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Fuzzy Map

Research Article

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Abstract: We introduce the new type of map called fuzzy map.

Keywords: Fuzzy map, fuzzy sets, fuzzy logic.

1. Introduction

In metric space and topological space the concept of functions plays an important role. In the year 1965 Lotfi A.Zadeh [2] introduced the concept of fuzzy sets and fuzzy logic. In the year 1968 C.L.Chang [1] introduced Fuzzy topological spaces. In the year 2014 We [3] introduced fuzzy sequences in a metric space.In the same year we [6] introduced fuzzy subsequences and limit points. In the same year we [4, 5] defined fuzzy net and fuzzy filter.

In this paper we introduce a new function called fuzzy map.

2. Fuzzy Map

Let X and Y be two non empty sets. A map or function is a rule which assigns to each element of X an unique element in Y. This is a crisp concept. Now we try to fuzzify this concept.

Definition 2.1. *Let X and Y be two non empty sets. A function $F : X \times Y \to [0, 1]$ is said to be a fuzzy map from X to Y. i.e., A fuzzy map from X to Y is a fuzzy set on $X \times Y$.*

Example 2.2. *$F : [0, 1] \times (-1/2, 1/2) \to [0, 1]$ defined as $F(a, b) = \mod (a + b)/2$ for all $(a, b) \in [0, 1] \times (-1/2, 1/2)$. Here F is a fuzzy map.*

$F : Q \times Q \to [0, 1]$ defined as $F(a, b) = a + b$ if $a + b \in [0, 1]$ and 1 otherwise. Here F is a fuzzy map.

Theorem 2.3. *Every crisp map is a fuzzy map.*

Proof. Let $f : X \times Y$ be a crisp map. This function f can also be defined as a function $F : X \times Y \to [0, 1]$ such that $F(x, y) = 1$ if $f(x) = y$ and 0 otherwise. Here F is a fuzzy map. Therefore every crisp map is a fuzzy map. □

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Result 2.4. *Converse is not true.*

Example 2.5. *Consider* $F : [0, 1] \times [0, 1] \to [0, 1]$ *defined as* $F(a, b) = (a + b)$ *if* $a + b \in [0, 1]$ *and 0 otherwise. Here F is a fuzzy map, but not a crisp map.*

Definition 2.6. *Let F be a fuzzy map from X to Y. Let* $x_0 \in X$*, then the image of* x_0 *denoted as* $F(x_0)$ *is defined as* $F(x_0) = \{y \in Y/F(x_0, y) = 1\}$.

Example 2.7. *Let* $X = \{1, 2, 3\}$ *and* $Y = \{4, 5, 6\}$*. Consider the fuzzy map F defined as* $F(a, b) = 1$ *if* $a + b \geq 7$ *and 0 otherwise.* $F(1) = \{6\}$, $F(2) = \{5, 6\}$, $F\{3\} = \{4, 5, 6\}$.

Definition 2.8. *Let F be a fuzzy map from X to Y. Let A be a non empty subset of X. Image of the set A denoted as F(A) is defined as* $F(A) = \{y \in Y/F(x_0, y) = 1 \text{ for some } x_0 \in A\}$.

Example 2.9. *Let* $X = \{1, 2, 3\}$ *and* $Y = \{4, 5, 6\}$*. Consider the fuzzy map F defined as* $F(a, b) = 1$ *if* $a + b \geq 7$ *and 0 otherwise. Let* $A = \{1, 2\}$, $F(A) = \{5, 6\}$.

Theorem 2.10. *Let F be a fuzzy map from X to Y. Let A and B be two non empty sets of X. Then* $F(A \cup B) = F(A) \cup F(B)$.

Proof. Clearly $F(A \cup B) \subset Y$ and $F(A) \cup F(B) \subset Y$. Let $y \in F(A \cup B)$. Then $\exists\ x \in A \cup B$ such that $F(x, y) = 1$. Since $x \in A \cup B$, $x \in A$ or $x \in B$. Therefore $y \in F(A)$ or $y \in F(B)$. Hence $y \in F(A) \cup F(B)$. Therefore $F(A \cup B) \subset F(A) \cup F(B)$. Now, Let $y \in F(A) \cup F(B)$ then $y \in F(A)$ or $y \in F(B)$. If $y \in F(A)$ then $\exists\ x_1 \in A$ such that $F(x_1, y) = 1$. Here $x_1 \in A$. Therefore $x_1 \in A \cup B$ such that $F(x_1, y) = 1$. Hence $y \in F(A \cup B)$. If $y \in F(B)$ then $\exists\ x_2 \in B$ such that $F(x_2, y) = 1$. Here $x_2 \in B$. $x_2 \in A \cup B$ such that $F(x_2, y) = 1$. Hence $y \in F(A \cup B)$. Therefore $F(A) \cup F(B) \subset F(A \cup B)$. Hence $F(A \cup B) = F(A) \cup F(B)$. □

Theorem 2.11. *Let F be a fuzzy map from X to Y. Let A and B be two non empty sets of X. Then* $F(A \cap B) \subset F(A) \cap F(B)$.

Proof. Let $y \in F(A \cap B)$. Then $\exists\ x \in A \cap B$ such that $F(x, y) = 1$. Since $x \in A \cap B$, $x \in A$ and $x \in B$. Hence $y \in F(A)$ and $y \in F(B)$. Therefore $y \in F(A) \cap F(B)$. Hence $y \in F(A \cap B) \Rightarrow y \in F(A) \cap F(B)$. Therefore $F(A \cap B) \subset F(A) \cap F(B)$. □

Result 2.12. *The equality does not hold.*

Example 2.13. *Let* $X = \{1, 2, \ldots 10\}$ *and* $Y = \{0, 1, 2, \ldots 10\}$*. Let F be a fuzzy map from X to Y such that* $F(2, 5) = 1$ *and* $F(8, 5) = 1$ *and when* $(a, b) \neq (2, 5)$ *and* $(a, b) \neq (8, 5)$, $F(a, b) = (a + b)/5$ *if* $(a + b)/5 \in [0, 1]$ *and 0 otherwise. Let* $A = \{1, 2, 3, 4, 5\}$ *and* $B = \{5, 6, 7, 8, 9, 10\}$*. Here* $A \cap B = \{5\}$ *and so* $F(A \cap B) = \{0\}$, $F(A) = \{0, 1, 2, 4, 5\}$ *and* $F(B) = \{0, 5\}$*. Here* $F(A) \cap F(B) = \{0, 5\}$*. Hence* $F(A \cap B) \neq F(A) \cap F(B)$.

Definition 2.14. *Let F be a fuzzy map from X to Y. Let* $y \in Y$*. Then the inverse image of y in F denoted as* $F^{-1}(y)$ *is defined as* $F^{-1}(y) = \{x \in X/F(x, y) = 1\}$.

Example 2.15. *Let* $X = \{1, 2, 3\}$ *and* $Y = \{4, 5, 6\}$*. Consider the fuzzy map F defined as* $F(a, b) = 1$ *if* $a + b \geq 7$ *and 0 otherwise. Let* $B = \{5, 6\}$, $F^{-1}(B) = \{1, 2, 3\}$.

Definition 2.16. *Let F be a fuzzy function from X to Y. Let B be a non empty subset of Y. Then* $F^{-1}(B) = \{x \in X/F(x, y) = 1 \text{ for some } y \in B\}$.

Example 2.17. *Let* $X = \{1, 2, 3\}$ *and* $Y = \{4, 5, 6\}$*. Consider the fuzzy map F defined as* $F(a, b) = 1$ *if* $a + b \geq 7$ *and 0 otherwise. Let* $B = \{5, 6\}$, $F^{-1}(B) = \{1, 2, 3\}$.

Theorem 2.18. *Let F be a fuzzy function from X to Y. Let B_1 and B_2 be two non empty subsets of Y. Then $F^{-1}(B_1 \cup B_2) = F^{-1}(B_1) \cup F^{-1}(B_2)$.*

Proof. Clearly $F^{-1}(B_1 \cup B_2) \subset X$ and $F^{-1}(B_1) \cup F^{-1}(B_2) \subset X$. Let $x \in F^{-1}(B_1 \cup B_2)$. Then $\exists\ y \in B_1 \cup B_2$ such that $F(x,y) = 1$. i.e., $y \in B_1$ or $y \in B_2$ such that $F(x,y) = 1$. Therefore $x \in F^{-1}(B_1)$ or $x \in F^{-1}(B_2)$. Therefore $x \in F^{-1}(B_1) \cup F^{-1}(B_2)$. Hence $x \in F^{-1}(B_1 \cup B_2) \Rightarrow x \in F^{-1}(B_1) \cup F^{-1}(B_2)$. Therefore $F^{-1}(B_1 \cup B_2) \subset F^{-1}(B_1) \cup F^{-1}(B_2)$. Now, Let $x \in F^{-1}(B_1) \cup F^{-1}(B_2)$. Then $x \in F^{-1}(B_1)$ or $x \in F^{-1}(B_2)$. If $x \in F^{-1}(B_1)$ then $\exists\ y_1 \in B_1$ such that $F(x,y_1) = 1$. Now $y_1 \in B_1 \cup B_2$ such that $F(x,y_1) = 1$. Therefore $x \in F^{-1}(B_1 \cup B_2)$. If $x \in F^{-1}(B_2)$ then $y_2 \in B_2$ such that $F(x,y_2) = 1$. Now $y_2 \in B_1 \cup B_2$ such that $F(x,y_2) = 1$. Therefore $x \in F^{-1}(B_1 \cup B_2)$. Hence $x \in F^{-1}(B_1) \cup F^{-1}(B_2) \Rightarrow x \in F^{-1}(B_1 \cup B_2)$. Therefore $F^{-1}(B_1) \cup F^{-1}(B_2) \subset F^{-1}(B_1 \cup B_2)$. Hence $F^{-1}(B_1 \cup B_2) = F^{-1}(B_1) \cup F^{-1}(B_2)$. □

Theorem 2.19. *Let F be a fuzzy map from X to Y. Let B_1 and B_2 be two non empty subsets of Y. Then $F^{-1}(B_1 \cap B_2) \subset F^{-1}(B_1) \cap F^{-1}(B_2)$.*

Proof. Clearly $F^{-1}(B_1 \cap B_2) \subset X$ and $F^{-1}(B_1) \cap F^{-1}(B_2) \subset X$. Let $x \in F^{-1}(B_1 \cap B_2)$. Then $\exists\ y \in B_1 \cap B_2$ such that $F(x,y) = 1$. Since $y \in B_1 \cap B_2$, $y \in B_1$ and $y \in B_2$. Therefore $x \in F^{-1}(B_1)$ and $x \in F^{-1}(B_2)$. Hence $x \in F^{-1}(B_1) \cap F^{-1}(B_2)$. Now $x \in F^{-1}(B_1 \cap B_2) \Rightarrow x \in F^{-1}(B_1) \cap F^{-1}(B_2)$. Hence $F^{-1}(B_1 \cap B_2) \subset F^{-1}(B_1) \cap F^{-1}(B_2)$. □

Result 2.20. *The equality does not hold in the above theorem*

Example 2.21. *Let $X = \{1, \ldots 10\}$ and $Y = \{1, \ldots 10\}$. Let F be a fuzzy map from X to Y such that $F(5,2) = 1$ and $F(5,8) = 1$ and when $(a,b) \neq (5,2)$ and $(a,b) \neq (5,8)$, $F(a,b) = (a+b)/5$ if $(a+b)/5 \in [0,1]$ and 0 otherwise. Let $B_1 = \{1,2,3,4,5\}$ and $B_2 = \{5,6,7,8,9,10\}$, $B_1 \cap B_2 = \{5\}$. $F^{-1}(B_1 \cap B_2) = \{0\}$, $F^{-1}(B_1) = \{5,4,3,2,1,0\}$ and $F^{-1}(B_2) = \{0,5\}$, $F^{-1}(B_1) \cap F^{-1}(B_2) = \{0,5\}$. Therefore $F^{-1}(B_1 \cap B_2) \neq F^{-1}(B_1) \cap F^{-1}(B_2)$.*

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