International Journal of Current Research in Science and Technology
Volume 1, Issue 5 (2015), 53–58.
ISSN: 2394-5745
Available Online: http://ijcrst.in/

International Journal of Current Research in Science and Technology

# $\pi$-Generalized Closed Sets with Respect to an Ideal

**Research Article**

**O. Ravi[1]*, M. Suresh[2] and A. Pandi[1]**

1 Department of Mathematics, P. M. Thevar College, Usilampatti, Madurai District, Tamil Nadu, India.

2 Department of Mathematics, RMD Engineering College, Kavaraipettai, Gummidipoondi, Thiruvallur District, Tamil Nadu, India.

**Abstract:** An ideal on a set X is a non empty collection of subsets of X with heredity property which is also closed under finite unions. The concept of generalized closed ($g$-closed) sets was introduced by Levine [16]. Quite Recently, Jafari and Rajesh [12] have introduced and studied the notion of generalized closed ($g$-closed) sets with respect to an ideal. Many variations of $g$-closed sets are being introduced and investigated by modern researchers. One among them is $\pi g$-closed sets which were introduced by Dontchev and Noiri [4]. In this paper, we introduce and investigate the concept of $\pi$-generalized closed ($\pi g$-closed) sets with respect to an ideal.

**Keywords:** Topological space, $\pi$-open set, $\pi$-generalized closed set, $g$-closed set, $\mathcal{I}_g$-closed set, $\mathcal{I}_{\pi g}$-closed set, ideal.



## 1. Introduction and Preliminaries

In 1968, Zaitsev [22] introduced the notion of $\pi$-open sets as a finite union of regular open sets. This notion received a proper attention and some research articles came to existence.

The notion of closed set is fundamental in the study of topological spaces. In 1970, Levine [16] introduced the concept of generalized closed sets in a topological space by comparing the closure of a subset with its open supersets. He defined a subset A of a topological space X to be generalized closed (briefly, $g$-closed) if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open. This notion has been studied extensively in recent years by many topologists. After advent of $g$-closed sets, many variations of $g$-closed sets are being introduced and investigated by modern topologists. One among them is $\pi g$-closed sets which were introduced by Dontchev and Noiri [4].

Dontchev and Noiri [4] introduced and investigated $\pi g$-closed sets, $\pi$-continuity and $\pi g$-continuity. Ekici and Baker [6] studied further properties of $\pi g$-closed sets and continuities. In 2007, Ekici [7] introduced and studied some new forms of continuities. In [14], Kalantan introduced and investigated $\pi$-normality. The digital n-space is not a metric space, since it is not $T_1$. But recently Takigawa and Maki [21] showed that in the digital n-space every closed set is $\pi$-open. Recently, Ekici [5] introduced and studied contra $\pi g$-continuous functions. In 2010, Caldas et. al. [3] introduced and studied contra $\pi gp$-continuity.

Indeed ideals are very important tools in General Topology. It was the works of Newcomb [17], Rancin [18], Samuels [19] and Hamlett and Jankovic (see [8–11, 13]) which motivated the research in applying topological ideals to generalize the most basic properties in General Topology. A nonempty collection $\mathcal{I}$ of subsets on a topological space (X, $\tau$) is called a topological ideal [15] if it satisfies the following two conditions:

* E-mail: siingam@yahoo.com

1. If A $\in \mathcal{I}$ and B $\subseteq$ A implies B $\in \mathcal{I}$ (heredity)
2. If A $\in \mathcal{I}$ and B $\in \mathcal{I}$, then A $\cup$ B $\in \mathcal{I}$ (finite additivity)

If A is a subset of a topological space (X, $\tau$), cl(A) and int(A) denote the closure of A and the interior of A, respectively. Let A $\subseteq$ B $\subseteq$ X. Then $\mathrm{cl}_B$(A) (resp. $\mathrm{int}_B$(A)) denotes closure of A (resp. interior of A) with respect to B.

In this paper, we introduce and study the concept of $\pi g$-closed sets with respect to an ideal, which is the extension of the concept of $\pi g$-closed sets.

The following Definitions and Remarks are useful in the sequel.

**Definition 1.1.** *A subset A of a topological space X is regular open [20] if A = int(cl(A)).*

**Definition 1.2.** *The finite union of regular open sets is called $\pi$-open [22]. The complement of $\pi$-open set is $\pi$-closed [22].*

**Definition 1.3.** *A subset A of a topological space X to be $\pi$-generalized closed (briefly, $\pi g$-closed) [4] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $\pi$-open.*

**Definition 1.4.** *A subset A of a topological space X to be generalized closed (briefly, g-closed) [16] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open.*

**Definition 1.5.** *Let (X, $\tau$) be a topological space and $\mathcal{I}$ be an ideal on X. A subset A of X is said to be generalized closed with respect to an ideal (briefly $\mathcal{I}_g$-closed) [12] if and only if cl(A)$-$B $\in \mathcal{I}$, whenever A $\subseteq$ B and B is open.*

**Remark 1.6.** *For a subset of a topological space, the following properties hold:*

1. *Every closed set is g-closed but not conversely [16].*
2. *Every g-closed set is $\pi g$-closed but not conversely [4].*

**Remark 1.7** ([12])**.** *Every g-closed set is $\mathcal{I}_g$-closed but not conversely.*

**Definition 1.8** ([1, 2])**.** *A function f : (X, $\tau$) $\to$ (Y, $\sigma$) is called $\pi$-irresolute if $f^{-1}$(V) is $\pi$-closed in X for every $\pi$-closed set V of Y.*

## 2. $\pi$-Generalized Closed Sets with Respect to an Ideal

**Definition 2.1.** *Let (X, $\tau$) be a topological space and $\mathcal{I}$ be an ideal on X. A subset A of X is said to be $\pi$-generalized closed with respect to an ideal (briefly $\mathcal{I}_{\pi g}$-closed) if and only if cl(A)$-$B $\in \mathcal{I}$, whenever A $\subseteq$ B and B is $\pi$-open.*

**Remark 2.2.** *Every $\pi g$-closed set is $\mathcal{I}_{\pi g}$-closed, but the converse need not be true, as this may be seen from the following Example.*

**Example 2.3.** *Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ and $\mathcal{I} = \{\emptyset, \{d\}\}$. Then $\{a\}$ is $\mathcal{I}_{\pi g}$-closed but not $\pi g$-closed.*

The following theorem gives a characterization of $\mathcal{I}_{\pi g}$-closed sets.

**Theorem 2.4.** *If a set A is $\mathcal{I}_{\pi g}$-closed in (X, $\tau$), then F $\subseteq$ cl(A)$-$A and F is $\pi$-closed in X implies F $\in \mathcal{I}$.*

*Proof.* Assume that A is $\mathcal{I}_{\pi g}$-closed. Let F $\subseteq$ cl(A)$-$A. Suppose F is $\pi$-closed. Then A $\subseteq$ X$-$F. By our assumption, cl(A)$-$(X$-$F) $\in \mathcal{I}$. But F $\subseteq$ cl(A)$-$(X$-$F) and hence F $\in \mathcal{I}$. □

**Remark 2.5.** *The intersection of $\pi$-closed set with closed set need not be a $\pi$-closed in general.*

**Example 2.6.** *Let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$. Then $\emptyset$, X, $\{d\}$, $\{a, d\}$, $\{c, d\}$, $\{a, c, d\}$, $\{b, c, d\}$ are closed sets and $\emptyset$, X, $\{d\}$, $\{a, d\}$, $\{b, c, d\}$ are $\pi$-closed sets. Clearly intersection of the closed set $\{c, d\}$ and the $\pi$-closed set X is $\{c, d\}$ which is closed set but not $\pi$-closed set.*

**Definition 2.7.** *A topological space $(X, \tau)$ is called $\pi$-C-O space if the intersection of $\pi$-closed set in X with closed set is $\pi$-closed set in X or the union of $\pi$-open set in X with open set is $\pi$-open set in X.*

**Example 2.8.** *Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$. Then $(X, \tau)$ is not a $\pi$-C-O space.*

**Example 2.9.** *Let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, X, \{a\}, \{b, c, d\}\}$. Then $(X, \tau)$ is a $\pi$-C-O space.*

**Theorem 2.10.** *Let $(X, \tau)$ be a $\pi$-C-O space and $A \subseteq X$. If $F \subseteq cl(A)-A$ and F is $\pi$-closed in X implies $F \in \mathcal{I}$, the set is $\mathcal{I}_{\pi g}$-closed in $(X, \tau)$.*

*Proof.* Assume that $F \subseteq cl(A)-A$ and F is $\pi$-closed in X implies that $F \in \mathcal{I}$. Suppose $A \subseteq U$ and U is $\pi$-open. Then $cl(A)-U = cl(A) \cap (X-U)$ is a $\pi$-closed set in X, that is contained in $cl(A)-A$. By assumption, $cl(A)-U \in \mathcal{I}$. This implies that A is $\mathcal{I}_{\pi g}$-closed. □

**Theorem 2.11.** *Let $(X, \tau)$ be a $\pi$-C-O space and $A \subseteq X$. A set A is $\mathcal{I}_{\pi g}$-closed in $(X, \tau)$ if and only if $F \subseteq cl(A)-A$ and F is $\pi$-closed in X implies $F \in \mathcal{I}$.*

*Proof.* It follows from Theorems 2.4 and 2.10. □

**Theorem 2.12.** *If A and B are $\mathcal{I}_{\pi g}$-closed sets of $(X, \tau)$, then their union $A \cup B$ is also $\mathcal{I}_{\pi g}$-closed.*

*Proof.* Suppose A and B are $\mathcal{I}_{\pi g}$-closed sets in $(X, \tau)$. If $A \cup B \subseteq U$ and U is $\pi$-open, then $A \subseteq U$ and $B \subseteq U$. By assumption, $cl(A)-U \in \mathcal{I}$ and $cl(B)-U \in \mathcal{I}$ and hence $cl(A \cup B)-U = (cl(A)-U) \cup (cl(B)-U) \in \mathcal{I}$. That is $A \cup B$ is $\mathcal{I}_{\pi g}$-closed. □

**Remark 2.13.** *The intersection of two $\mathcal{I}_{\pi g}$-closed sets need not be an $\mathcal{I}_{\pi g}$-closed as shown by the following Example.*

**Example 2.14.** *Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$ and $\mathcal{I} = \{\emptyset\}$. Then $A = \{a, c\}$ and $B = \{a, d\}$ are $\mathcal{I}_{\pi g}$-closed but their intersection $A \cap B = \{a\}$ is not $\mathcal{I}_{\pi g}$-closed.*

**Remark 2.15.** *Every $\mathcal{I}_g$-closed set is $\mathcal{I}_{\pi g}$-closed but not conversely.*

**Example 2.16.** *In Example 2.3, $\{b\}$ is $\mathcal{I}_{\pi g}$-closed but not $\mathcal{I}_g$-closed.*

**Remark 2.17.** *For several subsets defined above, we have the following implications.*

$$\begin{array}{ccccc} & & \mathcal{I}_g\text{-closed set} & \longrightarrow & \mathcal{I}_{\pi g}\text{-closed set} \\ & & \uparrow & & \uparrow \\ \text{closed set} & \longrightarrow & g\text{-closed set} & \longrightarrow & \pi g\text{-closed set} \end{array}$$

*The reverse implications are not true.*

**Theorem 2.18.** *If A is $\mathcal{I}_{\pi g}$-closed and $A \subseteq B \subseteq cl(A)$ in $(X, \tau)$, then B is $\mathcal{I}_{\pi g}$-closed in $(X, \tau)$.*

*Proof.* Suppose A is $\mathcal{I}_{\pi g}$-closed and A $\subseteq$ B $\subseteq$ cl(A) in (X, $\tau$). Suppose B $\subseteq$ U and U is $\pi$-open. Then A $\subseteq$ U. Since A is $\mathcal{I}_{\pi g}$-closed, we have cl(A)$-$U $\in \mathcal{I}$. Now B $\subseteq$ cl(A). This implies that cl(B)$-$U $\subseteq$ cl(A)$-$U $\in \mathcal{I}$. Hence B is $\mathcal{I}_{\pi g}$-closed in (X, $\tau$). ☐

**Theorem 2.19.** *Let $A \subseteq Y \subseteq X$ and suppose that $A$ is $\mathcal{I}_{\pi g}$-closed in $(X, \tau)$. Then $A$ is $\mathcal{I}_{\pi g}$-closed relative to the subspace $Y$ of $X$, with respect to the ideal $\mathcal{I}_Y = \{F \subseteq Y : F \in \mathcal{I}\}$.*

*Proof.* Suppose A $\subseteq$ U $\cap$ Y and U is $\pi$-open in (X, $\tau$), then A $\subseteq$ U. Since A is $\mathcal{I}_{\pi g}$-closed in (X, $\tau$), we have cl(A)$-$U $\in \mathcal{I}$. Now (cl(A) $\cap$ Y)$-$(U$\cap$Y) = (cl(A)$-$U) $\cap$ Y $\in \mathcal{I}$, whenever A $\subseteq$ U $\cap$ Y and U is $\pi$-open. Hence A is $\mathcal{I}_{\pi g}$-closed relative to the subspace Y. ☐

**Theorem 2.20.** *Let $A$ be an $\mathcal{I}_{\pi g}$-closed set and $F$ be a closed set in the $\pi$-C-O space $(X, \tau)$, then $A \cap F$ is an $\mathcal{I}_{\pi g}$-closed set in $(X, \tau)$.*

*Proof.* Let A $\cap$ F $\subseteq$ U and U is $\pi$-open. Then A $\subseteq$ U $\cup$ (X$-$F). Since A is $\mathcal{I}_{\pi g}$-closed, we have cl(A)$-$(U $\cup$ (X$-$F)) $\in \mathcal{I}$. Now, cl(A $\cap$ F) $\subseteq$ cl(A) $\cap$ F = (cl(A) $\cap$ F)$-$(X$-$F). Therefore, cl(A $\cap$ F)$-$U $\subseteq$ (cl(A) $\cap$ F)$-$(U $\cap$ (X$-$F)) $\subseteq$ cl(A)$-$(U $\cup$ (X$-$F)) $\in \mathcal{I}$. Hence A $\cap$ F is $\mathcal{I}_{\pi g}$-closed in (X, $\tau$). ☐

**Definition 2.21.** *Let $(X, \tau)$ be a topological space and $\mathcal{I}$ be an ideal on $X$. A subset $A \subseteq X$ is said to be $\pi$-generalized open with respect to an ideal (briefly $\mathcal{I}_{\pi g}$-open) if and only if $X-A$ is $\mathcal{I}_{\pi g}$-closed.*

**Theorem 2.22.** *A set $A$ is $\mathcal{I}_{\pi g}$-open in $(X, \tau)$ if and only if $F-U \subseteq$ int(A), for some $U \in \mathcal{I}$, whenever $F \subseteq A$ and $F$ is $\pi$-closed.*

*Proof.* Suppose A is $\mathcal{I}_{\pi g}$-open. Suppose F $\subseteq$ A and F is $\pi$-closed. We have X$-$A $\subseteq$ X$-$F. By assumption, cl(X$-$A) $\subseteq$ (X$-$F) $\cup$ U, for some U $\in \mathcal{I}$. This implies X$-$((X$-$F) $\cup$ U) $\subseteq$ X$-$ (cl(X$-$A)) and hence F$-$U $\subseteq$ int(A).
Conversely, assume that F $\subseteq$ A and F is $\pi$-closed. Then F$-$U $\subseteq$ int(A), for some U $\in \mathcal{I}$. Consider an $\pi$-open set G such that X$-$A $\subseteq$ G. Then X$-$G $\subseteq$ A. By assumption, (X$-$G)$-$U $\subseteq$ int(A) = X$-$cl(X$-$A). This gives that X$-$(G $\cup$ U) $\subseteq$ X$-$cl(X$-$A). Then, cl(X$-$A) $\subseteq$ G $\cup$ U, for some U $\in \mathcal{I}$. This shows that cl(X$-$A)$-$G $\in \mathcal{I}$. Hence X$-$A is $\mathcal{I}_{\pi g}$-closed.
Recall that the sets A and B are said to be separated if cl(A) $\cap$ B = $\emptyset$ and A $\cap$ cl(B) = $\emptyset$. ☐

**Theorem 2.23.** *If $A$ and $B$ are separated $\mathcal{I}_{\pi g}$-open sets in $\pi$-C-O space $(X, \tau)$, then $A \cup B$ is $\mathcal{I}_{\pi g}$-open.*

*Proof.* Suppose A and B are separated $\mathcal{I}_{\pi g}$-open sets in (X, $\tau$) and F be a $\pi$-closed subset of A $\cup$ B. Then F $\cap$ cl(A) $\subseteq$ A and F $\cap$ cl(B) $\subseteq$ B. By assumption, (F $\cap$ cl(A))$-U_1 \subseteq$ int(A) and (F $\cap$ cl(B))$-U_2 \subseteq$ int(B), for some $U_1$, $U_2 \in \mathcal{I}$. It means that ((F $\cap$ cl(A))$-$int(A)) $\in \mathcal{I}$ and ((F $\cap$ cl(B))$-$int(B)) $\in \mathcal{I}$. Then ((F $\cap$ cl(A))$-$int(A)) $\cup$ ((F $\cap$ cl(B))$-$int(B)) $\in \mathcal{I}$. Hence (F $\cap$ (cl(A) $\cup$ cl(B))$-$(int(A) $\cup$ int(B))) $\in \mathcal{I}$. But F = F $\cap$ (A $\cup$B) $\subseteq$ F $\cap$ cl(A $\cup$ B), and we have F$-$int(A $\cup$ B) $\subseteq$ (F $\cap$ cl(A $\cup$ B))$-$int(A $\cup$ B) $\subseteq$ (F $\cap$ cl(A $\cup$B))$-$(int(A) $\cup$ int(B)) $\in \mathcal{I}$. Hence, F$-$U $\subseteq$ int(A $\cup$ B), for some U $\in \mathcal{I}$. This proves that A $\cup$ B is $\mathcal{I}_{\pi g}$-open. ☐

**Corollary 2.24.** *Let $A$ and $B$ are $\mathcal{I}_{\pi g}$-closed sets and suppose $X-A$ and $X-B$ are separated in $(X, \tau)$. Then $A \cap B$ is $\mathcal{I}_{\pi g}$-closed.*

**Corollary 2.25.** *If $A$ and $B$ are $\mathcal{I}_{\pi g}$-open sets in $(X, \tau)$, then $A \cap B$ is $\mathcal{I}_{\pi g}$-open.*

*Proof.* If A and B are $\mathcal{I}_{\pi g}$-open, then X$-$A and X$-$B are $\mathcal{I}_{\pi g}$-closed. By Theorem 2.12, X$-$(A $\cap$ B) is $\mathcal{I}_{\pi g}$-closed, which implies A $\cap$ B is $\mathcal{I}_{\pi g}$-open. ☐

**Theorem 2.26.** *If $A \subseteq B \subseteq X$, $A$ is $\mathcal{I}_{\pi g}$-open relative to $B$ and $B$ is $\mathcal{I}_{\pi g}$-open relative to $X$, then $A$ is $\mathcal{I}_{\pi g}$-open relative to $X$.*

*Proof.* Suppose $A \subseteq B \subseteq X$, A is $\mathcal{I}_{\pi g}$-open relative to B and B is $\mathcal{I}_{\pi g}$-open relative to X. Suppose $F \subseteq A$ and F is $\pi$-closed. Since A is $\mathcal{I}_{\pi g}$-open relative to B, by Theorem 2.22, $F-U_1 \subseteq int_B(A)$, for some $U_1 \in \mathcal{I}$. This implies there exists an $\pi$-open set $G_1$ such that $F-U_1 \subseteq G_1 \cap B \subseteq A$, for some $U_1 \in \mathcal{I}$. Since B is $\mathcal{I}_{\pi g}$-open, $F \subseteq B$ and F is $\pi$-closed; we have $F-U_2 \subseteq int(B)$, for some $U_2 \in \mathcal{I}$. This implies there exists an $\pi$-open set $G_2$ such that $F-U_2 \subseteq G_2 \subseteq B$, for some $U_2 \in \mathcal{I}$. Now $F-(U_1 \cup U_2) \subseteq (F-U_1) \cap (F-U_2) \subseteq G_1 \cap G_2 \subseteq G_1 \cap B \subseteq A$. This implies that $F-(U_1 \cup U_2) \subseteq int(A)$, for some $U_1 \cup U_2 \in \mathcal{I}$ and hence A is $\mathcal{I}_{\pi g}$-open relative to X. □

**Theorem 2.27.** *If $int(A) \subseteq B \subseteq A$ and $A$ is $\mathcal{I}_{\pi g}$-open in $(X, \tau)$, then $B$ is $\mathcal{I}_{\pi g}$-open in $X$.*

*Proof.* Suppose $int(A) \subseteq B \subseteq A$ and A is $\mathcal{I}_{\pi g}$-open. Then $X-A \subseteq X-B \subseteq cl(X-A)$ and $X-A$ is $\mathcal{I}_{\pi g}$-closed. By Theorem 2.18, $X-B$ is $\mathcal{I}_{\pi g}$-closed and hence B is $\mathcal{I}_{\pi g}$-open. □

**Theorem 2.28.** *Let $(X, \tau)$ be a $\pi$-C-O space. Then a set $A$ is $\mathcal{I}_{\pi g}$-closed in $X$ if and only if $cl(A)-A$ is $\mathcal{I}_{\pi g}$-open in $X$.*

*Proof.* Necessity: Suppose $F \subseteq cl(A)-A$ and F be $\pi$-closed. Then by Theorem 2.4, $F \in \mathcal{I}$. This implies that $F-U = \emptyset$, for some $U \in \mathcal{I}$. Clearly, $F-U \subseteq int(cl(A)-A)$. By Theorem 2.22, $cl(A)-A$ is $\mathcal{I}_{\pi g}$-open.
Sufficiency: Suppose $A \subseteq G$ and G is $\pi$-open in $(X, \tau)$. Then $cl(A) \cap (X-G) \subseteq cl(A) \cap (X-A) = cl(A)-A$. By hypothesis, $(cl(A) \cap (X-G))-U \subseteq int(cl(A)-A) = \emptyset$, for some $U \in \mathcal{I}$. This implies that $cl(A) \cap (X-G) \subseteq U \in \mathcal{I}$ and hence $cl(A)-G \in \mathcal{I}$. Thus, A is $\mathcal{I}_{\pi g}$-closed. □

**Theorem 2.29.** *Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be $\pi$-irresolute and closed. If $A \subseteq X$ is $\mathcal{I}_{\pi g}$-closed in $X$, then $f(A)$ is $f(\mathcal{I})_{\pi g}$-closed in $(Y, \sigma)$, where $f(\mathcal{I}) = \{f(U) : U \in \mathcal{I}\}$.*

*Proof.* Suppose $A \subseteq X$ and A is $\mathcal{I}_{\pi g}$-closed. Suppose $f(A) \subseteq G$ and G is $\pi$-open. Then $A \subseteq f^{-1}(G)$. By definition, $cl(A)-f^{-1}(G) \in \mathcal{I}$ and hence $f(cl(A))-G \in f(\mathcal{I})$. Since f is closed, $cl(f(A)) \subseteq cl(f(cl(A))) = f(cl(A))$. Then $cl(f(A))-G \subseteq f(cl(A))-G \in f(\mathcal{I})$ and hence f(A) is $f(\mathcal{I})_{\pi g}$-closed. □

## References


[1] S.C.Arora, S.Tahiliani and H.Maki, *On $\pi$ generalized $\beta$-closed sets in topological spaces II*, Scientiae Mathematicae Japonicae Online, (e-2009), 637-648.

[2] G.Aslim, A.Cakshu Guler and T.Noiri, *On $\pi gs$-closed sets in topological spaces*, Acta Math. Hungar., 112(4)(2004), 275-283.

[3] M.Caldas, S.Jafari, K.Viswanathan and S.Krishnaprakash, *On contra $\pi gp$-continuous functions*, Kochi J. Math., 5(2010), 67-78.

[4] J.Dontchev and T.Noiri, *Quasi-normal spaces and $\pi g$-closed sets*, Acta Math. Hungar., 89(3)(2000), 211-219.

[5] E.Ekici, *On contra $\pi g$-continuous functions*, Chaos Solitons and Fractals, 35(2008), 71-81.

[6] E.Ekici and C.W.Baker, *On $\pi g$-closed sets and continuity*, Kochi J. Math., 2(2007), 35-42.

[7] E.Ekici, *On (g, s)-continuous and ($\pi g$, s)-continuous functions*, Sarajevo J. Math., 3(15)(2007), 99-113.

[8] T.R.Hamlett and D.Jankovic, *Compactness with respect to an ideal*, Boll. Un. Mat. Ita., 7 (4-B)(1990), 849-861.

[9] T.R.Hamlett and D.Jankovic, *Ideals in topological spaces and the set operator*, Boll. Un. Mat. Ita., 7(1990), 863-874.

[10] T.R.Hamlett and D.Jankovic, *Ideals in General Topology and Applications (Midletown, CT, 1988)*, Lecture Notes in Pure and Appl. Math. Dekker, New York, (1990), 115-125.

[11] T.R.Hamlett and D.Jankovic, *Compatible extensions of ideals*, Boll. Un. Mat. Ita., 7(1992), 453-465.

[12] S.Jafari and N.Rajesh, *Generalized closed sets with respect to an ideal*, European J. Pure Appl. Math., 4(2)(2011), 147-151.

[13] D.Jankovic and T.R.Hamlett, *New topologies from old via ideals*, Amer. Math. Month., 97(1990), 295-310.

[14] L.N.Kalantan, *$\pi$-normal topological spaces*, Filomat., 22(1)(2008), 173-181.

[15] K.Kuratowski, *Topologies I*, Warszawa, (1933).

[16] N.Levine, *Generalized closed sets in topology*, Rend. Circ. Mat. Palermo., 19(2)(1970), 89-96.

[17] R.L.Newcomb, *Topologies which are compact modulo an ideal*, Ph.D. Dissertation, Univ. Cal. at Santa Barbara, (1967).

[18] D.V.Rancin, *Compactness modulo an ideal*, Soviet Math. Dokl., 13(1972), 193-197.

[19] P.Samuels, *A topology from a given topology and ideal*, J. London Math. Soc., (2)(10)(1975), 409-416.

[20] M.H.Stone, *Applications of the theory of Boolean rings to general topology*, Trans. Amer. Math. Soc., 41(1937), 375-481.

[21] S.Takigawa and H.Maki, *Every nonempty open set of the digital n-space is expressible as the union of finitely many nonempty regular open sets*, Sci. Math. Jpn., 67(2008), 365-376.

[22] V.Zaitsev, *On certain classes of topological spaces and their bicompactifications*, Dokl. Akad. Nauk. SSSR, 178(1968), 778-779.