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Strongly Nano $g^{\star}$-Closed Sets

Research Article

M. Rameshpandi[1,*]

1 Department of Mathematics, P. M. Thevar College, Usilampatti, Madurai District, Tamil Nadu, India.

Abstract: Throughout this paper $(U, \tau_R(X))$ represent non empty nano topological spaces on which no separation axioms are assumed, unlesses otherwise mentioned. For a subset H of $(U, \tau_R(X))$, $Ncl(H)$ and $Nint(H)$ represent the nano closure of H with respect to $(U, \tau_R(X))$ and the nano interior of H with respect to $(U, \tau_R(X))$ respectively.

Keywords: Nano closed, nano $g^{\star}$-closed set and strongly nano $g^{\star}$-closed set.

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1. Introduction and Preliminaries

Lellis Thivagar et al [4] introduced a nano topological space with respect to a subset X of an universe which is defined in terms of lower approximation and upper approximation and boundary region. The classical nano topological space is based on an equivalence relation on a set, but in some situation, equivalence relations are nor suitable for coping with granularity, instead the classical nano topology is extend to general binary relation based covering nano topological space. Veerakumar introduced and investigated between closed sets and $g^{\star}$-closed sets. The aim of this paper is to introduce and study stronger form of generalized nano $g^{\star}$-closed sets in a nano topological space. Also we investigate nano topological properties of strongly nano $g^{\star}$-closed sets. Throughout this paper $(U, \tau_R(X))$ represent non empty nano topological spaces on which no separation axioms are assumed, unlesses otherwise mentioned.

Throughout this paper $(U, \tau_R(X))$ and (V, σ) (or X and Y) represent nano topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset H of a space $(U, \tau_R(X))$, $Ncl(H)$ and $Nint(H)$ denote the nano closure of H and the nano interior of H respectively. We recall the following definitions which are useful in the sequel.

Definition 1.1 ([7]). *Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq G$.*

(1). The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \subseteq X\}$, where $R(x)$ denotes the equivalence class determined by x.

(2). The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \cap X \neq \phi\}$.

* *E-mail: proframesh9@gmail.com*

(3). The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Proposition 1.2 ([4]). *If $(U; R)$ is an approximation space and $X; YU$; then*

(1). $L_R(X) \subseteq X \subseteq G_R(X)$;

(2). $L_R(\phi) = U_R(\phi) = \phi$ and $L_R(U) = U_R(U) = U$;

(3). $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$;

(4). $U_R(X \cap Y) \subseteq G_R(X) \cap U_R(Y)$;

(5). $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$;

(6). $L_R(X \cap Y) \subseteq L_R(X) \cap L_R(Y)$;

(7). $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq G_R(Y)$ whenever $X \subseteq Y$;

(8). $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$;

(9). $U_R U_R(X) = L_R U_R(X) = U_R(X)$;

(10). $L_R L_R(X) = U_R L_R(X) = L_R(X)$.

Definition 1.3 ([4]). *Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq G$. Then by the Property 1.2, $R(X)$ satisfies the following axioms:*

(1). U and $\phi \in \tau_R(X)$,

(2). The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,

(3). The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called as nano open sets and $[\tau_R(X)]^c$ is called as the dual nano topology of $[\tau_R(X)]$.

Remark 1.4 ([4]). *If $[\tau_R(X)]$ is the nano topology on U with respect to X, then the set $B = \{U, \phi, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.*

Definition 1.5 ([4]). *If $(U, \tau_R(X))$ is a nano topological space with respect to X and if $A \subseteq G$, then the nano interior of A is defined as the union of all nano open subsets of A and it is denoted by $Nint(H)$. That is, $Nint(H)$ is the largest nano open subset of A. The nano closure of A is defined as the intersection of all nano closed sets containing A and it is denoted by $Ncl(H)$. That is, $Ncl(H)$ is the smallest nano closed set containing H.*

Definition 1.6 ([4]). *A subset H of a nano topological space $(U, \tau_R(X))$ is called;*

(1). nano pre open set if $H \subseteq Nint(Ncl(H))$.

(2). nano semi open set if $H \subseteq Ncl(Nint(H))$.

(3). nano α-open set if $H \subseteq Nint(Ncl(Nint(H)))$.

The complements of the above mentioned sets are called their respective closed sets.

Definition 1.7. *A subset H of a nano topological space $(U, \tau_R(X))$ is called;*

(1). nano g-closed [1] if $Ncl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano open.

(2). nano sg-closed set [2] if $Nscl(H) \subseteq G$, whenever $H \subseteq G$, G is nano semi open.

(3). nano gs-closed set [2] if $Nscl(H) \subseteq G$ whenever $H \subseteq G$, G is nano open.

(4). nano $g\alpha$-closed [10] if $N\alpha cl(H) \subseteq G$ whenever $H \subseteq G$ and G is nano α-open.

(5). nano αg-closed set [10] if $N\alpha cl(H) \subseteq G$ whenever $H \subseteq G$ and G is nano open.

(6). nano rg-closed set [9] if $Ncl(H) \subseteq G$ whenever $H \subseteq G$ and G is nano regular open.

(7). nano gp-closed set [3] if $Npcl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano open.

(8). nano gpr-closed set [6] if $Npcl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano regular open.

(9). nano $g^\star$-closed set [8] if $Ncl(H) \subseteq G$ whenever $H \subseteq G$ and G is g-open.

2. Strongly Nano $g^\star$-closed Sets

Definition 2.1. *A subset H of a space $(U, \tau_R(X))$ is called strongly nano $g^\star$-closed set if $Ncl(Nint(H)) \subseteq G$ whenever $H \subseteq G$ and G is nano g-open.*

Example 2.2. *Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b, c\}, \{d\}\}$ and $X = \{b, d\}$. Then the nano topology $\tau_R(X) = \{\phi, \{d\}, \{b, c\}, \{b, c, d\}, U\}$.*

(1). then $\{c\}$ is strongly nano $g^\star$-closed.

(2). then $\{d\}$ is not strongly nano $g^\star$-closed.

Theorem 2.3. *In a space $(U, \tau_R(X))$, every nano closed set is strongly nano $g^\star$-closed set.*

Proof. The proof is immediate from the definition of nano closed set. □

Remark 2.4. *The converse of the Theorem 2.3 need not be true from the following Example.*

Example 2.5. *In Example 2.2, $\{c\}$ is strongly nano $g^\star$-closed set but not nano closed set.*

Theorem 2.6. *If a subset H of a nano topological space $(U, \tau_R(X))$ is nano $g^\star$-closed then it is strongly nano $g^\star$-closed in $(U, \tau_R(X))$ but not conversely.*

Proof. Suppose H is nano $g^\star$-closed. Let G be an nano open set containing H in $(U, \tau_R(X))$. Then G contains $Ncl(H)$. Now $G \supseteq Ncl(H) \supseteq Ncl(Nint(H))$. Thus H is strongly nano $g^\star$-closed. □

Remark 2.7. *The converse of the Theorem 2.6 need not be true from the following example.*

Example 2.8. *In Example 2.2, $\{b\}$ is strongly nano $g^\star$-closed but not nano $g^\star$-closed set.*

Theorem 2.9. *If H is a subset of a nano topological space $(U, \tau_R(X))$ is nano open and strongly nano $g^\star$-closed then it is nano closed.*

Proof. Suppose a subset H of $(U, \tau_R(X))$ is both nano open and strongly nano $g^{\star}$-closed. Now $H \supseteq Ncl(Nint(H)) \supseteq Ncl(H)$. Therefore $H \supseteq Ncl(H)$. Since $Ncl(H) \supseteq H$. We have $H \subseteq Ncl(H)$. Thus H is nano closed. □

Corollary 2.10. *If H is both nano open and strongly nano $g^{\star}$-closed then it is both nano regular open and nano regular closed.*

Proof. As H is nano open $H = Nint(H) = Nint(Ncl(H))$, since H is nano closed. Thus H is nano regular open . Again H is nano open in $(U, \tau_R(X))$, $Ncl(Nint(H)) = Ncl(H)$. As H is nano closed $Ncl(Nint(H)) = H$. Thus H is nano regular closed. □

Corollary 2.11. *If H is both nano open and strongly nano $g^{\star}$-closed then it is nano rg-closed*

Theorem 2.12. *If a subset H of a nano topological space $(U, \tau_R(X))$ is both strongly nano $g^{\star}$-closed and nano semi open then it is nano $g^{\star}$-closed.*

Proof. Suppose H is both strongly nano $g^{\star}$-closed and nano semi open in $(U, \tau_R(X))$, Let G be an nano open set containing H. As H is strongly nano $g^{\star}$-closed , $G \supseteq Ncl(Nint(H))$. Now $G \supseteq Ncl(H)$. since H is nano semi open . Thus H is nano $g^{\star}$-closed in $(U, \tau_R(X))$. □

Corollary 2.13. *If H subset H of a nano topological space $(U, \tau_R(X))$ is both strongly nano $g^{\star}$-closed and nano open then it is nano $g^{\star}$-closed set.*

Proof. As every nano open set is nano semi open by the above theorem the proof follows. Strongly nano $g^{\star}$-closed sets in nano topological spaces. □

Theorem 2.14. *A set H is strongly nano $g^{\star}$-closed $\iff$ $Ncl(Nint(H)) - H$ contains no non empty nano closed set.*

Proof. Necessary : Suppose that F is non empty nano closed subset of $Ncl(Nint(H))$. Now $F \subseteq Ncl(Nint(H)) - H$ implies $F \subseteq Ncl(Nint(H)) \cap H^c$, since $Ncl(Nint(H)) - H = Ncl(Nint(H)) \cap H^c$. Thus $F \subseteq Ncl(Nint(H))$. Now $F \subseteq H^c$ implies $H \subseteq F^c$. Here F^c is nano g-open and H is strongly nano $g^{\star}$-closed, we have $Ncl(Nint(H)) \subseteq F^c$. Thus $F \subseteq (Ncl(Nint(H)))^c$. Hence $F \subseteq (Ncl(Nint(H))) \cap (Ncl(Nint(H)))^c = \phi$. Therefore $F = \phi \Rightarrow Ncl(Nint(H)) - H$ contains no non empty nano closed sets.

Sufficient: Let $H \subseteq G$, G is nano g-open. suppose that $Ncl(Nint(H))$ is not contained in G then $(Ncl(Nint(H)))^c$ is a non empty nano closed set of $Ncl(Nint(H)) - H$ which is a contradiction. Therefore $Ncl(Nint(H)) \subseteq G$ and hence H is strongly nano $g^{\star}$ -closed. □

Corollary 2.15. *A strongly nano $g^{\star}$-closed set H is nano regular closed $\iff$ $Ncl(Nint(H)) - H$ is nano closed and $Ncl(Nint(H)) \supseteq H$.*

Proof. Assume H that H is nano regular closed. Since $Ncl(Nint(H)) = H$, $Ncl(Nint(H)) - H = \phi$ is nano regular closed and hence nano closed.

conversely assume that $Ncl(Nint(H)) - H$ is nano closed. By the above theorem $Ncl(Nint(H)) - H$ contains no nonempty nano closed set. Therefore $Ncl(Nint(H)) - H = \phi$. Thus H is nano regular closed. □

Theorem 2.16. *Suppose that $K \subseteq H \subseteq U$, H is strongly nano $g^{\star}$-closed set-closed set relative to H and that both nano open and strongly nano $g^{\star}$-closed subset of U then H is strongly nano $g^{\star}$-closed set relative to U.*

Proof. Let $H \subseteq G$ and G be an nano open set in U. But given that $K \subseteq H \subseteq U$, therefore $K \subseteq H$ and $K \subseteq G$. This implies $K \subseteq H \subseteq G$. Since K is strongly nano $g^{\star}$- closed relative to H, $Ncl(Nint(K)) \subseteq H \cap G$. (ie) $H \cap Ncl(Nint(K)) \subseteq H \cap G$. This implies $H \cap (Ncl(Nint(K))) \subseteq G$. Thus $(H \cap (Ncl(Nint(K)))) \cup (Ncl(Nint(K)))^c \subseteq G \cup (Ncl(Nint(K)))^c$ implies $A \cup (Ncl(Nint(K)))^c \subseteq G \cup (Ncl(Nint(K)))^c$. since H is strongly nano $g^{\star}$closed in U, we have $(Ncl(Nint(H))) \subseteq G \cup (Ncl(Nint(K)))^c$. Also $K \subseteq H \Rightarrow Ncl(Nint(K)) \subseteq Ncl(Nint(H))$. Thus $Ncl(Nint(K)) \subseteq Ncl(Nint(H)) \subseteq G \cup (Ncl(Nint(K)))^c$. Therefore K is strongly nano $g^{\star}$-closed set relative to U. □

Corollary 2.17. *Let H be strongly nano $g^{\star}$-closed and suppose that F is closed then $H \cap F$ is strongly nano $g^{\star}$-closed set.*

Proof. To show that $H \cap F$ is strongly nano $g^{\star}$-closed, we have to show $Ncl(Nint(K \cap F)) \subseteq G$ whenever $H \cap F \subseteq G$ and G is nano g-open. $H \cap F$ is nano closed in H and so strongly nano $g^{\star}$-closed in H. By the above theorem $H \cap F$ is strongly nano $g^{\star}$-closed in U. Since $H \cap F \subseteq H \subseteq U$. □

Theorem 2.18. *If H is strongly nano $g^{\star}$-closed and $H \subseteq K \subseteq Ncl(Nint(H))$ then K is strongly nano $g^{\star}$-closed .*

Proof. Given that $K \subseteq Ncl(Nint(H))$ then $Ncl(Nint(K)) \subseteq Ncl(Nint(H))$,
$Ncl(Nint(K)) - K \subseteq Ncl(Nint(H)) - H$. Since $H \subseteq K$. As H is strongly nano $g^{\star}$-closed by the above theorem $Ncl(Nint(H)) - H$ contains no non empty closed set, $Ncl(Nint(K)) - K$ contains no empty nano closed set. Again by theorem 3.13, K is strongly nano $g^{\star}$-closed set. □

Theorem 2.19. *Let $H \subseteq V \subseteq U$ and suppose that H is strongly nano $g^{\star}$-closed in U then H is strongly nano $g^{\star}$-closed relative to V.*

Proof. Given that $H \subseteq V \subseteq U$ and H is strongly nano $g^{\star}$-closed in U. To show that H is strongly nano $g^{\star}$-closed relative to V, let $H \subseteq V \cap G$, where G is nano g-open in U. Since H is strongly nano $g^{\star}$-closed in U, $H \subseteq G$ implies $Ncl(Nint(H)) \subseteq G$.
(ie) $V \cap Ncl(Nint(H)) \subseteq V \cap G$, where $V \cap Ncl(Nint(H))$ is nano closure of nano interior of H in V. Thus H is strongly nano $g^{\star}$-closed relative to V. □

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