remark 2.5 ([5]). *(1). Every t-set is an $\alpha^\star$-set but not conversely.*

(2). An open set need not be an $\alpha^\star$-set.

(3). The union of two $\alpha^\star$-sets need not be an $\alpha^\star$-set.

(4). Arbitrary intersection of $\alpha^\star$-sets is an $\alpha^\star$-set.

Definition 2.6. *A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be*

(1). α-continuous [9] if for each open set V of Y, $f^{-1}(V)$ is α-open in X.

(2). $\breve{g}$-continuous [16] if for each open set V of Y, $f^{-1}(V)$ is $\breve{g}$-open in X.

(3). $\breve{g}_\alpha$-continuous [16](resp. $\breve{g}_p$-continuous [19]) if for each open set V of Y, $f^{-1}(V)$ is $\breve{g}_\alpha$-open (resp. $\breve{g}_p$-open) set in X.

3. On $\breve{g}_t$-sets and $\breve{g}_{\alpha^\star}$-sets

Definition 3.1. *A subset S of a space (X, τ) is called*

1. *an $\breve{g}_t$-set if $S = M \cap N$, where M is $\breve{g}$-open in X and N is a t-set in X.*
2. *an $\breve{g}_{\alpha^\star}$-set if $S = M \cap N$, where M is $\breve{g}$-open in X and N is an $\alpha^\star$-set in X.*

The family of all $\breve{g}_t$-sets (resp. $\breve{g}_{\alpha^\star}$-sets) in a space (X, τ) is denoted by $\breve{g}_t$(X, σ)(resp. $\breve{g}_{\alpha^\star}$(X, τ)).

Proposition 3.2. *Let S be a subset of X. Then*

(1). if S is a t-set, then $S \in \breve{g}_t(X, \tau)$.

(2). if S is an $\alpha^\star$-set, then $S \in \breve{g}_{\alpha^\star}(X, \tau)$.

(3). if S is an $\breve{g}$-open set in X, then $S \in \breve{g}_t(X, \tau)$ and $S \in \breve{g}_{\alpha^\star}(X, \tau)$.

Proposition 3.3. *In a space X, every $\breve{g}_t$-set is an $\breve{g}_{\alpha^\star}$-set but not conversely.*

Example 3.4. *Let $X = \{a, b, c\}$ with $\tau = \{\phi, \{a, b\}, X\}$. Then $\{a, c\}$ is $\breve{g}_{\alpha^\star}$-set but it is not an $\breve{g}_t$-set.*

Remark 3.5. *The following examples show that*

(1). the converse of Proposition 3.2 need not be true.

(2). the concepts of $\breve{g}_t$-sets and $\breve{g}_p$-open sets are independent.

(3). the concepts of $\breve{g}_{\alpha^\star}$-sets and $\breve{g}_\alpha$-open sets are independent.

Example 3.6. *Let $X = \{a, b, c\}$ with $\tau = \{\phi, \{a, b\}, X\}$. Then $\{a\}$ is $\breve{g}_t$-set but not a t-set and the set $\{a, b\}$ is an $\breve{g}_{\alpha^\star}$-set but it is not an $\alpha^\star$-set.*

Example 3.7. *Let $X = \{a, b, c\}$ with $\tau = \{\phi, \{a, b\}, X\}$. Then $\{c\}$ is both $\breve{g}_t$-set and $\breve{g}_{\alpha^\star}$-set but it is not an $\breve{g}$-open set.*

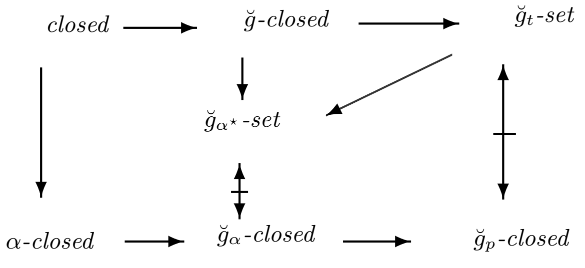
Example 3.8. *Let $X = \{a, b, c\}$ with $\tau = \{\phi, \{a, b\}, X\}$. Then the set $\{c\}$ is an $\breve{g}_t$-set but not a $\breve{g}_p$-open set whereas the set $\{b, c\}$ is a $\breve{g}_p$-open set but not an $\breve{g}_t$-set.*

Example 3.9. *Let $X = \{a, b, c\}$ with $\tau = \{\phi, \{a, b\}, X\}$. Then $\{b, c\}$ is an $\breve{g}_{\alpha^\star}$-set but not an $\breve{g}_\alpha$-open set.*

Example 3.10. *Let $X = \{a, b, c\}$ with $\tau = \{\phi, \{a\}, X\}$. Then the set $\{a, b\}$ is an $\breve{g}_\alpha$-open set but not an $\breve{g}_{\alpha^\star}$-set.*

Example 3.11. *Let $X = \{a, b, c\}$ with $\tau = \{\phi, \{a, b\}, X\}$. Then $\{a\}$ is $\breve{g}_{\alpha^\star}$-set and $\breve{g}_t$-set but it is not an $\breve{g}$-closed.*

Remark 3.12. *From the above discussions, we have the following diagram of implications where $A \longrightarrow B$ (resp. $A \nleftrightarrow B$) represents A implies B but not conversely (resp. A and B are independent of each other).*



Remark 3.13.

(1). The union of two $\breve{g}_t$-sets need not be an $\breve{g}_t$-set.

(2). The union of two $\breve{g}_{\alpha^\star}$-sets need not be an $\breve{g}_{\alpha^\star}$-set.

In Example 3.4, {b} and {c} are $\breve{g}_t$-sets but {b}∪{c} = {b, c} is not an $\breve{g}_t$-set. In Example 3.10, {a} and {b} are $\breve{g}_{\alpha^\star}$-sets but {a}∪{b} = {a, b} is not an $\breve{g}_{\alpha^\star}$-set.

Remark 3.14.

(1). The intersection of any numbers of $\breve{g}_t$-sets belongs to $\breve{g}_t(X, \tau)$.

(2). The intersection of any numbers of $\breve{g}_{\alpha^\star}$-sets belongs to $\breve{g}_{\alpha^\star}(X, \tau)$.

Lemma 3.15.

(1). A subset S of (X, τ) is $\breve{g}$-open [15] if and only if F⊆int(S) whenever F⊆S and F is B-closed in X.

(2). A subset S of (X, τ) is $\breve{g}_\alpha$-open [17] if and only if F⊆αint(S) whenever F⊆S and F is B-closed in X.

(3). A subset S of (X, τ) is $\breve{g}_p$-open [18] if and only if F⊆pint(S)whenever F⊆S and F is B-closed in X.

Theorem 3.16. *A subset S is $\breve{g}$-open in (X, τ) if and only if it is both $\breve{g}_\alpha$-open and $\breve{g}_{\alpha^\star}$-set in (X, τ).*

Proof. Necessity. The proof is obvious.

Sufficiency. Let S be both $\breve{g}_\alpha$-open set and $\breve{g}_{\alpha^\star}$-set. Since S is an $\breve{g}_{\alpha^\star}$-set, S = A∩B, where A is $\breve{g}$-open and B is an $\alpha^\star$-set. Assume that F⊆S, where F is B-closed in X. Since A is $\breve{g}$-open, by Lemma 3.15 (1), F⊆int(A). Since S is $\breve{g}_{\alpha^\star}$-open in X, by Lemma 3.15 (2), F⊆αint(S) = S∩int(cl(int(S))) = (A∩B)∩int(cl(int(A∩B)))⊆A∩B∩ int(cl(int(A)))∩ int(cl(int(B))) = A∩B∩int(cl(int(A)))∩int(B)⊆int(B). Therefore, we obtain F⊆int(B) and hence F⊆int(A)∩int(B) = int(S). Hence S is $\breve{g}$-open, by Lemma 3.14 (1). □

Theorem 3.17. *A subset S is $\breve{g}$-open in (X, τ) if and only if it is both $\breve{g}_p$-open and $\breve{g}_t$-set in (X, τ).*

Proof. Similar to Theorem 3.16. □

4. Decompositions of $\breve{g}$-continuity

Definition 4.1. *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be*

(1). $\breve{g}$-continuous if for each open set V of Y, $f^{-1}(V) \in \breve{g}_t(X, \tau)$.

(2). $\breve{g}_{\alpha^\star}$-continuous if for each open set V of Y, $f^{-1}(V) \in \breve{g}_{\alpha^\star}(X, \tau)$.

Proposition 4.2. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following implications hold:*

(1). $\breve{g}$-continuity $\Rightarrow$ $\breve{g}_t$-continuity.

(2). $\breve{g}$-continuity $\Rightarrow$ $\breve{g}_{\alpha^\star}$-continuity.

(3). $\breve{g}$-continuity $\Rightarrow$ $\breve{g}_\alpha$-continuity $\Rightarrow$ $\breve{g}_p$-continuity.

The reverse implications in Proposition 4.2 are not true as shown in the following examples.

Example 4.3. *Let $X = Y = \{a, b, c\}$ with $\tau = \{\phi, \{a, b\}, X\}$ and $\sigma = \{\phi, \{c\}, Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity function. Then f is $\breve{g}_t$-continuous. However, f is neither $\breve{g}$-continuous nor $\breve{g}_p$-continuous.*

Example 4.4. *Let $X = Y = \{a, b, c\}$ with $\tau = \{\phi, \{a, b\}, X\}$ and $\sigma = \{\phi, \{c\}, Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity function. Then f is $\breve{g}_{\alpha^\star}$-continuous. However, f is neither $\breve{g}$-continuous nor $\breve{g}_\alpha$-continuous.*

The following Example 4.5 and Example 4.4 show that the concepts of $\breve{g}_{\alpha^\star}$-continuity and $\breve{g}_\alpha$-continuity are independent.

Example 4.5. *Let $X = Y = \{a, b, c\}$ with $\tau = \{\phi, \{a\}, X\}$ and $\sigma = \{\phi, \{a, b\}, Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity function. Then f is $\breve{g}_\alpha$-continuous but it is not $\breve{g}_{\alpha^\star}$-continuous.*

Examples 4.3 and 4.6 show that $\breve{g}_t$-continuity and $\breve{g}_p$-continuity are independent.

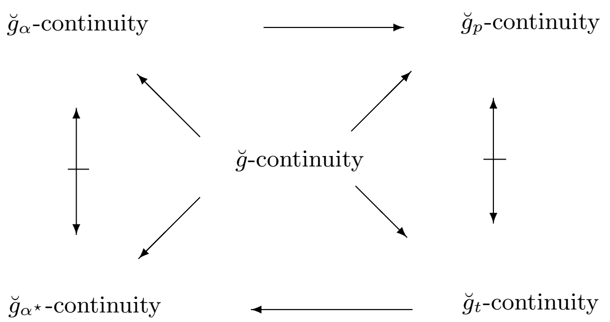
Example 4.6. *Let $X = Y = \{a, b, c\}$ with $\tau = \{\phi, \{a\}, X\}$ and $\sigma = \{\phi, \{a, b\}, Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity function. Then f is $\breve{g}_p$-continuous but it is not $\breve{g}_t$-continuous.*

Example 4.7. *Let $X = Y = \{a, b, c\}$ with $\tau = \{\phi, \{a\}, X\}$ and $\sigma = \{\phi, \{a, b\}, Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity function. Then f is $\breve{g}_\alpha$-continuous and $\breve{g}_p$-continuous but it is not $\breve{g}$-continuous.*

Example 4.8. *Let $X = Y = \{a, b, c\}$ with $\tau = \{\phi, \{a, b\}, X\}$ and $\sigma = \{\phi, \{b\}, \{a, b\}, \{b, c\}, Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity function. Then f is $\breve{g}_p$-continuous but it is not $\breve{g}_\alpha$-continuous.*

Example 4.9. *Let $X = Y = \{a, b, c\}$ with $\tau = \{\phi, \{a, b\}, X\}$ and $\sigma = \{\phi, \{b\}, \{a, b\}, \{b, c\}, Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity function. Then f is $\breve{g}_{\alpha^\star}$-continuous but it is not $\breve{g}_t$-continuous.*

Remark 4.10. *From the above discussions, we have the following diagram of implications where $A \to B$ (resp. $A \not\leftrightarrow B$) represents A implies B but not conversely (resp. A and B are independent of each other).*



Theorem 4.11. *A function $f : (X, \tau) \to (Y, \sigma)$ is $\breve{g}$-continuous if and only if it is both $\breve{g}_\alpha$-continuous and $\breve{g}_{\alpha^\star}$-continuous.*

Proof. The proof follows immediately from Theorem 3.16. □

Theorem 4.12. *A function $f : (X, \tau) \to (Y, \sigma)$ is $\breve{g}$-continuous if and only if it is both $\breve{g}_p$-continuous and $\breve{g}_t$-continuous.*

Proof. The proof follows immediately from Theorem 3.17. □

References

[1] P.Bhattacharya and B.K.Lahiri, *Semi-generalized closed sets in topology*, Indian J. Math., 29(3)(1987), 375-382.

[2] S.G.Crossley and S.K.Hildebrand, *Semi-closure*, Texas J. Sci., 22(1971), 99-112.

[3] J.Dontchev and M.Przemski, *On the various decompositions of continuous and weakly continuous functions*, Acta Math. Hungar.,71(1-2)(1996), 109-120.

[4] M.Ganster and I.L.Reilly, *A decomposition of continuity*, Acta Math. Hungar., 56(1990), 299-301.

[5] E.Hatir, T.Noiri and S.Yuksel, *A decomposition of continuity*, Acta Math. Hungar., 70(1-2)(1996), 145-150.

[6] N.Levine, *A decomposition of continuity in topological spaces*, Amer. Math. Monthly, 68(1961), 44-46.

[7] N.Levine, *Semi-open sets and semi-continuity in topological spaces*, Amer. Math. Monthly, 70(1963), 36-41.

[8] A.S.Mashhour, M.E.Abd El-Monsef and S.N.El-Deeb, *On precontinuous and weak pre continuous mappings*, Proc. Math. and Phys. Soc. Egypt, 53(1982), 47-53.

[9] A.S.Mashhour, I.A.Hasanein and S.N.El-Deeb, *α-continuous and α-open mappings*, Acta Math. Hungar., 41(1983), 213-218.

[10] O.Njastad, *On some classes of nearly open sets*, Pacific J. Math., 15(1965), 961-970.

[11] T.Noiri, H.Maki and J.Umehara, *Generalized preclosed functions*, Mem. Fac. Sci. Kochi Univ. Math., 19(1998), 13-20.

[12] T.Noiri, M.Rajamani and P.Sundaram, *A decomposition of a weaker form of continuity*, Acta Math. Hungar., 93(1-2)(2001), 109-114.

[13] M.Przemski, *A decomposition of continuity and α-continuity*, Acta Math. Hungar., 61(1991), 93-98.

[14] O.Ravi and S.Ganesan, *$\ddot{g}$-closed sets in topology*, International Journal of computer science and emerging technologies, 2(3)(2011), 330-337.

[15] O.Ravi, S.Padmasekaran, S.Usharani and I.Rajasekaran, *$\breve{g}$-closed sets in topological spaces* (submitted).

[16] O.Ravi, S.Padmasekaran, S.Usharani and I.Rajasekaran, *$\breve{g}_\alpha$-continuous functions in topological spaces* (submitted).

[17] O.Ravi, S.Padmasekaran, S.Usharani and I.Rajasekaran, *$\breve{g}_\alpha$-closed sets in topological spaces* (submitted).

[18] O.Ravi, S.Padmasekaran, S.Usharani and I.Rajasekaran, *On $\breve{g}_p$-closed sets in topological spaces* (submitted).

[19] O.Ravi, S.Padmasekaran, S.Usharani and I.Rajasekaran, *On $\breve{g}_p$-continuous functions in topological spaces* (submitted).

[20] D.Rose, *On Levine's decomposition of continuity*, Canad. Math. Bull., 21(1978), 477-481.

[21] J.Tong, *A decomposition of continuity*, Acta Math. Hungar., 48(1986), 11-15.

[22] J.Tong, *On decomposition of continuity in topological spaces*, Acta Math. Hungar., 54(1989), 51-55.