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Contra g^*-continuity and Separation Axioms

Research Article

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Abstract: The main results of this paper are several properties concerning contra g*-continuous maps. Furthermore, the relationships between the contra g*-continuity and some topological maps as well as Separation axioms are investigated.

Keywords: Contra g-continuity, contra g*-continuity, g*-connected space.

1. Introduction and Preliminaries

In the literature there are many types of continuities introduced by various authors. Quite recently, Jafari and Noiri introduced and investigated the notions of contra-precontinuity, contra-α-continuity, contra-g-continuity and contra-super-continuity as a continuation of research done by Dontchev [8], and Dontchev and Noiri [7] on the interesting notions of contra-continuity and contra-semi-continuity.

This paper devotes to introduce and investigate a new class of maps called contra g*-continuous maps which are weaker than contra-continuity and stronger than contra g-continuity, contra sg-continuity and contra gs-continuity. The main results of this paper are that several properties concerning contra g*-continuous maps. Furthermore, the relationships between the contra g*-continuity and some topological maps as well as Separation axioms are investigated.

Throughout the paper, (X, τ), (Y, σ) and (Z, ρ) (briefly X, Y and Z) represent topological spaces on which no separation axioms are assumed unless or otherwise mentioned. For a subset A of a space X, cl(A), int(A) and C(A) denotes the closure of A, the interior of A and the complement of A respectively. We recall the following definitions which are useful in the sequel.

Definition 1.1. *A subset A of a space X is called*

1. *a semi-open [13] if A $\subseteq$ cl(int(A));*
2. *a preopen [15] if A $\subseteq$ int(cl(A));*
3. *an α-open [18] if A $\subseteq$ int(cl(int(A))) and*
4. *a semi-closed [5] (resp. an α-closed [16], a preclosed [15]) if C(A) is a semi-open (resp. an α-open, a preopen).*

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The semi-closure (resp. α-closure, preclosure) of a subset A of X, denoted by scl(A) (resp. $\alpha cl(A)$, pcl(A)), is the intersection of all semi-closed (resp. α-closed, preclosed) sets containing A.

Definition 1.2. *A subset A of a space X is called*

1. *g-closed [12] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X;*
2. *sg-closed [3] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in X;*
3. *gs-closed [1] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X;*
4. *αg-closed [14] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X;*
5. *g^*-closed [19] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in X and*
6. *g-open [12] (resp. sg-open [3], gs-open [1], αg-open [14] and g^*-open [19]) if C(A) is g-closed (resp. sg-closed, gs-closed, αg-closed and g^*-closed).*

Definition 1.3. *A map f: $X \to Y$ is said to be*

1. *contra-continuous [8] if $f^{-1}(V)$ is closed in X for every open set V of Y;*
2. *contra semi-continuous [7] if $f^{-1}(V)$ is semi-closed in X for every open set V of Y;*
3. *contra α-continuous [10] if $f^{-1}(V)$ is α-closed in X for every open set V of Y;*
4. *contra g-continuous [11] if $f^{-1}(V)$ is g-closed in X for every open set V of Y;*
5. *contra sg-continuous [7] if $f^{-1}(V)$ is sg-closed in X for every open set V of Y;*
6. *contra gs-continuous [7] if $f^{-1}(V)$ is gs-closed in X for every open set V of Y;*
7. *g^*-irresolute [19] if $f^{-1}(V)$ is g^*-closed in X for every g^*-closed set V of Y;*
8. *gc-irresolute [2] if $f^{-1}(V)$ is g-closed in X for every g-closed set V of Y;*
9. *αg-irresolute [14] if $f^{-1}(V)$ is αg-closed in X for every αg-closed set V of Y;*
10. *gs-irresolute [4] if $f^{-1}(V)$ is gs-closed in X for every gs-closed set V of Y;*
11. *pre g^*-closed [19] if f(V) is g^*-closed in Y for every g^*-closed set V of X;*
12. *g^*-continuous [19] if $f^{-1}(V)$ is g^*-closed in X for every closed set V of Y and*
13. *preclosed [9] if f(V) is preclosed in Y for every closed set V of X.*

Theorem 1.4. *[19] In a topological space X, the followings hold:*

1. *Every closed set is g^*-closed.*
2. *every g^*-closed set is g-closed and hence gs-closed.*

The converses of the above statements are not true in general.

Definition 1.5. *A space X is called*

1. *an* T_c *space if every gs-closed set in it is* g^**-closed* [19];

2. *an* ${}_{\alpha}T_c$ *space if every* α*g-closed set in it is* g^**-closed* [19] *and*

3. T_b *space if every gs-closed set in it is closed* [6].

Definition 1.6. [17] *A space X is called a locally indiscrete if every open set in it is closed.*

2. Properties of contra g*-continuous maps

Definition 2.1. *A map f: X → Y is called contra* g^**-continuous if* $f^{-1}(V)$ *is* g^**-closed set of X for every open set V of Y.*

Theorem 2.2.

1. *Every contra-continuous map is contra* g^**-continuous.*

2. *Every contra* g^**-continuous map is contra g-continuous and hence contra gs-continuous.*

The converses of the above Theorem are not true as per the following examples.

Example 2.3. *Let* $X = Y = \{a, b, c\}$*. Let* $\tau = \{\emptyset, X, \{a, b\}\}$ *and* $\sigma = \{\emptyset, Y, \{a, c\}\}$*. Let f: X → Y be the identity map. Then f is contra* g^**-continuous map but it is not contra-continuous.*

Example 2.4. *Let* $X = Y = \{a, b, c\}$*. Let* $\tau = \{\emptyset, X, \{a\}, \{b, c\}\}$ *and* $\sigma = \{\emptyset, Y, \{b\}\}$*. Let f: X → Y be the identity map. Then f is contra g-continuous map and hence contra gs-continuous map but it is not contra* g^**-continuous.*

Theorem 2.5. *The composition of two contra* g^**-continuous maps need not be contra* g^**-continuous map.*

The following example supports the above theorem.

Example 2.6. *Let* $X = Y = Z = \{a, b, c\}$*. Let* $\tau = \{\emptyset, X, \{a, b\}\}$*,* $\sigma = \{\emptyset, Y, \{a, c\}\}$ *and* $\rho = \{\emptyset, Z, \{b\}\}$*. Let f: X → Y be the identity map and g: Y → Z be the identity map. Then f is contra* g^**-continuous map and g is contra* g^**-continuous map. But their composition g o f: X → Z is not contra* g^**-continuous.*

Theorem 2.7. *Let f:X → Y be a map. Then the following statements are equivalent.*

1. *f is contra* g^**-continuous.*

2. *The inverse image of each open set in Y is* g^**-closed in X.*

3. *The inverse image of each closed set in Y is* g^**-open in X.*

Proof. (1) ⇒ (2) : Let G be any open set in Y. By the assumption of (1), f^{-1}(G) is g*-closed in X.
(2) ⇒ (3) : Let G be any closed set in Y. Then Y − G is open set in Y. By the assumption of (2), f^{-1}(Y − G) = X − f^{-1}(G) is g*-closed in X. Therefore f^{-1}(G) is g*-open in X.
(3) ⇒ (1) : Let G be any open set in Y. Then Y − G is closed in Y. By the assumption of (3), f^{-1}(Y − G) = X − f^{-1}(G) is g*-open in X. Therefore f^{-1}(G) is g*-closed in X. Thus f is contra g*-continuous map. □

Theorem 2.8. *Let f: X → Y be surjective,* g^**-irresolute and pre* g^**-closed, and g: Y → Z be any map. Then g o f: X → Z is contra* g^**-continuous if and only if g is contra* g^**-continuous.*

Proof. Let g o f: X→Z be contra g*-continuous map. Let F be an open subset of Z. Then (g o f)$^{-1}$(F) = $f^{-1}(g^{-1}$(F)) is a g*-closed subset of X. Since f is pre g*-closed, $f(f^{-1}(g^{-1}$(F))) = g^{-1}(F) is g*-closed in Y. Thus g is contra g*-continuous map.

Conversely, let g: Y→ Z be contra g*-continuous map. Let G be an open subset of Z. Since g is contra g*-continuous, g^{-1}(G) is g*-closed in Y. Since f is g*-irresolute, $f^{-1}(g^{-1}$(G)) = (g o f)$^{-1}$ (G) is g*-closed in X. Hence g o f is contra g*-continuous map. □

3. Relationship with other maps

Theorem 3.1. *If f: X → Y is g^*-irresolute map and g: Y →Z is contra-continuous map, then the composition gof: X → Z is contra g^*-continuous map.*

Proof. Let G be an open set in Z. Since g is contra-continuous, g^{-1}(G) is closed in Y. It implies that g^{-1}(G) is g*-closed in Y. Since f is g*-irresolute, $f^{-1}(g^{-1}$(G)) = (g o f)$^{-1}$(G) is g*-closed in X. Therefore g o f is contra g*-continuous map. □

Corollary 3.2. *If f: X → Y is g^*-irresolute map and g: Y → Z is contra g^*-continuous map, then g o f: X → Z is contra g^*-continuous map.*

Theorem 3.3. *If f: X → Y is gc-irresolute map and g: Y → Z is contra g^*-continuous map, then g o f: X → Z is contra g-continuous map.*

Proof. Let G be an open set in Z. Since g is contra g*-continuous map, g^{-1}(G) is g*-closed in Y. It implies that g-closed in Y. Since f is gc-irresolute, $f^{-1}(g^{-1}$(G)) = (g o f)$^{-1}$(G) is g-closed in X. Thus g o f is contra g-continuous map. □

Corollary 3.4. *If f: X → Y is gs-irresolute map and g: Y → Z is contra g^*-continuous map, then g o f: X → Z is contra gs-continuous map.*

Theorem 3.5. *Let $\{X_\lambda \mid \lambda \in \Omega\}$ be any family of topological spaces. If f: X → Π X_λ is a contra g^*-continuous map, then Pr_λ o f: X → X_λ is contra g^*-continuous for each $\lambda \in \Omega$, where Pr_λ is the projection of ΠX_λ onto X_λ.*

Proof. We shall consider a fixed $\lambda \in \Omega$. Suppose U_λ is an arbitrary open set in X_λ. Then $Pr_\lambda{}^{-1}(U_\lambda)$ is open in ΠX_λ. Since f is contra g*-continuous, we have by definition $f^{-1}(Pr_\lambda{}^{-1}(U_\lambda))$ = $(Pr_\lambda$ o f$)^{-1}(U_\lambda)$ is g*-closed in X. Therefore Pr_λ o f is contra g*-continuous. □

Theorem 3.6. *Let f: X → Y be a map and g: X → X × Y the graph function of f, defined by g(x) = (x, f(x)) for every $x \in X$. If g is contra g^*-continuous, then f is contra g^*-continuous.*

Proof. Let U be an open set in Y. Then X × U is an open set in X × Y. It follows from Theorem 2.7 that f^{-1}(U) = g^{-1}(X × U) is g*-closed in X. Thus, f is contra g*-continuous. □

Definition 3.7. *A space X is called ${}_\alpha T^*_{1/2}$ if every g^*-closed set is α-closed.*

4. Relation with Separation Axioms

Theorem 4.1. *Let f: X → Y be a contra g^*-continuous map. If X is an ${}_\alpha T^*_{1/2}$ space, then f is contra α-continuous map.*

Proof. Let V be an open set of Y. Since f is contra g*-continuous, f^{-1}(V) is a g*-closed set of X. Since X is an ${}_\alpha T^*_{1/2}$ space, f^{-1}(V) is an α-closed set of X. Therefore f is a contra α-continuous map. □

Theorem 4.2. *Let f: X → Y be a contra semi-continuous map. If X is an T_c space, then f is contra g^*-continuous map.*

Proof. Let V be an open set of Y. Since f is contra semi-continuous, f^{-1}(V) is a semi-closed set of X and hence gs-closed in X. Since X is an T_c space, f^{-1}(V) is a g^*-closed set of X. Therefore f is a contra g^*-continuous map. □

Theorem 4.3. *Let f: X → Y be a contra α-continuous map. If X is an $_\alpha T_c$ space, then f is contra g^*-continuous map.*

Proof. Let V be an open set of Y. Since f is contra α-continuous, f^{-1}(V) is an α-closed set of X and hence αg-closed in X. Since X is an $_\alpha T_c$ space, f^{-1}(V) is a g^*-closed set of X. Therefore f is contra g^*-continuous map. □

Theorem 4.4. *Let f: X → Y be a contra gs-continuous map. If X is $_a T_b$ space, then f is contra g^*-continuous map.*

Proof. Let V be an open set of Y. Since f is contra gs-continuous, f^{-1}(V) is gs-closed set of X. Since X is T_b space, it is a closed set of X. It implies that f^{-1}(V) is g^*-closed set of X. Therefore f is a contra g^*-continuous map. □

Theorem 4.5. *Let f: X → Y be a surjective, preclosed, contra g^*-continuous map and X be T_b space, then Y is locally indiscrete.*

Proof. Suppose V is open set in Y. By hypothesis, f is contra g^*-continuous map, f^{-1}(V) is g^*-closed and hence gs-closed in X. Since X is T_b space, f^{-1}(V) is closed in X. Since f is preclosed, V is preclosed in Y. Now we have cl(V) = cl(int(V)) ⊆ V. This means that V is closed in Y. Thus Y is locally indiscrete. □

Theorem 4.6. *Let X and Z be any topological spaces and Y be T_b space. If f: X → Y is g^*-continuous map and g: Y → Z is contra gs-continuous map, then g o f: X→ Z is contra g^*-continuous map.*

Proof. Let G be an open set in Z. Since g is contra gs-continuous, g^{-1}(G) is gs-closed in Y. But Y is T_b space, g^{-1}(G) is closed in Y. Since f is g^*-continuous, $f^{-1}(g^{-1}$(G)) = (g o f)$^{-1}$(G) is g^*-closed in X. Therefore gof is contra g^*-continuous map. □

Corollary 4.7. *Let X and Z be any topological spaces and Y be T_b space. If f: X → Y is g^*-irresolute map and g: Y → Z is contra gs-continuous map, then g o f: X → Z is contra g^*-continuous map.*

Theorem 4.8. *Let X and Z be any topological spaces and Y be a T_c space. If f: X → Y is g^*-irresolute map and g: Y → Z is contra-continuous map, then g o f: X → Z is contra g^*-continuous map.*

Proof. Let G be an open set in Z. Since g is contra-continuous, g^{-1}(G) is closed and hence gs-closed in Y. But Y is a T_c space, g^{-1}(G) is g^*-closed in Y. Since f is g^*-irresolute, $f^{-1}(g^{-1}$(G)) = (g o f)$^{-1}$(G) is g^*-closed in X. Therefore g o f is contra g^*-continuous map. □

Corollary 4.9. *Let X and Z be any topological spaces and Y be an $_\alpha T_c$ space. If f: X → Y is g^*-irresolute map and g: Y → Z is contra-continuous map, then g o f: X → Z is contra g^*-continuous map.*

Definition 4.10. *A space X is called g^*-connected provided that X is not the union of two disjoint non-empty g^*-open sets.*

Theorem 4.11. *If f: X → Y is contra g^*-continuous surjection and X is g^*-connected, then Y is connected.*

Proof. Suppose that Y is not connected space. There exist non-empty disjoint open sets V_1 and V_2 such that Y = $V_1 \cup V_2$. Therefore V_1 and V_2 are clopen in Y. Since f is contra g^*-continuous, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are g^*-open in X. Moreover, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are non-empty disjoint and X = $f^{-1}(V_1) \cup f^{-1}(V_2)$. This shows that X is not g^*-connected. This contradicts that Y is not connected assumed. Hence Y is connected. □

Definition 4.12. *A space X is said to be*

1. *g^*-compact (strongly S-closed [8]) if every g^*-open (respectively closed) cover of X has a finite subcover;*
2. *countably g^*-compact (strongly countably S-closed) if every countable cover of X by g^*-open (respectively closed) sets has a finite subcover;*
3. *g^*-Lindelof (strongly S- Lindelof) if every g^*-open (respectively closed) cover of X has a countable subcover.*

Theorem 4.13. *The contra g^*-continuous images of g^*-compact (g^*-Lindelof, countably g^*-compact) spaces are strongly S-closed (respectively strongly S- Lindelof, strongly countably S-closed).*

Proof. Suppose that f: X → Y is a contra g*-continuous surjection. Let $\{V_\alpha : \alpha \in I\}$ be any closed cover of Y. Since f is contra g*-continuous, then $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is an g*-open cover of X and hence there exists a finite subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Therefore, we have $Y = \cup\{V_\alpha : \alpha \in I_0\}$ and Y is strongly S-closed.
The other proofs can be obtained similarly. □

Definition 4.14. *A space X is said to be*

1. *g^*-closed-compact if every g^*-closed cover of X has a finite subcover;*
2. *countably g^*-closed-compact if every countable cover of X by g^*-closed sets has a finite subcover;*
3. *g^*-closed-Lindelof if every g^*-closed cover of X has a countable subcover.*

Theorem 4.15. *The contra g^*-continuous images of g^*-closed-compact (g^*-closed-Lindelof, countably g^*-closed-compact) spaces are compact (respectively Lindelof, countably compact).*

Proof. Suppose that f: X → Y is a contra g*-continuous surjection. Let $\{V_\alpha : \alpha \in I\}$ be any open cover of Y. Since f is contra g*-continuous, then $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is a g*-closed cover of X. Since X is g*-closed-compact, there exists a finite subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Therefore, we have $Y = \cup\{V_\alpha : \alpha \in I_0\}$ and Y is compact.
The other proofs can be obtained similarly. □

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