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Neighborhood Indices of Nanostructures

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Abstract: A topological index is a numerical parameter mathematically derived from the graph structure. In this study, we propose the modified first neighborhood index, neighborhood inverse degree, neighborhood zeroth order index, F-neighborhood index and general first neighborhood index of a graph. Also we introduce the first neighborhood polynomial, total neighborhood polynomial and F-neighborhood polynomial of a graph. Furthermore we compote exact formulas for line graphs of subdivision graphs of 2D-lattice, nanotube and nantorus of $TUC_4C_8[p, q]$.

MSC: 05C05, 05C012, 05C090.

Keywords: F-neighborhood index, general first neighborhood index, nanostructures.

1. Introduction

A molecular graph is graph such that its vertices correspond to the atoms and the edges to the bonds. Chemical Graph Theory is a branch of Mathematical Chemistry which has an important effect on the development of the Chemical Sciences. Several topological indices have found many applications, especially, in QSPR/QSAR study, see [1, 2]. Let G be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. The edge connecting the vertices u and v will be denoted by uv. Let $N_G(v) = \{u : uv \in E(G)\}$. Let $S_G(v) = \sum_{u \in N_G(v)} d_G(u)$ be the degree sum of neighbor vertices. The line graph $L(G)$ of a graph G is the graph whose vertex set corresponds to the edges of G such that two vertices of $L(G)$ are adjacent if the corresponding edges of G are adjacent. The subdivision graph $S(G)$ of G is the graph obtained from G by replacing each of its edges by a path of length two. For undefined term and notation, we refer the reader to [3]. We need the following results.

Lemma 1.1. *Let G be a (p, q) graph. Then $S(G)$ has $p + q$ vertices and $2q$ edges.*

Lemma 1.2. *Let G be a (p, q) graph. Then $L(G)$ has q vertices and $\frac{1}{2}\sum_{i=1}^{p} d_G(u)^2 - q$ edges.*

In [4], Graovac introduced the fifth M_1 and M_2 Zagreb indices, defined as

$$M_1G_5(G) = \sum_{uv \in E(G)} [S_G(u) + S_G(v)], \quad M_2G_5(G) = \sum_{uv \in E(G)} S_G(u) S_G(v).$$

In [5], Kulli proposed the fifth hyper M_1 and M_2 Zagreb indices, defined as

$$HM_1G_5(G) = \sum_{uv \in E(G)} [S_G(u) + S_G(v)]^2, \quad HM_2G_5(G) = \sum_{uv \in E(G)} [S_G(u) S_G(v)]^2.$$

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Recently, the fifth multiplicative Zagreb indices [6], fifth multiplicative hyper Zagreb indices [7], fifth multiplicative connectivity indices [7] were introduced and studied. Recently, the first neighborhood Zagreb index was introduced and studied by Basavanagoud [8] and Mondal [9], defined as

$$NM_1(G) = \sum_{u \in V(G)} S_G(u)^2.$$

The total neighborhood index of a graph G is expressed as [8, 9]

$$T_n(G) = \sum_{u \in V(G)} S_G(u).$$

We introduce the following the neighborhood Zagreb indices. The modified first neighborhood index of a graph G is defined as

$$^{m}NM_1(G) = \sum_{u \in V(G)} \frac{1}{S_G(u)^2}.$$

The neighborhood inverse degree of a graph G is defined as

$$NID(G) = \sum_{u \in V(G)} \frac{1}{S_G(u)}.$$

The neighborhood zeroth order index of a graph G is defined as

$$NZ(G) = \sum_{u \in V(G)} \frac{1}{\sqrt{S_G(u)}}.$$

The F-neighborhood index of a graph G is defined as

$$FN(G) = \sum_{u \in V(G)} S_G(u)^3.$$

The general first neighborhood index of a graph G is defined as

$$NM_1^a(G) = \sum_{u \in V(G)} S_G(u)^a \tag{1}$$

where a is a real number. We also introduce the first neighborhood polynomial, total neighborhood polynomial, F-neighborhood polynomial of a graph, defined as

$$NM_1(G, x) = \sum_{u \in V(G)} x^{S_G(u)^2}. \tag{2}$$

$$T_n(G, x) = \sum_{u \in V(G)} x^{S_G(u)}. \tag{3}$$

$$FN(G, x) = \sum_{u \in V(G)} x^{S_G(u)^3}. \tag{4}$$

For a graph G, the modified version of neighborhood connectivity index and its polynomial are defined as

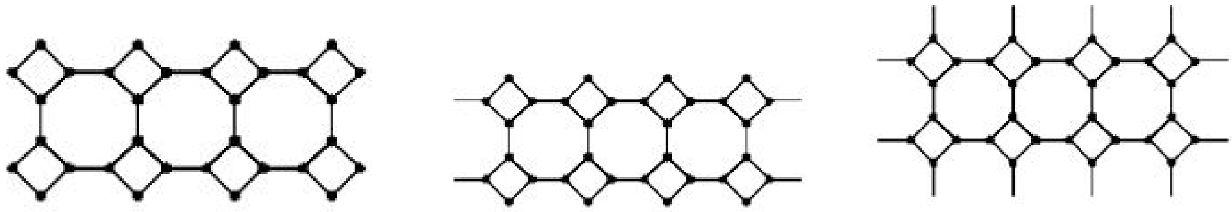
$$NC(G) = \sum_{u \in V(G)} d_G(u) S_G(u), \tag{5}$$

$$NC(G, x) = \sum_{u \in V(G)} S_G(u) x^{d_G(u)}. \tag{6}$$

In this paper, we deduce explicit formulas for determining the modified first neighborhood index, neighborhood inverse degree, F-neighborhood index and general first neighborhood index of line graphs of subdivision graphs of 2-D lattice, nanotube and nanotorus of $TUC_4C_8[p, q]$. For more results on topological indices of line graphs of subdivision graphs see [10, 11, 12, 13, 14].

2. 2-D lattice, Nanotube and Nanotorus of $TUC_4C_8[p,q]$

In this section, we consider the graph of 2-D lattice, nanotube and nanotorus of $TUC_4C_8[p,q]$, where p is the number of squares in a row and q is the number of rows of squares. These graphs are presented in Figure 1.

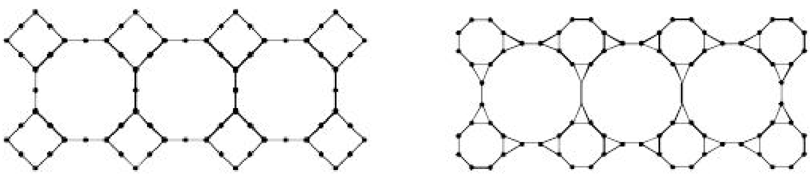


(a) 2D-lattice of $TUC_4C_8[4,2]$ (b) $TUC_4C_8[4,2]$ nanotube (c) $TUC_4C_8[4,2]$ nanotorus

Figure 1:

3. Results for 2-*D* Lattice of $TUC_4C_8[p,q]$

The line graph of subdivision graph of 2-D lattice of $TUC_4C_8[p,q]$ is shown in Figure 2(b).



(a) Subdivision graph of 2-D lattice of $TUC_4C_8[4,2]$ (b) Line graph of subdivision graph of 2-D lattice of $TUC_4C_8[4,2]$

Figure 2:

Let G be a line graph of subdivision graph of 2-D lattice of $TUC_4C_8[p,q]$. The 2-D lattice of $TUC_4C_8[p,q]$ is a graph with $4pq$ vertices and $6pq-p-q$ edges. By Lemma 1.1, the subdivision graph of 2-D lattice of $TUC_4C_8[p,q]$ is a graph with $10pq-p-q$ vertices and $2(6pq-p-q)$ edges, Thus by Lemma 1.2, G has $2(6pq-p-q)$ vertices and $18pq-5p-5q$ edges. Clearly, the vertices of G are either of degree 2 or 3, see Figure 2(b). The vertex partition based on the degree sum of neighbor vertices is obtained as given in Table 1 and Table 2.

$S_G(u) \setminus u \in V(G)$	4	5	8	9
Number of vertices	8	$4(p+q-2)$	$4(p+q-2)$	$2(6pq-5p-5q+4)$

Table 1: Vertex partition of G when $p>1$, $q>1$

$S_G(u) \setminus u \in V(G)$	4	5	8	9
Number of vertices	8	$4(p-1)$	$4(p-1)$	$2(p-1)$

Table 2: Vertex partition of G when $p>1$, $q=1$

Theorem 3.1. *Let G be a line graph of subdivision graph of 2D-lattice of $TUC_4C_8[p,q]$. Then the general first neighborhood index of G is*

$$NM_1^a II\left(G\right) = 8\times 4^a + \left(5^a+8^a\right)4\left(p+q-2\right) + 2\left(6pq-5p-5q+4\right)9^a, \quad \textit{if } p>1,\ q>1, \tag{7}$$

$$= 8\times 4^a + \left(4\times 5^a + 4\times 8^a + 2\times 9^a\right)\left(p-1\right), \quad \textit{if } p>1,\ q=1. \tag{8}$$

Proof. **Case 1:** Let $p>1$ and $q>1$.

From equation (1) and by using Table 1, we obtain

$$NM_1^a\left(G\right) = \sum_{u\in V(G)} S_G\left(u\right)^a$$

$$= 8\times 4^a + 4\left(p+q-2\right)5^a + 4\left(p+q-2\right)8^a + 2\left(6pq-5p-5q+4\right)9^a$$

Thus, $NM_1^a\left(G\right) = 8\times 4^a + 4\left(p+q-2\right)\left(5^a+8^a\right) + 2\left(6pq-5p-5q+4\right)9^a$.

Case 2: Let $p>1$ and $q=1$.

From equation (1) and by using Table 2, we obtain

$$NM_1^a\left(G\right) = \sum_{u\in V(G)} S_G\left(u\right)^a$$

$$= 8\times 4^a + 4\left(p-1\right)5^a + 4\left(p-1\right)8^a + 2\left(p-1\right)9^a$$

Thus, $NM_1^a\left(G\right) = 8\times 4^a + \left(4\times 5^a + 4\times 8^a + 2\times 9^a\right)\left(p-1\right)$. □

We establish the following results by using Theorem 3.1.

Corollary 3.2 ([8]). *The first neighborhood Zagreb index of G is*

$$NM_1(G) = 972pq - 454(p+q) + 64, \quad \textit{when } p>1,\ q>1,$$

$$= 518p - 390, \quad \textit{when } p>1,\ q=1.$$

Proof. Put $a=2$ in equations (7) and (8), we get the desired results. □

Corollary 3.3. *The total neighborhood index of G is*

$$T_n(G) = 108pq - 38(p+q) + 72, \quad \textit{if } p>1,\ q>1,$$

$$= 70p - 38, \quad \textit{if } p>1,\ q=1.$$

Proof. Put $a=1$ in equations (7) and (8), we get the desired results. □

Corollary 3.4. *The modified first neighborhood index of G is*

$${}^mNM_1\left(G\right) = \frac{2}{81}\left(6pq-5p-5q+4\right) + \left(\frac{1}{25}+\frac{1}{64}\right)4\left(p+q-2\right) + \frac{1}{2}, \quad \textit{if } p>1,\ q>1,$$

$$= \left(\frac{4}{25}+\frac{4}{64}+\frac{2}{81}\right)p + \frac{1}{2} - \left(\frac{4}{25}+\frac{4}{64}+\frac{2}{81}\right), \quad \textit{if } p>1,\ q=1.$$

Proof. Put $a=-2$ in equations (7) and (8), we get desired results. □

Corollary 3.5. *The neighborhood inverse degree of G is*

$$\begin{aligned} NID(G) &= \frac{2}{9}(6pq-5p-5q+4)+\frac{52}{40}+(p+q-2)+2, && \text{if } p>1,\ q>1,\\ &= \frac{141}{90}p+\frac{13}{30}, && \text{if } p>1,\ q=1. \end{aligned}$$

Proof. Put $a=-1$ in equations (7) and (8), we obtain the desired results. □

Corollary 3.6. *The neighborhood zeroth order index of G is*

$$\begin{aligned} NZ(G) &= \frac{2}{3}(6pq-5p-5q+4)+\left(\frac{1}{\sqrt{5}}+\frac{1}{\sqrt{8}}\right)4(p+q-2)+4, && \text{if } p>1,\ q>1,\\ &= \left(\frac{4}{\sqrt{5}}+\frac{4}{\sqrt{8}}+\frac{2}{3}\right)p+4-\left(\frac{4}{\sqrt{5}}+\frac{4}{\sqrt{8}}+\frac{2}{3}\right), && \text{if } p>1,\ q=1. \end{aligned}$$

Proof. Put $a=-\frac{1}{2}$ in equations (7) and (8), we get the desired results. □

Corollary 3.7. *The F-neighborhood index of G is*

$$\begin{aligned} FN(G) &= 8748pq-4742(p+q)+1248, && \text{when } p>1,\ q>1,\\ &= 4006p-3494, && \text{when } p>1,\ q=1. \end{aligned}$$

Proof. Put $a=3$ in equations (7) and (8), we obtain the desired results. □

Theorem 3.8. *Let G be a line graph of subdivision graph of 2D-lattice of $TUC_4C_8[p,q]$. Then*

$$\begin{aligned} NM_1(G,x) &= 8\times x^{16}+4(p+q-2)x^{25}+4(p+q-2)x^{64}+2(6pq-5p-5q+4)x^{81}, && \text{if } p>1,\ q>1,\\ &= 8\times x^{16}+4(p-1)x^{25}+4(p-1)x^{64}+2(p-1)x^{81}, && \text{if } p>1,\ q=1. \end{aligned}$$

Proof. **Case 1:** Let $p>1$ and $q>1$.

From equation (2) and by using Table 1, we have

$$\begin{aligned} NM_1(G,x) &= \sum_{u\in V(G)} x^{S_G(u)^2}\\ &= 8\times x^{16}+4(p+q-2)x^{25}+4(p+q-2)x^{64}+2(6pq-5p-5q+4)x^{81}. \end{aligned}$$

Case 2: Let $p>1$ and $q=1$.

From equation (2) and by using Table 2, we obtain

$$\begin{aligned} NM_1(G,x) &= \sum_{u\in V(G)} x^{S_G(u)^2}\\ &= 8\times x^{16}\times 4(p-1)x^{25}+4(p-1)x^{64}+2(p-1)x^{81}. \end{aligned}$$

□

Theorem 3.9. *Let G be a line graph of subdivision graph of 2D-lattice of $TUC_4C_8[p,q]$. Then*

$$\begin{aligned} T_n(G,x) &= 8\times x^{4}+4(p+q-2)x^{5}+4(p+q-2)x^{8}+2(6pq-5p-5q+4)x^{9}, && \text{if } p>1,\ q>1,\\ &= 8\times x^{4}+4(p-1)x^{5}+4(p-1)x^{8}+2(p-1)x^{9}, && \text{if } p>1,\ q=1. \end{aligned}$$

Proof. **Case 1:** Suppose $p > 1$and $q > 1$.

From equation (3) and by using Table 1, we deduce

$$\begin{aligned} T_n(G,x) &= \sum_{u \in V(G)} x^{S_G(u)} \\ &= 8 \times x^4 + 4(p+q-2)x^5 + 4(p+q-2)x^8 + 2(6pq - 5p - 5q + 4)x^9. \end{aligned}$$

Case 2: Suppose $p > 1$ and $q = 1$.

By using equation (3) and using Table 2, we derive

$$\begin{aligned} T_n(G,x) &= \sum_{u \in V(G)} x^{S_G(u)} \\ &= 8 \times x^4 + 4(p-1)x^5 + 4(p-1)x^8 + 2(p-1)x^9. \end{aligned}$$

□

Theorem 3.10. *Let G be a line graph of subdivision graph of 2-D lattice of $TUC_4C_8[p,q]$. Then*

$$\begin{aligned} FN(G,x) &= 8x^{64} + 4(p+q-2)x^{125} + 4(p+q-2)x^{512} + 2(6pq - 5p - 5q + 4)x^{729}, && \text{if } p > 1,\ q > 1, \\ &= 8x^{64} + 4(p-1)x^{125} + 4(p-1)x^{512} + 2(p-1)x^{729}, && \text{if } p > 1,\ q = 1. \end{aligned}$$

Proof. **Case 1:** Suppose $p > 1$, $q > 1$.

By using equation (4) and Table 1, we have

$$\begin{aligned} FN(G,x) &= \sum_{u \in V(G)} x^{S_G(u)^3} \\ &= 8x^{4^3} + 4(p+q-2)x^{5^3} + 4(p+q-2)x^{8^3} + 2(6pq - 5p - 5q + 4)x^{9^3}. \\ &= 8x^{64} + 4(p+q-2)x^{125} + 4(p+q-2)x^{512} + 2(6pq - 5p - 5q + 4)x^{729}. \end{aligned}$$

Case 2: Suppose $p > 1$, $q = 1$.

From equation (4) and by using Table 2, we obtain

$$\begin{aligned} FN(G,x) &= \sum_{u \in V(G)} x^{S_G(u)^3} \\ &= 8 \times x^{64} + 4(p-1)x^{125} + 4(p-1)x^{512} + 2(p-1)x^{729}. \end{aligned}$$

□

Theorem 3.11. *Let G be a line graph of subdivision graph 2D-lattice of $TUC_4C_8[p,q]$. Then the modified version of neighborhood connectivity index and its polynomial of G are given by*

$$\begin{aligned} NC(G) &= 324pq - 134(p+q) + 8, && \text{if } p > 1,\ q > 1, \\ &= 190p - 126, && \text{if } p > 1,\ q = 1. \\ NC(G,x) &= [20(p+q) - 8]x^2 + [108pq - 58(p+q) + 8]x^3, && \text{if } p > 1,\ q > 1, \\ &= (20p - 12)x^2 + (42p - 42)x^3, && \text{if } p > 1,\ q = 1. \end{aligned}$$

Proof. Consider the graph G which is shown in Figure 2(b). The vertex partitions of G are given in Table 3 and Table 4.

$d_G(u)$, $S_G(v)$	(2, 4)	(2, 5)	(3, 8)	(3, 9)
Number of vertices	8	$4(p+q-2)$	$4(p+q-2)$	$2(6pq-5p-5q+4)$

Table 3: Vertex partition of G when $p>1$, $q>1$

$d_G(u)$, $S_G(v)$	(2, 4)	(2, 5)	(3,8)	(3, 9)
Number of vertices	8	$4(p-1)$	$4(p-1)$	$2(p-1)$

Table 4: Vertex partition of G when $p>1$, $q=1$

Case 1: Suppose $p>1$, $q>1$.

(i). By using equation (5) and Table 3, we derive

$$\begin{aligned} NC(G) &= \sum_{u \in V(G)} d_G(u) S_G(u) \\ &= (2\times 4)\,8 + (2\times 5)\,4(p+q-2) + (3\times 8)4(p+q-2) + (3\times 9)\,2(6pq-5p-5q+4) \\ &= 324pq - 134(p+q) + 8 \end{aligned}$$

(ii). From equation (6) and using Table 3, we deduce

$$\begin{aligned} NC(G,x) &= \sum_{u \in V(G)} S_G(u)\, x^{d_G(u)} \\ &= 8\times 4x^2 + 4(p+q-2)\,5x^2 + 4(p+q-2)\,8x^3 + 2(6pq-5p-5q+4)\,9x^3 \\ &= [20(p+q)-8]\,x^2 + [108pq - 58(p+q)+8]\,x^3. \end{aligned}$$

Case 2: Suppose $p>1$, $q=1$.

(i). From equation (5) and by using Table 4, we have

$$\begin{aligned} NC(G) &= \sum_{u \in V(G)} d_G(u) S_G(u) \\ &= (2\times 4)\,8 + (2\times 5)\,4(p-1) + (3\times 8)\,4(p-1) + (3\times 9)\,2(p-1) \\ &= 190p - 126. \end{aligned}$$

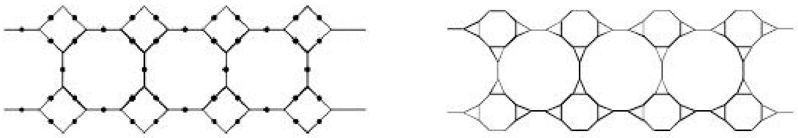
(ii). From equation (6) an by using Table 4, we obtain

$$\begin{aligned} NC(G,x) &= \sum_{u \in V(G)} S_G(u)\, x^{d_G(u)} \\ &= 8\times 4x^2 + 4(p-1)\,5x^2 + 4(p-1)\,8x^3 + 2(p-1)\,9x^3 \\ &= (20p-12)\,x^2 + (42p-42)\,x^3. \end{aligned}$$

□

4. Results for $TUC_4C_8[p,q]$ Nanotube

The line graph of subdivision graph of $TUC_4C_8[p,q]$ nanotube is shown in Figure 3 (b).



(a) Subdivision graph of $TUC_4C_8[4,2]$ nanotube

(b) Line graph of subdivision graph of $TUC_4C_8[4,2]$ nanotube

Figure 3:

Let H be a line graph of subdivision graph of $TUC_4C_8[p,q]$ nanotube. The graph of $TUC_4C_8[p,q]$ nanotube has $4pq$ vertices and $6pq-p$ edges. By Lemma 2.1, the subdivision graph of $TUC_4C_8[p,q]$ nanotube is a graph with $10pq-p$ vertices and $12pq-2p$ edges. Thus by Lemma 1.2, H has $12pq-2p$ vertices and $18pq-5p$ edges. Clearly the vertices of H are either of degree 2 or 3. The vertex partition based on the degree sum of neighbor vertices of each vertex is given in Table 5 and Table 6.

$S_H(u) \setminus u \in V(H)$	5	8	9
Number of vertices	$4p$	$4p$	$12pq-10p$

Table 5: Vertex partition of H if $p>1,\ q>1$

$S_H(u) \setminus u \in V(H)$	5	8	9
Number of vertices	$4p$	$4p$	$2p$

Table 6: Vertex partition of H if $p>1,\ q=1$

Theorem 4.1. *Let H be a line graph of subdivision graph of $TUC_4C_8[p,q]$ nanotube. Then the general first neighborhood index of H is*

$$NM_1^a(H) = (5^a+8^a)\,4p + 9^a\,(12pq-10p)\,, \quad \textit{if } p>1,\ q>1, \tag{9}$$

$$= (5^a+8^a)\,4p + 9^a \times 2p, \quad \textit{if } p>1,\ q=1. \tag{10}$$

Proof. **Case 1:** Suppose $p>1$ and $q>1$.

From equation (1) and by using Table 5, we deduce

$$\begin{aligned} NM_1^a(H) &= \sum_{u\in V(H)} S_H(u)^a \\ &= 4p\times 5^a \times 4p \times 8^a + (12pq-10p)\times 9^a \\ &= (5^a+8^a)\,4p + 9^a\,(12pq-10p)\,. \end{aligned}$$

Case 2: Suppose $p>1$ and $q=1$.

By using equation (1) and Table 6, we derive

$$\begin{aligned} NM_1^a(H) &= \sum_{u \in V(H)} S_H(u)^a \\ &= 4p \times 5^a \times 4p \times 8^a + 2p \times 9^a \\ &= (5^a + 8^a)\, 4p + 9^a \times 2p. \end{aligned}$$

□

We obtain the following results by Theorem 4.1.

Corollary 4.2 ([8]). . *The first neighborhood Zagreb index of H is*

$$\begin{aligned} NM_1(H) &= 972pq - 454p, \quad \text{if } p > 1,\ q > 1, \\ &= 518p, \quad \text{if } p > 1,\ q = 1. \end{aligned}$$

Proof. Put $a = 2$ in equations (9) and (10), we get the desired results. □

Corollary 4.3. *The total neighborhood index of H is*

$$\begin{aligned} T_n(H) &= 108pq - 38p, \quad \text{if } p > 1,\ q > 1, \\ &= 70p, \quad \text{if } p > 1,\ q = 1. \end{aligned}$$

Proof. Put $a = 1$ in equations (9) and (10), we get the desired results. □

Corollary 4.4. *The modified first neighborhood index of H is*

$$\begin{aligned} {}^mNM_1(H) &= \frac{4}{27}pq + \left(\frac{4}{25} + \frac{4}{64} - \frac{10}{81}\right)p, \quad \text{if } p > 1,\ q > 1, \\ &= \left(\frac{4}{25} + \frac{4}{64} + \frac{2}{81}\right)p, \quad \text{if } p > 1,\ q = 1. \end{aligned}$$

Proof. Put $a = -2$ in equations (9) and (10), we get the desired results. □

Corollary 4.5. *The neighborhood inverse degree of H is*

$$\begin{aligned} NID(H) &= \frac{4}{3}pq + \frac{17}{90}p, \quad \text{if } p > 1,\ q > 1, \\ &= \frac{137}{90}p, \quad \text{if } p > 1,\ q = 1. \end{aligned}$$

Proof. Put $a = -1$ in equations (9) and (10), we get the desired results. □

Corollary 4.6. *The neighborhood zeroth order index of H is*

$$\begin{aligned} NZ(H) &= 4pq + \left(\frac{4}{\sqrt{5}} + \frac{4}{\sqrt{8}} - \frac{10}{3}\right)p, \quad \text{if } p > 1,\ q > 1, \\ &= \left(\frac{4}{\sqrt{5}} + \frac{4}{\sqrt{8}} + \frac{2}{3}\right)p, \quad \text{if } p > 1,\ q = 1. \end{aligned}$$

Proof. Put $a = -\frac{1}{2}$ in equations (9) and (10), we get the desired results. □

Corollary 4.7. *The F-neighborhood index of H is*

$$\begin{aligned} FN(H) &= 8748pq - 4742p, \quad && \text{if } p > 1,\ q > 1, \\ &= 4006p, && \text{if } p > 1,\ q = 1. \end{aligned}$$

Proof. Put $a = 3$ in equations (9) and (10), we get the desired results. ☐

Theorem 4.8. *Let H be a line graph of subdivision graph of $TUC_4C_8[p, q]$ nanotube. Then*

$$\begin{aligned} NM_1(H, x) &= 4px^{25} + 4px^{64} + (12pq - 10p)\,x^{81}, \quad && \text{if } p > 1,\ q > 1, \\ &= 4px^{25} + 4px^{64} + 2px^{81}, && \text{if } p > 1,\ q = 1. \end{aligned}$$

Proof. **Case 1:** Suppose $p > 1$ and $q > 1$.

By using equation (2) and Table 5, we obtain

$$\begin{aligned} NM_1(H, x) &= \sum_{u \in V(H)} x^{S_H(u)^2} \\ &= 4px^{5^2} \times 4px^{8^2} + (12pq - 10p)\,x^{9^2} \\ &= 4px^{25} + 4px^{64} + (12pq - 10p)x^{81}. \end{aligned}$$

Case 2: Suppose $p > 1$, $q = 1$.

From equation (2) and by using Table 6, we obtain

$$\begin{aligned} NM_1(H, x) &= \sum_{u \in V(H)} x^{S_H(u)^2} \\ &= 4px^{5^2} + 4px^{8^2} + 2px^{9^2} \\ &= 4px^{25} + 4px^{64} + 2px^{81}. \end{aligned}$$

☐

Theorem 4.9. *Let H be a line graph of subdivision graph of $TUC_4C_8[p, q]$ nanotube. Then*

$$\begin{aligned} T_n(H, x) &= 4px^{5} + 4px^{8} + (12pq - 10p)\,x^{9}, \quad && \text{if } p > 1,\ q > 1, \\ &= 4px^{5} + 4px^{8} + 2px^{9}, && \text{if } p > 1,\ q = 1. \end{aligned}$$

Proof. **Case 1:** Suppose $p > 1$, $q > 1$.

From equation (3) and by using Table 5, we deduce

$$\begin{aligned} T_n(H, x) &= \sum_{u \in V(H)} x^{S_H(u)} \\ &= 4px^{5} + 4px^{8} + (12pq - 10p)\,x^{9}. \end{aligned}$$

Case 2: Suppose $p > 1$, $q = 1$.

By using equation (3) and Table 6, we derive

$$\begin{aligned} T_n(H, x) &= \sum_{u \in V(H)} x^{S_H(u)} \\ &= 4px^{5} + 4px^{8} + 2px^{9}. \end{aligned}$$

☐

Theorem 4.10. *Let H be a line graph of subdivision graph of $TUC_4C_8[p,q]$ nanotube. Then*

$$\begin{aligned} FN(H,x) &= 4px^{125} + 4px^{512} + (12pq-10p)\,x^{729}, && \text{if } p>1,\ q>1, \\ &= 4px^{125} + 4px^{512} + 2px^{729}, && \text{if } p>1,\ q=1. \end{aligned}$$

Proof. **Case 1:** Suppose $p>1$, $q>1$.

By using equation (4) and Table 5, we have

$$\begin{aligned} FN(H,x) &= \sum_{u\in V(H)} x^{S_H(u)^3} \\ &= 4px^{5^3} + 4px^{8^3} + (12pq-10p)\,x^{9^3} \\ &= 4px^{125} + 4px^{512} + (12pq-10p)\,x^{729}. \end{aligned}$$

Case 2: Suppose $p>1$, $q=1$.

From equation (4) and by using Table 6, we obtain

$$\begin{aligned} FN(H,x) &= \sum_{u\in V(H)} x^{S_H(u)^3} \\ &= 4px^{5^3} + 4px^{8^3} + 2px^{9^3} \\ &= 4px^{125} + 4px^{512} + 2px^{729}. \end{aligned}$$

□

Theorem 4.11. *Let H be a line graph of subdivision graph of $TUC_4C_8[p,q]$ nanotube. Then the modified version of neighborhood connectivity index and its polynomial of H are given by*

$$\begin{aligned} NC(H) &= 324pq - 134p, && \text{if } p>1,\ q>1, \\ &= 190p, && \text{if } p>1,\ q=1. \\ NC(H,x) &= 20px^2 + (108pq-58p)\,x^3, && \text{if } p>1,\ q>1, \\ &= 20px^2 + 50px^3, && \text{if } p>1,\ q=1. \end{aligned}$$

Proof. Consider the graph H which is shown in Figure 3(b). The vertex partitions of H are given in Table 7 and Table 8.

$d_H(u)$, $S_H(u)$	(2, 5)	(3, 8)	(3,9)
Number of vertices	$4p$	$4p$	$12pq-10p$

Table 7: Vertex partition of H when $p>1$, $q>1$

$d_H(u)$, $S_H(u)$	(2, 5)	(3, 8)	(3,9)
Number of vertices	$4p$	$4p$	$2p$

Table 8: Vertex partition of H when $p>1$, $q=1$

Case 1: Suppose $p>1$, $q>1$.

(i). From equation (5) and by using Table 7, we obtain

$$\begin{aligned}NC(H) &= \sum_{u\in V(H)} d_H(u)\, S_H(u)\\ &= (2\times 5)\,4p + (3\times 8)\,4p + (3\times 9)\,(12pq - 10p)\\ &= 324pq - 134p.\end{aligned}$$

(ii). From equation (6) and using Table 7, we have

$$\begin{aligned}NC(H,x) &= \sum_{u\in V(H)} S_H(u)\, x^{d_H(u)}\\ &= 4p\times 5x^2 + 4p\times 8x^3 + (12pq - 10p)\times 9x^3\\ &= 20px^2 + (108pq - 58p)\,x^3.\end{aligned}$$

Case 2: Suppose $p > 1$, $q = 1$.

(i). By using equation (5) and Table 8, we deduce

$$\begin{aligned}NC(H) &= \sum_{u\in V(H)} d_H(u)\, S_H(u)\\ &= (2\times 5)\,4p + (3\times 8)\,4p + (3\times 9)\,2p\\ &= 190p.\end{aligned}$$

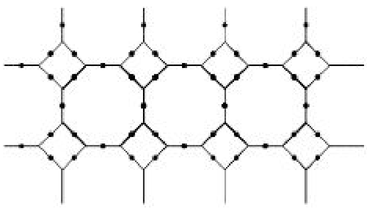
(ii). By using equation (6) and Table 8, we derive

$$\begin{aligned}NC(H,x) &= \sum_{u\in V(H)} S_H(u)\, x^{d_H(u)}\\ &= 4p\times 4x^2 + 4p\times 8x^3 + 2p\times 9x^3\\ &= 20px^2 + 50px^3.\end{aligned}$$

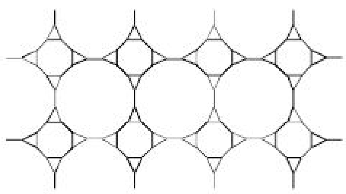
□

5. Results for $TUC_4C_8[p,q]$ Nanotorus

The line graph of subdivision graph of $TUC_4C_8[p,q]$ nanotorus is shown in Figure 4(b).



(a) Subdivision graph of $TUC_4C_8[p,q]$ nanotorus



(b) Line graph of subdivision graph of $TUC_4C_8[p,q]$ nanotorus

Figure 4:

Let K be a line graph of subdivision graph of $TUC_4C_8[p,q]$ nanotorus. A graph of $TUC_4C_8[p,q]$ nanotorus has $4pq$ vertices and $6pq$ edges. By Lemma 1, the subdivision graph of $TUC_4C_8[p,q]$ nanotorus is a graph with $10pq$ vertices and $12pq$ edges. Thus by Lemma 1.2, K has $12pq$ vertices and $18pq$ edges. Clearly the degree of each vertex is 3. The vertex partition based on the degree sum of neighbor vertices of each vertex is as given in Table 9.

$d_K(u), S_K(u) \setminus u \in V(K)$	(3, 9)
Number of vertices	$12pq$

Table 9: Vertex partition of K

Theorem 5.1. *Let K be a line graph of subdivision graph of $TUC_4C_8[p,q]$ nanotorus. Then*

(1). $NM_1^a(K) = 9^a \times 12pq$.

(2). $NM_1(K) = 972pq$ *[8].*

(3). $T_n(K) = 108pq$.

(4). ${}^mNM_1(K) = \frac{4}{27}pq$.

(5). $NID(K) = \frac{4}{3}pq$.

(6). $NZ(K) = 4pq$.

(7). $FN(K) = 729pq$.

(8). $NM_1(K,x) = 12pqx^{81}$.

(9). $T_n(K,x) = 12pqx^9$.

(10). $FN(K,x) = 12pqx^{729}$.

(11). $NC(K) = 324pq$.

(12). $NC(K,x) = 108pqx^3$.

Proof. By using definitions and Table 9, we obtain the desired results. □

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