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On $(1,2)^{\star}$-$g^{\star}$-continuous functions

Research Article

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Abstract: The aim of this paper is to study $(1,2)^{\star}$-$g^{\star}$-continuous functions in bitopological spaces and investigate their relations with various generalized $(1,2)^{\star}$-continuous functions. We also discuss some properties of $(1,2)^{\star}$-$g^{\star}$-continuous functions. We also introduce $(1,2)^{\star}$-$g^{\star}$-irresolute functions and study some of its applications. Finally using $(1,2)^{\star}$-$g^{\star}$-continuous functions we obtain a decomposition of $(1,2)^{\star}$-continuity.

MSC: 54E55.

Keywords: $(1,2)^{\star}$-T$_{1/2}$-space, $(1,2)^{\star}$-α-space, $(1,2)^{\star}$-αg-irresolute function, $(1,2)^{\star}$-$g^{\star}$-continuous function, $(1,2)^{\star}$-$g^{\star}_{\alpha}$-continuous function, $(1,2)^{\star}$-$g$sp-continuous function.

1. Introduction

Several authors [1, 12, 13, 39] working in the field of general topology have shown more interest in studying the concepts of generalizations of continuous functions. A weak form of continuous functions called g-continuous functions were introduced by Balachandran et al [6]. Recently Sheik John [36] have introduced and studied another form of generalized continuous functions called ω-continuous functions respectively.

In this paper, we first study $(1,2)^{\star}$-$g^{\star}$-continuous functions and investigate their relations with various generalized $(1,2)^{\star}$-continuous functions. We also discuss some properties of $(1,2)^{\star}$-$g^{\star}$-continuous functions. We also introduce $(1,2)^{\star}$-$g^{\star}$-irresolute functions and study some of its applications. Finally using $(1,2)^{\star}$-$g^{\star}$-continuous functions we obtain a decomposition of $(1,2)^{\star}$-continuity.

2. Preliminaries

Throughout this paper, X, Y and Z denote bitopological spaces (X, τ_1, τ_2), (Y, σ_1, σ_2) and (Z, η_1, η_2) respectively.

Definition 2.1. *Let A be a subset of a bitopological space X. Then A is called $\tau_{1,2}$-open [18] if $A = P \cup Q$, for some $P \in \tau_1$ and $Q \in \tau_2$. The complement of $\tau_{1,2}$-open set is called $\tau_{1,2}$-closed. The family of all $\tau_{1,2}$-open (resp. $\tau_{1,2}$-closed) sets of X is denoted by $(1,2)^{\star}$-$O(X)$ (resp. $(1,2)^{\star}$-$C(X)$).*

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Definition 2.2 ([18]). *Let A be a subset of a bitopological space X. Then*

1. *the $\tau_{1,2}$-interior of A, denoted by $\tau_{1,2}$-int(A), is defined by $\cup$ { U : $U \subseteq A$ and U is $\tau_{1,2}$-open};*

2. *the $\tau_{1,2}$-closure of A, denoted by $\tau_{1,2}$-cl(A), is defined by $\cap$ { U : $A \subseteq U$ and U is $\tau_{1,2}$-closed}.*

Remark 2.3 ([18]). *Notice that $\tau_{1,2}$-open subsets of X need not necessarily form a topology.*

Definition 2.4. *Let A be a subset of a bitopological space X. Then A is called*

1. *$(1,2)^{\star}$-semi-open set [18] if $A \subseteq \tau_{1,2}$-cl($\tau_{1,2}$-int(A)).*

2. *$(1,2)^{\star}$-preopen set [18] if $A \subseteq \tau_{1,2}$-int($\tau_{1,2}$-cl(A)).*

3. *$(1,2)^{\star}$-α-open set [18] if $A \subseteq \tau_{1,2}$-int($\tau_{1,2}$-cl($\tau_{1,2}$-int(A))).*

4. *$(1,2)^{\star}$-β-open set [31] if $A \subseteq \tau_{1,2}$-cl($\tau_{1,2}$-int($\tau_{1,2}$-cl(A))).*

5. *$(1,2)^{\star}$-regular open set [29] if $A = \tau_{1,2}$-int($\tau_{1,2}$-cl(A)).*

The complements of the above mentioned open sets are called their respective closed sets.

The $(1,2)^{\star}$-preclosure [26] (resp. $(1,2)^{\star}$-semi-closure [26], $(1,2)^{\star}$-α-closure [26], $(1,2)^{\star}$-β-closure [31]) of a subset A of X, denoted by $(1,2)^{\star}$-pcl(A) (resp. $(1,2)^{\star}$-scl(A), $(1,2)^{\star}$-αcl(A), $(1,2)^{\star}$-βcl(A)) is defined to be the intersection of all $(1,2)^{\star}$-preclosed (resp. $(1,2)^{\star}$-semi-closed, $(1,2)^{\star}$-α-closed, $(1,2)^{\star}$-β-closed) sets of X containing A. It is known that $(1,2)^{\star}$-pcl(A) (resp. $(1,2)^{\star}$-scl(A), $(1,2)^{\star}$-αcl(A), $(1,2)^{\star}$-βcl(A)) is a $(1,2)^{\star}$-preclosed (resp. $(1,2)^{\star}$-semi-closed, $(1,2)^{\star}$-α-closed, $(1,2)^{\star}$-β-closed) set.

Definition 2.5. *Let A be a subset of a bitopological space X. Then A is called*

1. *a $(1,2)^{\star}$-generalized closed (briefly, $(1,2)^{\star}$-g-closed) set [34] if $\tau_{1,2}$-cl$(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_{1,2}$-open in X. The complement of $(1,2)^{\star}$-g-closed set is called $(1,2)^{\star}$-g-open set.*

2. *a $(1,2)^{\star}$-semi-generalized closed (briefly, $(1,2)^{\star}$-sg-closed) set [3] if $(1,2)^{\star}$-scl$(A) \subseteq U$ whenever $A \subseteq U$ and U is $(1,2)^{\star}$-semi-open in X. The complement of $(1,2)^{\star}$-sg-closed set is called $(1,2)^{\star}$-sg-open set.*

3. *a $(1,2)^{\star}$-generalized semi-closed (briefly, $(1,2)^{\star}$-gs-closed) set [3] if $(1,2)^{\star}$-scl$(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_{1,2}$-open in X. The complement of $(1,2)^{\star}$-gs-closed set is called $(1,2)^{\star}$-gs-open set.*

4. *an $(1,2)^{\star}$-α-generalized closed (briefly, $(1,2)^{\star}$-αg-closed) set [15] if $(1,2)^{\star}$-αcl$(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_{1,2}$-open in X. The complement of $(1,2)^{\star}$-αg-closed set is called $(1,2)^{\star}$-αg-open set.*

5. *a $(1,2)^{\star}$-generalized semi-preclosed (briefly, $(1,2)^{\star}$-gsp-closed) set [15] if $(1,2)^{\star}$-βcl$(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_{1,2}$-open in X. The complement of $(1,2)^{\star}$-gsp-closed set is called $(1,2)^{\star}$-gsp-open set.*

6. *$(1,2)^{\star}$-$g^{\star}$-closed set [28] if $\tau_{1,2}$-cl$(A) \subseteq U$ whenever $A \subseteq U$ and U is $(1,2)^{\star}$-g-open in X. The complement of $(1,2)^{\star}$-$g^{\star}$-closed set is called $(1,2)^{\star}$-$g^{\star}$-open.*

7. *$(1,2)^{\star}$-$g^{\star}_{\alpha}$-closed set [28] if $(1,2)^{\star}$-αcl$(A) \subseteq U$ whenever $A \subseteq U$ and U is $(1,2)^{\star}$-g-open in X. The complement of $(1,2)^{\star}$-$g^{\star}_{\alpha}$-closed set is called $(1,2)^{\star}$-$g^{\star}_{\alpha}$-open.*

Remark 2.6. *The collection of all $(1,2)^\star$-$g^\star$-closed (resp. $(1,2)^\star$-$g^\star_\alpha$-closed, $(1,2)^\star$-g-closed, $(1,2)^\star$-gs-closed, $(1,2)^\star$-gsp-closed, $(1,2)^\star$-αg-closed, $(1,2)^\star$-sg-closed, $(1,2)^\star$-α-closed, $(1,2)^\star$-semi-closed) sets is denoted by $(1,2)^\star$-$G^\star C(X)$ (resp. $(1,2)^\star$-$G^\star_\alpha C(X)$, $(1,2)^\star$-GC(X), $(1,2)^\star$-GSC(X), $(1,2)^\star$-GSPC(X), $(1,2)^\star$-$\alpha GC(X)$, $(1,2)^\star$-SGC(X), $(1,2)^\star$-$\alpha C(X)$, $(1,2)^\star$-SC(X)).*

We denote the power set of X by P(X).

Definition 2.7. *A bitopological space X is called:*

1. *$(1,2)^\star$-$T_{1/2}$-space [32] if every $(1,2)^\star$-g-closed set in it is $\tau_{1,2}$-closed.*
2. *$(1,2)^\star$-$T^\star_g$-space [28] if every $(1,2)^\star$-$g^\star$-closed set in it is $\tau_{1,2}$-closed.*
3. *$(1,2)^\star$-${}_\alpha T_b$-space [31] if every $(1,2)^\star$-αg-closed set in it is $\tau_{1,2}$-closed.*

Remark 2.8. *In a bitopological space, the following holds:*

1. *Every $\tau_{1,2}$-closed set is $(1,2)^\star$-$g^\star$-closed set but not conversely.*
2. *Every $(1,2)^\star$-$g^\star$-closed set is $(1,2)^\star$-$g^\star_\alpha$-closed set but not conversely.*
3. *Every $(1,2)^\star$-$g^\star$-closed set is $(1,2)^\star$-g-closed set but not conversely.*
4. *Every $(1,2)^\star$-$g^\star$-closed set is $(1,2)^\star$-αg-closed set but not conversely.*
5. *Every $(1,2)^\star$-$g^\star$-closed set is $(1,2)^\star$-gs-closed set but not conversely.*
6. *Every $(1,2)^\star$-$g^\star$-closed set is $(1,2)^\star$-gsp-closed set but not conversely.*

Definition 2.9. *A function $f: X \to Y$ is called:*

1. *$(1,2)^\star$-g-continuous [16] if $f^{-1}(V)$ is a $(1,2)^\star$-g-closed set in X for every $\sigma_{1,2}$-closed set V of Y.*
2. *$(1,2)^\star$-αg-continuous [31] if $f^{-1}(V)$ is an $(1,2)^\star$-αg-closed set in X for every $\sigma_{1,2}$-closed set V of Y.*
3. *$(1,2)^\star$-gs-continuous [31] if $f^{-1}(V)$ is a $(1,2)^\star$-gs-closed set in X for every $\sigma_{1,2}$-closed set V of Y.*
4. *$(1,2)^\star$-gsp-continuous [31] if $f^{-1}(V)$ is a $(1,2)^\star$-gsp-closed set in X for every $\sigma_{1,2}$-closed set V of Y.*
5. *$(1,2)^\star$-sg-continuous [34] if $f^{-1}(V)$ is a $(1,2)^\star$-sg-closed set in X for every $\sigma_{1,2}$-closed set V of Y.*
6. *$(1,2)^\star$-semi-continuous [26] if $f^{-1}(V)$ is a $(1,2)^\star$-semi-open set in X for every $\sigma_{1,2}$-open set V of Y.*
7. *$(1,2)^\star$-α-continuous [26] if $f^{-1}(V)$ is an $(1,2)^\star$-α-closed set in X for every $\sigma_{1,2}$-closed set V of Y.*

Definition 2.10. *A function $f: X \to Y$ is called:*

1. *$(1,2)^\star$-g-irresolute [16] if the inverse image of every $(1,2)^\star$-g-closed set in Y is $(1,2)^\star$-g-closed in X.*
2. *$(1,2)^\star$-sg-irresolute [31] if the inverse image of every $(1,2)^\star$-sg-closed (resp. $(1,2)^\star$-sg-open) set in Y is $(1,2)^\star$-sg-closed (resp. $(1,2)^\star$-sg-open) in X.*

3. $(1,2)^\star$-$g^\star$-continuous Functions

We introduce the following definitions:

Definition 3.1. *A function $f: X \to Y$ is called:*

1. *$(1,2)^\star$-$g^\star$-continuous if the inverse image of every $\sigma_{1,2}$-closed set in Y is $(1,2)^\star$-$g^\star$-closed set in X.*
2. *$(1,2)^\star$-$g^\star_\alpha$-continuous if $f^{-1}(V)$ is an $(1,2)^\star$-$g^\star_\alpha$-closed set in X for every $\sigma_{1,2}$-closed set V of Y.*
3. *strongly $(1,2)^\star$-$g^\star$-continuous if the inverse image of every $(1,2)^\star$-$g^\star$-open set in Y is $\tau_{1,2}$-open in X.*

Example 3.2. *Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{c\}, X\}$ and $\tau_2 = \{\phi, \{a, c\}, X\}$. Then the sets in $\{\phi, \{c\}, \{a, c\}, X\}$ are called $\tau_{1,2}$-open and the sets in $\{\phi, \{b\}, \{a, b\}, X\}$ are called $\tau_{1,2}$-closed. Let $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, Y\}$ and $\sigma_2 = \{\phi, \{a\}, Y\}$. Then the sets in $\{\phi, \{a\}, Y\}$ are called $\sigma_{1,2}$-open and the sets in $\{\phi, \{b, c\}, Y\}$ are called $\sigma_{1,2}$-closed. We have $(1,2)^\star$-$G^\star C(X) = \{\phi, \{b\}, \{a, b\}, \{b, c\}, X\}$. Let $f: X \to Y$ be the identity function. Then f is $(1,2)^\star$-$g^\star$-continuous.*

Proposition 3.3. *Every $(1,2)^\star$-continuous function is $(1,2)^\star$-$g^\star$-continuous but not conversely.*

Example 3.4. *The function f in Example 3.2 is $(1,2)^\star$-$g^\star$-continuous but not $(1,2)^\star$-continuous, since $f^{-1}(\{b, c\}) = \{b, c\}$ is not $\tau_{1,2}$-closed in X.*

Proposition 3.5. *Every $(1,2)^\star$-$g^\star$-continuous function is $(1,2)^\star$-$g^\star_\alpha$-continuous but not conversely.*

Example 3.6. *Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X\}$ and $\tau_2 = \{\phi, \{b\}, X\}$. Then the sets in $\{\phi, \{b\}, X\}$ are called $\tau_{1,2}$-open and the sets in $\{\phi, \{a, c\}, X\}$ are called $\tau_{1,2}$-closed. Let $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, Y\}$ and $\sigma_2 = \{\phi, \{b, c\}, Y\}$. Then the sets in $\{\phi, \{b, c\}, Y\}$ are called $\sigma_{1,2}$-open and the sets in $\{\phi, \{a\}, Y\}$ are called $\sigma_{1,2}$-closed. We have $(1,2)^\star$-$G^\star C(X) = \{\phi, \{a, c\}, X\}$ and $(1,2)^\star$-$G^\star_\alpha C(X) = \{\phi, \{a\}, \{c\}, \{a, c\}, X\}$. Let $f: X \to Y$ be the identity function. Then f is $(1,2)^\star$-$g^\star_\alpha$-continuous but not $(1,2)^\star$-$g^\star$-continuous, since $f^{-1}(\{a\}) = \{a\}$ is not $(1,2)^\star$-$g^\star$-closed in X.*

Proposition 3.7. *Every $(1,2)^\star$-$g^\star$-continuous function is $(1,2)^\star$-g-continuous but not conversely.*

Example 3.8. *Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, X\}$ and $\tau_2 = \{\phi, \{b, c\}, X\}$. Then the sets in $\{\phi, \{a\}, \{b, c\}, X\}$ are called $\tau_{1,2}$-open and the sets in $\{\phi, \{a\}, \{b, c\}, X\}$ are called $\tau_{1,2}$-closed. Let $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, Y\}$ and $\sigma_2 = \{\phi, \{c\}, Y\}$. Then the sets in $\{\phi, \{c\}, Y\}$ are called $\sigma_{1,2}$-open and the sets in $\{\phi, \{a, b\}, Y\}$ are called $\sigma_{1,2}$-closed. We have $(1,2)^\star$-$G^\star C(X) = \{\phi, \{a\}, \{b, c\}, X\}$ and $(1,2)^\star$-$GC(X) = P(X)$. Let $f: X \to Y$ be the identity function. Then f is $(1,2)^\star$-g-continuous but not $(1,2)^\star$-$g^\star$-continuous, since $f^{-1}(\{a, b\}) = \{a, b\}$ is not $(1,2)^\star$-$g^\star$-closed in X.*

Proposition 3.9. *Every $(1,2)^\star$-$g^\star$-continuous function is $(1,2)^\star$-αg-continuous but not conversely.*

Example 3.10. *Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, X\}$ and $\tau_2 = \{\phi, \{b, c\}, X\}$. Then the sets in $\{\phi, \{a\}, \{b, c\}, X\}$ are called $\tau_{1,2}$-open and the sets in $\{\phi, \{a\}, \{b, c\}, X\}$ are called $\tau_{1,2}$-closed. Let $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, Y\}$ and $\sigma_2 = \{\phi, \{b\}, Y\}$. Then the sets in $\{\phi, \{b\}, Y\}$ are called $\sigma_{1,2}$-open and the sets in $\{\phi, \{a, c\}, Y\}$ are called $\sigma_{1,2}$-closed. We have $(1,2)^\star$-$G^\star C(X) = \{\phi, \{a\}, \{b, c\}, X\}$ and $(1,2)^\star$-$\alpha GC(X) = P(X)$. Let $f: X \to Y$ be the identity function. Then f is $(1,2)^\star$-αg-continuous but not $(1,2)^\star$-$g^\star$-continuous, since $f^{-1}(\{a, c\}) = \{a, c\}$ is not $(1,2)^\star$-$g^\star$-closed in X.*

Proposition 3.11. *Every $(1,2)^\star$-$g^\star$-continuous function is $(1,2)^\star$-gs-continuous but not conversely.*

Example 3.12. *Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X\}$ and $\tau_2 = \{\phi, \{a\}, X\}$. Then the sets in $\{\phi, \{a\}, X\}$ are called $\tau_{1,2}$-open and the sets in $\{\phi, \{b, c\}, X\}$ are called $\tau_{1,2}$-closed. Let $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, Y\}$ and $\sigma_2 = \{\phi, \{a, b\}, Y\}$. Then the* *sets in $\{\phi, \{a, b\}, Y\}$ are called $\sigma_{1,2}$-open and the sets in $\{\phi, \{c\}, Y\}$ are called $\sigma_{1,2}$-closed. We have $(1,2)^\star$-$G^\star C(X) = \{\phi, \{b, c\}, X\}$ and $(1,2)^\star$-$GSC(X) = \{\phi, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$. Let $f : X \to Y$ be the identity function. Then f is $(1,2)^\star$-gs-continuous but not $(1,2)^\star$-$g^\star$-continuous, since $f^{-1}(\{c\}) = \{c\}$ is not $(1,2)^\star$-$g^\star$-closed in X.*

Proposition 3.13. *Every $(1,2)^\star$-$g^\star$-continuous function is $(1,2)^\star$-gsp-continuous but not conversely.*

Example 3.14. *Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X\}$ and $\tau_2 = \{\phi, \{b\}, X\}$. Then the sets in $\{\phi, \{b\}, X\}$ are called $\tau_{1,2}$-open and the sets in $\{\phi, \{a, c\}, X\}$ are called $\tau_{1,2}$-closed. Let $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, Y\}$ and $\sigma_2 = \{\phi, \{a, b\}, Y\}$. Then the sets in $\{\phi, \{a, b\}, Y\}$ are called $\sigma_{1,2}$-open and the sets in $\{\phi, \{c\}, Y\}$ are called $\sigma_{1,2}$-closed. We have $(1,2)^\star$-$G^\star C(X) = \{\phi, \{a, c\}, X\}$ and $(1,2)^\star$-$GSPC(X) = \{\phi, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$. Let $f : X \to Y$ be the identity function. Then f is $(1,2)^\star$-gsp-continuous but not $(1,2)^\star$-$g^\star$-continuous, since $f^{-1}(\{c\}) = \{c\}$ is not $(1,2)^\star$-$g^\star$-closed in X.*

Remark 3.15. *The following examples show that $(1,2)^\star$-$g^\star$-continuity is independent of $(1,2)^\star$-α-continuity and $(1,2)^\star$-semi-continuity.*

Example 3.16. *Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X\}$ and $\tau_2 = \{\phi, \{a, b\}, X\}$. Then the sets in $\{\phi, \{a, b\}, X\}$ are called $\tau_{1,2}$-open and the sets in $\{\phi, \{c\}, X\}$ are called $\tau_{1,2}$-closed. Let $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, Y\}$ and $\sigma_2 = \{\phi, \{a\}, Y\}$. Then the sets in $\{\phi, \{a\}, Y\}$ are called $\sigma_{1,2}$-open and the sets in $\{\phi, \{b, c\}, Y\}$ are called $\sigma_{1,2}$-closed. We have $(1,2)^\star$-$G^\star C(X) = \{\phi, \{c\}, \{a, c\}, \{b, c\}, X\}$ and $(1,2)^\star$-$\alpha C(X) = (1,2)^\star$-$SC(X) = \{\phi, \{c\}, X\}$. Let $f : X \to Y$ be the identity function. Then f is $(1,2)^\star$-$g^\star$-continuous but it is neither $(1,2)^\star$-α-continuous nor $(1,2)^\star$-semi-continuous, since $f^{-1}(\{b, c\}) = \{b, c\}$ is neither $(1,2)^\star$-α-closed nor $(1,2)^\star$-semi-closed in X.*

Example 3.17. *In Example 3.12, we have $(1,2)^\star$-$G^\star C(X) = \{\phi, \{b, c\}, X\}$ and $(1,2)^\star$-$\alpha C(X) = (1,2)^\star$-$SC(X) = \{\phi, \{b\}, \{c\}, \{b, c\}, X\}$. Let $f : X \to Y$ be the identity function. Then f is both $(1,2)^\star$-α-continuous and $(1,2)^\star$-semi-continuous but it is not $(1,2)^\star$-$g^\star$-continuous, since $f^{-1}(\{c\}) = \{c\}$ is not $(1,2)^\star$-$g^\star$-closed in X.*

Proposition 3.18. *A function $f : X \to Y$ is $(1,2)^\star$-$g^\star$-continuous if and only if $f^{-1}(U)$ is $(1,2)^\star$-$g^\star$-open in X for every $\sigma_{1,2}$-open set U in Y.*

Proof. Let f : X → Y be $(1,2)^\star$-$g^\star$-continuous and U be an $\sigma_{1,2}$-open set in Y. Then U^c is $\sigma_{1,2}$-closed in Y and since f is $(1,2)^\star$-$g^\star$-continuous, $f^{-1}(U^c)$ is $(1,2)^\star$-$g^\star$-closed in X. But $f^{-1}(U^c) = (f^{-1}(U))^c$ and so $f^{-1}(U)$ is $(1,2)^\star$-$g^\star$-open in X. Conversely, assume that $f^{-1}(U)$ is $(1,2)^\star$-$g^\star$-open in X for each $\sigma_{1,2}$-open set U in Y. Let F be a $\sigma_{1,2}$-closed set in Y. Then F^c is $\sigma_{1,2}$-open in Y and by assumption, $f^{-1}(F^c)$ is $(1,2)^\star$-$g^\star$-open in X. Since $f^{-1}(F^c) = (f^{-1}(F))^c$, we have $f^{-1}(F)$ is $(1,2)^\star$-$g^\star$-closed in X and so f is $(1,2)^\star$-$g^\star$-continuous. □

Remark 3.19. *The composition of two $(1,2)^\star$-$g^\star$-continuous functions need not be a $(1,2)^\star$-$g^\star$-continuous function as shown in the following example.*

Example 3.20. *Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, \{a, c\}, X\}$ and $\tau_2 = \{\phi, \{a, b\}, X\}$. Then the sets in $\{\phi, \{a\}, \{a, b\}, \{a, c\}, X\}$ are called $\tau_{1,2}$-open and the sets in $\{\phi, \{b\}, \{c\}, \{b, c\}, X\}$ are called $\tau_{1,2}$-closed. Let $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, Y\}$ and $\sigma_2 = \{\phi, \{a, b\}, Y\}$. Then the sets in $\{\phi, \{a, b\}, Y\}$ are called $\sigma_{1,2}$-open and the sets in $\{\phi, \{c\}, Y\}$ are called $\sigma_{1,2}$-closed. Let $Z = \{a, b, c\}$, $\eta_1 = \{\phi, Z\}$ and $\eta_2 = \{\phi, \{b\}, Z\}$. Then the sets in $\{\phi, \{b\}, Z\}$ are called $\eta_{1,2}$-open and the sets in $\{\phi, \{a, c\}, Z\}$ are called $\eta_{1,2}$-closed. Let $f : X \to Y$ and $g : Y \to Z$ be the identity functions. Then f and g are $(1,2)^\star$-$g^\star$-continuous but $g \circ f : X \to Z$ is not $(1,2)^\star$-$g^\star$-continuous, since for the set $V = \{a, c\}$ is $\eta_{1,2}$-closed in Z, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V)) = f^{-1}(g^{-1}(\{a, c\})) = f^{-1}(\{a, c\}) = \{a, c\}$ is not $(1,2)^\star$-$g^\star$-closed in X.*

Proposition 3.21. *Let X and Z be bitopological spaces and Y be a $(1,2)^\star$-$T_g^\star$-space. Then the composition $g \circ f : X \to Z$ of the $(1,2)^\star$-$g^\star$-continuous functions $f : X \to Y$ and $g : Y \to Z$ is $(1,2)^\star$-$g^\star$-continuous.*

Proof. Let F be any $\eta_{1,2}$-closed set of Z. Then g^{-1}(F) is $(1,2)^\star$-$g^\star$-closed in Y, since g is $(1,2)^\star$-$g^\star$-continuous. Since Y is a $(1,2)^\star$-$T^\star_g$-space, g^{-1}(F) is $\sigma_{1,2}$-closed in Y. Since f is $(1,2)^\star$-$g^\star$-continuous, $f^{-1}(g^{-1}(F))$ is $(1,2)^\star$-$g^\star$-closed in X. But $f^{-1}(g^{-1}(F)) = (g\ o\ f)^{-1}(F)$ and so g o f is $(1,2)^\star$-$g^\star$-continuous. □

Proposition 3.22. *Let X and Z be bitopological spaces and Y be a $(1,2)^\star$-$T_{1/2}$-space (resp. $(1,2)^\star$-T_b-space, $(1,2)^\star$-${}_\alpha T_b$-space). Then the composition $g\ o\ f : X \to Z$ of the $(1,2)^\star$-$g^\star$-continuous function $f : X \to Y$ and the $(1,2)^\star$-g-continuous (resp. $(1,2)^\star$-gs-continuous, $(1,2)^\star$-αg-continuous) function $g : Y \to Z$ is $(1,2)^\star$-$g^\star$-continuous.*

Proof. Similar to Proposition 3.21. □

Proposition 3.23. *If $f : X \to Y$ is $(1,2)^\star$-$g^\star$-continuous and $g : Y \to Z$ is $(1,2)^\star$-continuous, then their composition $g\ o\ f : X \to Z$ is $(1,2)^\star$-$g^\star$-continuous.*

Proof. Let F be any $\eta_{1,2}$-closed set in Z. Since g : Y → Z is $(1,2)^\star$-continuous, g^{-1}(F) is $\sigma_{1,2}$-closed in Y. Since f : X → Y is $(1,2)^\star$-$g^\star$-continuous, $f^{-1}(g^{-1}(F)) = (g\ o\ f)^{-1}(F)$ is $(1,2)^\star$-$g^\star$-closed in X and so g o f is $(1,2)^\star$-$g^\star$-continuous. □

Proposition 3.24. *Let A be $(1,2)^\star$-$g^\star$-closed in X. If $f : X \to Y$ is $(1,2)^\star$-g-irresolute and $(1,2)^\star$-closed, then f(A) is $(1,2)^\star$-$g^\star$-closed in Y.*

Proof. Let U be any $(1,2)^\star$-g-open in Y such that f(A) ⊆ U. Then A ⊆ f^{-1}(U) and by hypothesis, $\tau_{1,2}$-cl(A) ⊆ f^{-1}(U). Thus f($\tau_{1,2}$-cl(A)) ⊆ U and f($\tau_{1,2}$-cl(A)) is a $\sigma_{1,2}$-closed set. Now, $\sigma_{1,2}$-cl(f(A)) ⊆ $\sigma_{1,2}$-cl(f($\tau_{1,2}$-cl(A))) = f($\tau_{1,2}$-cl(A)) ⊆ U. i.e., $\sigma_{1,2}$-cl(f(A)) ⊆ U and so f(A) is $(1,2)^\star$-$g^\star$-closed. □

4. $(1,2)^\star$-$g^\star$-irresolute Functions

We introduce the following definition.

Definition 4.1. *A function $f : X \to Y$ is called an $(1,2)^\star$-$g^\star$-irresolute if the inverse image of every $(1,2)^\star$-$g^\star$-closed set in Y is $(1,2)^\star$-$g^\star$-closed in X.*

Remark 4.2. *The following examples show that the notions of $(1,2)^\star$-sg-irresolute functions and $(1,2)^\star$-$g^\star$-irresolute functions are independent.*

Example 4.3. *Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X\}$ and $\tau_2 = \{\phi, \{a, b\}, X\}$. Then the sets in $\{\phi, \{a, b\}, X\}$ are called $\tau_{1,2}$-open and the sets in $\{\phi, \{c\}, X\}$ are called $\tau_{1,2}$-closed. Let $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, \{a\}, \{a, b\}, Y\}$ and $\sigma_2 = \{\phi, \{b\}, Y\}$. Then the sets in $\{\phi, \{a\}, \{b\}, \{a, b\}, Y\}$ are called $\sigma_{1,2}$-open and the sets in $\{\phi, \{c\}, \{a, c\}, \{b, c\}, Y\}$ are called $\sigma_{1,2}$-closed. We have $(1,2)^\star$-$G^\star C(X) = \{\phi, \{c\}, \{a, c\}, \{b, c\}, X\}$, $SGC(X) = \{\phi, \{c\}, \{a, c\}, \{b, c\}, X\}$, $(1,2)^\star$-$G^\star C(Y) = \{\phi, \{c\}, \{a, c\}, \{b, c\}, Y\}$ and $(1,2)^\star$-$SGC(Y) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}, Y\}$. Let $f : X \to Y$ be the identity function. Then f is $(1,2)^\star$-$g^\star$-irresolute but it is not $(1,2)^\star$-sg-irresolute, since $f^{-1}(\{b\}) = \{b\}$ is not $(1,2)^\star$-sg-closed in X.*

Example 4.4. *Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, \{a, b\}, X\}$ and $\tau_2 = \{\phi, \{b\}, X\}$. Then the sets in $\{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ are called $\tau_{1,2}$-open and the sets in $\{\phi, \{c\}, \{a, c\}, \{b, c\}, X\}$ are called $\tau_{1,2}$-closed. Let $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, \{b\}, Y\}$ and $\sigma_2 = \{\phi, \{b, c\}, Y\}$. Then the sets in $\{\phi, \{b\}, \{b, c\}, Y\}$ are called $\sigma_{1,2}$-open and the sets in $\{\phi, \{a\}, \{a, c\}, Y\}$ are called $\sigma_{1,2}$-closed. We have $(1,2)^\star$-$G^\star C(X) = \{\phi, \{c\}, \{a, c\}, \{b, c\}, X\}$ and $(1,2)^\star$-$SGC(X) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}, X\}$, $(1,2)^\star$-$G^\star C(Y) = \{\phi, \{a\}, \{a, b\}, \{a, c\}, Y\}$ and $(1,2)^\star$-$SGC(Y) = \{\phi, \{a\}, \{c\}, \{a, c\}, Y\}$. Let $f : X \to Y$ be the identity function. Then f is $(1,2)^\star$-sg-irresolute but it is not $(1,2)^\star$-$g^\star$-irresolute, since $f^{-1}(\{a\}) = \{a\}$ is not $(1,2)^\star$-$g^\star$-closed in X.*

Proposition 4.5. *A function $f : X \to Y$ is $(1,2)^{\star}$-$g^{\star}$-irresolute if and only if the inverse of every $(1,2)^{\star}$-$g^{\star}$-open set in Y is $(1,2)^{\star}$-$g^{\star}$-open in X.*

Proof. Similar to Proposition 3.18. □

Proposition 4.6. *If a function $f : X \to Y$ is $(1,2)^{\star}$-$g^{\star}$-irresolute then it is $(1,2)^{\star}$-$g^{\star}$-continuous but not conversely.*

Example 4.7. *Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X\}$ and $\tau_2 = \{\phi, \{b\}, X\}$. Then the sets in $\{\phi, \{b\}, X\}$ are called $\tau_{1,2}$-open and the sets in $\{\phi, \{a, c\}, X\}$ are called $\tau_{1,2}$-closed. Let $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, Y\}$ and $\sigma_2 = \{\phi, \{a, b\}, Y\}$. Then the sets in $\{\phi, \{a, b\}, Y\}$ are called $\sigma_{1,2}$-open and the sets in $\{\phi, \{c\}, Y\}$ are called $\sigma_{1,2}$-closed. We have $(1,2)^{\star}$-$G^{\star}C(X)$ $= \{\phi, \{a, c\}, X\}$ and $(1,2)^{\star}$-$G^{\star}C(Y) = \{\phi, \{c\}, \{a, c\}, \{b, c\}, Y\}$. Let $f : X \to Y$ be the identity function. Then f is $(1,2)^{\star}$-$g^{\star}$-continuous but it is not $(1,2)^{\star}$-$g^{\star}$-irresolute, since $f^{-1}(\{a\}) = \{a\}$ is not $(1,2)^{\star}$-$g^{\star}$-open in X.*

Proposition 4.8. *Let X be any bitopological space, Y be a $(1,2)^{\star}$-$T_g^{\star}$-space and $f : X \to Y$ be a function. Then the following are equivalent:*

1. *f is $(1,2)^{\star}$-$g^{\star}$-irresolute.*
2. *f is $(1,2)^{\star}$-$g^{\star}$-continuous.*

Proof.

(1) ⇒ (2) Follows from Proposition 4.6.

(2) ⇒ (1) Let F be a $(1,2)^{\star}$-$g^{\star}$-closed set in Y. Since Y is a $(1,2)^{\star}$-$T_g^{\star}$-space, F is a $\sigma_{1,2}$-closed set in Y and by hypothesis, f^{-1}(F) is $(1,2)^{\star}$-$g^{\star}$-closed in X. Therefore f is $(1,2)^{\star}$-$g^{\star}$-irresolute. □

Definition 4.9. *A function $f : X \to Y$ is called pre-$(1,2)^{\star}$-g-open if f(U) is $(1,2)^{\star}$-g-open in Y, for each $(1,2)^{\star}$-g-open set U in X.*

Proposition 4.10. *If $f : X \to Y$ is bijective pre-$(1,2)^{\star}$-g-open and $(1,2)^{\star}$-$g^{\star}$-continuous then f is $(1,2)^{\star}$-$g^{\star}$-irresolute.*

Proof. Let A be $(1,2)^{\star}$-$g^{\star}$-closed set in Y. Let U be any $(1,2)^{\star}$-g-open set in X such that $\mathrm{f}^{-1}(\mathrm{A}) \subseteq$ U. Then A $\subseteq$ f(U). Since A is $(1,2)^{\star}$-$g^{\star}$-closed and f(U) is $(1,2)^{\star}$-g-open in Y, $\sigma_{1,2}$-cl(A) $\subseteq$ f(U) holds and hence $\mathrm{f}^{-1}(\sigma_{1,2}\text{-cl(A)}) \subseteq$ U. Since f is $(1,2)^{\star}$-$g^{\star}$-continuous and $\sigma_{1,2}$-cl(A) is $\sigma_{1,2}$-closed in Y, $\mathrm{f}^{-1}(\sigma_{1,2}\text{-cl(A)})$ is $(1,2)^{\star}$-$g^{\star}$-closed and hence $\tau_{1,2}\text{-cl}(\mathrm{f}^{-1}(\sigma_{1,2}\text{-cl(A)}))$ $\subseteq$ U and so $\tau_{1,2}\text{-cl}(\mathrm{f}^{-1}(\mathrm{A})) \subseteq$ U. Therefore, f^{-1}(A) is $(1,2)^{\star}$-$g^{\star}$-closed in X and hence f is $(1,2)^{\star}$-$g^{\star}$-irresolute. □

The following examples show that no assumption of Proposition 4.10 can be removed.

Example 4.11. *The identity function defined in Example 4.7 is $(1,2)^{\star}$-$g^{\star}$-continuous and bijective but not pre-$(1,2)^{\star}$-g-open and so f is not $(1,2)^{\star}$-$g^{\star}$-irresolute.*

Example 4.12. *Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, \{a, b\}, X\}$ and $\tau_2 = \{\phi, \{b\}, X\}$. Then the sets in $\{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ are called $\tau_{1,2}$-open and the sets in $\{\phi, \{c\}, \{a, c\}, \{b, c\}, X\}$ are called $\tau_{1,2}$-closed. Let $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, \{a\}, Y\}$ and $\sigma_2 = \{\phi, \{b, c\}, Y\}$. Then the sets in $\{\phi, \{a\}, \{b, c\}, Y\}$ are called $\sigma_{1,2}$-open and the sets in $\{\phi, \{a\}, \{b, c\}, Y\}$ are called $\sigma_{1,2}$-closed. We have $(1,2)^{\star}$-$G^{\star}C(X) = \{\phi, \{c\}, \{a, c\}, \{b, c\}, X\}$ and $(1,2)^{\star}$-GC(X) $= \{\phi, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}, X\}$, $(1,2)^{\star}$-$G^{\star}C(Y) = \{\phi, \{a\}, \{b, c\}, Y\}$ and $(1,2)^{\star}$-GC(Y) = P(Y). Let $f : X \to Y$ be the identity function. Then f is bijective and pre-$(1,2)^{\star}$-g-open but not $(1,2)^{\star}$-$g^{\star}$-continuous and so f is not $(1,2)^{\star}$-$g^{\star}$-irresolute, since $f^{-1}(\{a\}) = \{a\}$ is not $(1,2)^{\star}$-$g^{\star}$-closed in X.*

Proposition 4.13. *If $f : X \to Y$ is bijective $(1,2)^{\star}$-closed and $(1,2)^{\star}$-g-irresolute then the inverse function $f^{-1} : Y \to X$ is $(1,2)^{\star}$-$g^{\star}$-irresolute.*

Proof. Let A be $(1,2)^\star$-$g^\star$-closed in X. Let $(f^{-1})^{-1}(A) = f(A) \subseteq U$ where U is $(1,2)^\star$-g-open in Y. Then $A \subseteq f^{-1}(U)$ holds. Since $f^{-1}(U)$ is $(1,2)^\star$-g-open in X and A is $(1,2)^\star$-$g^\star$-closed in X, $\tau_{1,2}$-cl(A) $\subseteq f^{-1}(U)$ and hence f($\tau_{1,2}$-cl(A)) $\subseteq$ U. Since f is $(1,2)^\star$-closed and $\tau_{1,2}$-cl(A) is closed in X, f($\tau_{1,2}$-cl(A)) is $\sigma_{1,2}$-closed in Y and so f($\tau_{1,2}$-cl(A)) is $(1,2)^\star$-$g^\star$-closed in Y. Therefore $\sigma_{1,2}$-cl(f($\tau_{1,2}$-cl(A))) $\subseteq$ U and hence $\sigma_{1,2}$-cl(f(A)) $\subseteq$ U. Thus f(A) is $(1,2)^\star$-$g^\star$-closed in Y and so f^{-1} is $(1,2)^\star$-$g^\star$-irresolute. □

5. Applications

To obtain a decomposition of $(1,2)^\star$-continuity, we introduce the notion of $(1,2)^\star$-$glc^\star$-continuous function in bitopological spaces and prove that a function is $(1,2)^\star$-continuous if and only if it is both $(1,2)^\star$-$g^\star$-continuous and $(1,2)^\star$-$glc^\star$-continuous.

Definition 5.1. *A subset A of a bitopological space X is called $(1,2)^\star$-$glc^\star$-set if $A = M \cap N$, where M is $(1,2)^\star$-g-open and N is $\tau_{1,2}$-closed in X.*

The family of all $(1,2)^\star$-$glc^\star$-sets in a space X is denoted by $(1,2)^\star$-$glc^\star$(X).

Example 5.2. *Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X\}$ and $\tau_2 = \{\phi, \{c\}, X\}$. Then the sets in $\{\phi, \{c\}, X\}$ are called $\tau_{1,2}$-open and the sets in $\{\phi, \{a, b\}, X\}$ are called $\tau_{1,2}$-closed. Then $\{a\}$ is $(1,2)^\star$-$glc^\star$-set in X.*

Remark 5.3. *Every $\tau_{1,2}$-closed set is $(1,2)^\star$-$glc^\star$-set but not conversely.*

Example 5.4. *Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X\}$ and $\tau_2 = \{\phi, \{a\}, X\}$. Then the sets in $\{\phi, \{a\}, X\}$ are called $\tau_{1,2}$-open and the sets in $\{\phi, \{b, c\}, X\}$ are called $\tau_{1,2}$-closed. Then $\{a, b\}$ is $(1,2)^\star$-$glc^\star$-set but not $\tau_{1,2}$-closed in X.*

Remark 5.5. *$(1,2)^\star$-$g^\star$-closed sets and $(1,2)^\star$-$glc^\star$-sets are independent of each other.*

Example 5.6. *Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X\}$ and $\tau_2 = \{\phi, \{a, c\}, X\}$. Then the sets in $\{\phi, \{a, c\}, X\}$ are called $\tau_{1,2}$-open and the sets in $\{\phi, \{b\}, X\}$ are called $\tau_{1,2}$-closed. Then $\{b, c\}$ is a $(1,2)^\star$-$g^\star$-closed set but not $(1,2)^\star$-$glc^\star$-set in X.*

Example 5.7. *Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X\}$ and $\tau_2 = \{\phi, \{b\}, X\}$. Then the sets in $\{\phi, \{b\}, X\}$ are called $\tau_{1,2}$-open and the sets in $\{\phi, \{a, c\}, X\}$ are called $\tau_{1,2}$-closed. Then $\{a, b\}$ is an $(1,2)^\star$-$glc^\star$-set but not $(1,2)^\star$-$g^\star$-closed set in X.*

Proposition 5.8. *Let X be a bitopological space. Then a subset A of X is $\tau_{1,2}$-closed if and only if it is both $(1,2)^\star$-$g^\star$-closed and $(1,2)^\star$-$glc^\star$-set.*

Proof. Necessity is trivial. To prove the sufficiency, assume that A is both $(1,2)^\star$-$g^\star$-closed and $(1,2)^\star$-$glc^\star$-set. Then A $= M \cap N$, where M is $(1,2)^\star$-g-open and N is $\tau_{1,2}$-closed in X. Therefore, $A \subseteq M$ and $A \subseteq N$ and so by hypothesis, $\tau_{1,2}$-cl(A) $\subseteq$ M and $\tau_{1,2}$-cl(A) $\subseteq$ N. Thus $\tau_{1,2}$-cl(A) $\subseteq M \cap N = A$ and hence $\tau_{1,2}$-cl(A) = A i.e., A is $\tau_{1,2}$-closed in X. □

We introduce the following definition.

Definition 5.9. *A function $f : X \to Y$ is said to be $(1,2)^\star$-$glc^\star$-continuous if for each $\sigma_{1,2}$-closed set V of Y, $f^{-1}(V)$ is an $(1,2)^\star$-$glc^\star$-set in X.*

Example 5.10. *Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X\}$ and $\tau_2 = \{\phi, \{a\}, X\}$. Then the sets in $\{\phi, \{a\}, X\}$ are called $\tau_{1,2}$-open and the sets in $\{\phi, \{b, c\}, X\}$ are called $\tau_{1,2}$-closed. Let $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, \{a\}, Y\}$ and $\sigma_2 = \{\phi, \{b, c\}, Y\}$. Then the sets in $\{\phi, \{a\}, \{b, c\}, Y\}$ are called $\sigma_{1,2}$-open and the sets in $\{\phi, \{a\}, \{b, c\}, Y\}$ are called $\sigma_{1,2}$-closed. Let $f : X \to Y$ be the identity function. Then f is $(1,2)^\star$-$glc^\star$-continuous function.*

Remark 5.11. *From the definitions it is clear that every* $(1,2)^{\star}$*-continuous function is* $(1,2)^{\star}$*-glc***-continuous but not conversely.*

Example 5.12. *Let* $X = \{a, b, c\}$, $\tau_1 = \{\phi, X\}$ *and* $\tau_2 = \{\phi, \{b\}, X\}$. *Then the sets in* $\{\phi, \{b\}, X\}$ *are called* $\tau_{1,2}$*-open and the sets in* $\{\phi, \{a, c\}, X\}$ *are called* $\tau_{1,2}$*-closed. Let* $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, \{b\}, Y\}$ *and* $\sigma_2 = \{\phi, \{a, c\}, Y\}$. *Then the sets in* $\{\phi, \{b\}, \{a, c\}, Y\}$ *are called* $\sigma_{1,2}$*-open and the sets in* $\{\phi, \{b\}, \{a, c\}, Y\}$ *are called* $\sigma_{1,2}$*-closed. Let* $f : X \to Y$ *be the identity function. Then f is* $(1,2)^{\star}$*-glc***-continuous function but not* $(1,2)^{\star}$*-continuous. Since for the* $\sigma_{1,2}$*-closed set* $\{b\}$ *in Y,* $f^{-1}(\{b\}) = \{b\}$, *which is not* $\tau_{1,2}$*-closed in X.*

Remark 5.13. $(1,2)^{\star}$*-g***-continuity and* $(1,2)^{\star}$*-glc***-continuity are independent of each other.*

Example 5.14. *Let* $X = \{a, b, c\}$, $\tau_1 = \{\phi, X\}$ *and* $\tau_2 = \{\phi, \{a, b\}, X\}$. *Then the sets in* $\{\phi, \{a, b\}, X\}$ *are called* $\tau_{1,2}$*-open and the sets in* $\{\phi, \{c\}, X\}$ *are called* $\tau_{1,2}$*-closed. Let* $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, Y\}$ *and* $\sigma_2 = \{\phi, \{a\}, Y\}$. *Then the sets in* $\{\phi, \{a\}, Y\}$ *are called* $\sigma_{1,2}$*-open and the sets in* $\{\phi, \{b, c\}, Y\}$ *are called* $\sigma_{1,2}$*-closed. Let* $f : X \to Y$ *be the identity function. Then f is* $(1,2)^{\star}$*-g***-continuous function but not* $(1,2)^{\star}$*-glc***-continuous.*

Example 5.15. *Let* $X = \{a, b, c\}$, $\tau_1 = \{\phi, X\}$ *and* $\tau_2 = \{\phi, \{a\}, X\}$. *Then the sets in* $\{\phi, \{a\}, X\}$ *are called* $\tau_{1,2}$*-open and the sets in* $\{\phi, \{b, c\}, X\}$ *are called* $\tau_{1,2}$*-closed. Let* $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, Y\}$ *and* $\sigma_2 = \{\phi, \{b, c\}, Y\}$. *Then the sets in* $\{\phi, \{b, c\}, Y\}$ *are called* $\sigma_{1,2}$*-open and the sets in* $\{\phi, \{a\}, Y\}$ *are called* $\sigma_{1,2}$*-closed. Let* $f : X \to Y$ *be the identity function. Then f is* $(1,2)^{\star}$*-glc***-continuous function but not* $(1,2)^{\star}$*-g***-continuous.*

We have the following decomposition for $(1,2)^{\star}$-continuity.

Theorem 5.16. *A function* $f : X \to Y$ *is* $(1,2)^{\star}$*-continuous if and only if it is both* $(1,2)^{\star}$*-g***-continuous and* $(1,2)^{\star}$*-glc***-continuous.*

Proof. Assume that f is $(1,2)^{\star}$-continuous. Then by Proposition 3.3 and Remark 5.11, f is both $(1,2)^{\star}$-$g^{\star}$-continuous and $(1,2)^{\star}$-$glc^{\star}$-continuous.

Conversely, assume that f is both $(1,2)^{\star}$-$g^{\star}$-continuous and $(1,2)^{\star}$-$glc^{\star}$-continuous. Let V be a $\sigma_{1,2}$-closed subset of Y. Then f^{-1}(V) is both $(1,2)^{\star}$-$g^{\star}$-closed set and $(1,2)^{\star}$-$glc^{\star}$-set. By Proposition 5.8, f^{-1}(V) is a $\tau_{1,2}$-closed set in X and so f is $(1,2)^{\star}$-continuous. □

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