Geometric-Arithmetic Reverse and Sum Connectivity Reverse Indices of Silicate and Hexagonal Networks

Research Article

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Abstract: We introduce a new index known as geometric-arithmetic reverse index of a molecular graph. In this paper, we compute geometric-arithmetic reverse index and sum connectivity reverse index of different chemically interesting networks like silicate networks and hexagonal networks.

MSC: 05C05, 05C12, 05C35.

Keywords: Geometric-arithmetic reverse index, sum connectivity reverse index, silicate network, hexagonal network.

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1.   Introduction

Let G be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. Let $d_G(v)$ denote the degree of a vertex v in G. Let $\Delta(G)$ denote the maximum degree among the vertices of G. The reverse vertex degree of a vertex u in G is defined as $c_u = \Delta(G) - d_G(u) + 1$. The reverse edge connecting the reverse vertices u and v will be denoted by uv. For all further notation and terminology we refer to reader to [1]. Chemical graph theory is a branch of Mathematical Chemistry which has an important effect on the development of the chemical sciences. A topological index is a numerical parameter mathematically derived from the graph structure. Numerous such topological indices have been considered in Theoretical Chemistry. Recently we introduced the atom bond connectivity reverse index [2] of a graph G as

$$ABCC(G) = \sum_{uv \in E(G)} \sqrt{\frac{c_u + c_v - 2}{c_u c_v}}.$$

The sum connectivity reverse index was introduced by Kulli in [3]. The sum connectivity reverse index of a graph G is defined as

$$SC(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{c_u + c_v}}.$$

Recently some reverse indices were studied, for example, in [4–6]. We now introduce the geometric-arithmetic reverse index of a graph G as

$$GAC(G) = \sum_{uv \in E(G)} \frac{2\sqrt{c_u c_v}}{c_u + c_v}. \tag{1}$$

Recently several topological indices were studied, for example, in [7–17]. In this paper, the geometric-arithmetic reverse index and sum connectivity reverse index of silicate networks and hexagonal networks are computed. For silicate networks and hexagonal networks see [18].

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2.   Results for Silicate Networks

Silicates are obtained by fusing metal oxide or metal carbonates with sand. A silicate network is symbolized by SL_n where n is the number of hexagons between the center and boundary of SL_n. A 2-dimensional silicate network is depicted in Figure 1.
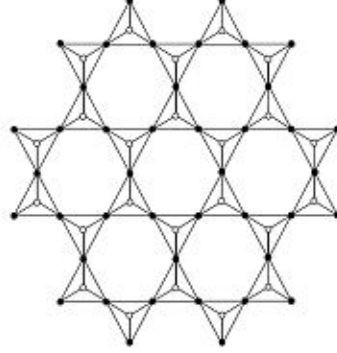

Figure 1: A 2-dimensional silicate network

Let G be the graph of silicate network SL_n. From Figure 1, it is easy to see that the vertices of SL_n are either of degree 3 or 6. Then $\Delta(G) = 6$. By algebraic method, we obtain that $|V(SL_n)| = 15n^2 + 3n$ and $|E(SL_n)| = 36n^2$. In SL_n, by algebraic method, there are three types of edges based on the degree of the end vertices of each edge as follows:

$$E_{33} = \{uv \in E(G)|d_G(u) = d_G(v) = 3\}, \qquad |E_{33}| = 6n.$$

$$E_{36} = \{uv \in E(G)|d_G(u) = 3, d_G(v) = 6\}, \quad |E_{36}| = 18n^2 + 6n.$$

$$E_{66} = \{uv \in E(G)|d_G(u) = d_G(v) = 6\}, \qquad |E_{66}| = 18n^2 - 12n.$$

We have $c_u = \Delta(G) - d_G(u) + 1 = 7 - d_G(u)$. Thus there are three types of reverse edges of follows:

$$CE_{44} = \{uv \in E(G)|c_u = c_v = 4\}, \qquad |CE_{44}| = 6n.$$

$$CE_{41} = \{uv \in E(G)|c_u = 4, c_v = 1\}, \quad |CE_{41}| = 18n^2 + 6n.$$

$$CE_{11} = \{uv \in E(G)|c_u = c_v = 1\}, \qquad |CE_{11}| = 18n^2 - 12n.$$

In the following theorem, we compute the geometric-arithmetic reverse index of silicate networks.

Theorem 2.1. *The geometric-arithmetic reverse index of silicate networks is given by*

$$GAC(SL_n) = \frac{162}{5}n^2 - \frac{6}{5}n.$$

Proof.   By definition, we have

$$GAC(G) = \sum_{uv \in E(G)} \frac{2\sqrt{c_u c_v}}{c_u + c_v}$$

Thus

$$
\begin{aligned}
GAC(SL_n) &= \sum_{CE_{44}} \frac{2\sqrt{c_u c_v}}{c_u + c_v} + \sum_{CE_{41}} \frac{2\sqrt{c_u c_v}}{c_u + c_v} + \sum_{CE_{11}} \frac{2\sqrt{c_u c_v}}{c_u + c_v} \\
&= \left(\frac{2\sqrt{4 \times 4}}{4 + 4}\right) 6n + \left(\frac{2\sqrt{4 \times 1}}{4 + 1}\right)(18n^2 + 6n) + \left(\frac{2\sqrt{1 \times 1}}{1 + 1}\right)(18n^2 - 12n) \\
&= \frac{162}{5}n^2 - \frac{6}{5}n.
\end{aligned}
$$

$\square$

In the following theorem, we compute the sum connectivity reverse index of silicate networks.

Theorem 2.2. *The sum connectivity reverse index of silicate networks is given by*

$$SC\left(SL_n\right) = \left(\frac{18}{\sqrt{5}} + \frac{18}{\sqrt{2}}\right)n^2 + \left(\frac{6}{\sqrt{5}} - \frac{9}{\sqrt{2}}\right)n.$$

Proof. By definition, we have

$$SC\left(G\right) = \sum_{uv \in E(G)} \frac{1}{\sqrt{c_u + c_v}}.$$

Thus

$$SC\left(SL_n\right) = \sum_{CE_{44}} \frac{1}{\sqrt{c_u + c_v}} + \sum_{CE_{41}} \frac{1}{\sqrt{c_u + c_v}} + \sum_{CE_{11}} \frac{1}{\sqrt{c_u + c_v}}$$

$$= \left(\frac{1}{\sqrt{4+4}}\right)6n + \left(\frac{1}{\sqrt{4+1}}\right)\left(18n^2 + 6n\right) + \left(\frac{1}{\sqrt{1+1}}\right)\left(18n^2 - 12n\right)$$

$$= \left(\frac{18}{\sqrt{5}} + \frac{18}{\sqrt{2}}\right)n^2 + \left(\frac{6}{\sqrt{5}} - \frac{9}{\sqrt{2}}\right)n.$$

$\square$

3. Results for Hexagonal Networks

A triangular tiling is used in the construction of hexagonal networks. A hexagonal network is symbolized by HX_n where n is the number of vertices in each side of hexagon. A hexagonal network of dimension six is depicted in Figure 2.
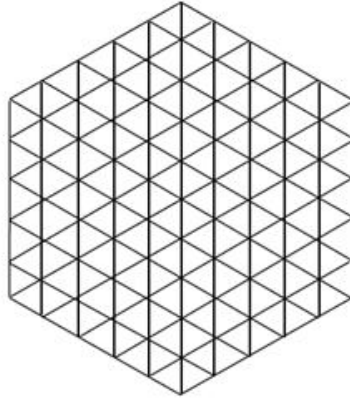

Figure 2: A 6-dimensional hexagonal network

Let H be the graph of hexagonal network HX_n. By calculation, we obtain that $|V(HX_n)| = 3n^2 - 3n + 1$ and $|E(HX_n)| = 9n^2 - 15n + 6$. From Figure 2, one can see that the vertices of HX_n are either of degree 3, 4 or 6. Then $\Delta(H) = 6$. In H, by algebraic method, there are five types of edges based on the degree of the end vertices of each edge as follows:

$$E_{34} = \{uv \in E(H)|d_H(u) = 3, d_H(v) = 4\}, \quad |E_{34}| = 12.$$

$$E_{36} = \{uv \in E(H)|d_H(u) = 3, d_H(v) = 6\}, \quad |E_{36}| = 6.$$

$$E_{44} = \{uv \in E(H)|d_H(u) = d_H(v) = 4\}, \quad |E_{44}| = 6n - 18.$$

$$E_{46} = \{uv \in E(H)|d_G(u) = 4, d_G(v) = 6\}, \quad |E_{46}| = 12n - 24.$$

$$E_{66} = \{uv \in E(H)|d_G(u) = d_G(v) = 6\}, \quad |E_{66}| = 9n^2 - 33n + 30.$$

Clearly, we have $c_u = \Delta(H) - d_H(u) + 1 = 7 - d_H(u)$. Thus there are five types of reverse edges as follows:

$$CE_{43} = \{uv \in E(H)|c_u = 4, c_v = 3\}, \quad |CE_{43}| = 12.$$

$$CE_{41} = \{uv \in E(H)|c_u = 4, c_v = 1\}, \quad |CE_{41}| = 6.$$

$$CE_{33} = \{uv \in E(H)|c_u = c_v = 3\}, \qquad |CE_{33}| = 6n - 18.$$

$$CE_{31} = \{uv \in E(H)|c_u = 3, c_v = 1\}, \quad |CE_{31}| = 12n - 24.$$

$$CE_{11} = \{uv \in E(H)|c_u = c_v = 1\}, \qquad |CE_{11}| = 9n^2 - 33n + 30.$$

In the following theorem, we compute the geometric-arithmetic reverse index of hexagonal networks.

Theorem 3.1. *The geometric-arithmetic reverse index of hexgonal networks is given by*

$$GAC(HX_n) = 9n^2 + \left(6\sqrt{3} - 27\right)n + \left(\frac{48\sqrt{3}}{7} + \frac{24}{5} + 12\right).$$

Proof. By definition, we have

$$GAC(H) = \sum_{uv \in E(H)} \frac{2\sqrt{c_u c_v}}{c_u + c_v}.$$

Thus

$$
\begin{aligned}
GAC(HX_n) &= \sum_{CE_{43}} \frac{2\sqrt{c_u c_v}}{c_u + c_v} + \sum_{CE_{41}} \frac{2\sqrt{c_u c_v}}{c_u + c_v} + \sum_{CE_{33}} \frac{2\sqrt{c_u c_v}}{c_u + c_v} + \sum_{CE_{31}} \frac{2\sqrt{c_u c_v}}{c_u + c_v} + \sum_{CE_{11}} \frac{2\sqrt{c_u c_v}}{c_u + c_v} \\
&= \left(\frac{2\sqrt{4 \times 3}}{4 + 3}\right) 12 + \left(\frac{2\sqrt{4 \times 1}}{4 + 1}\right) 6 + \left(\frac{2\sqrt{3 \times 3}}{3 + 3}\right)(6n - 18) \\
&\quad + \left(\frac{2\sqrt{3 \times 1}}{3 + 1}\right)(12n - 24) + \left(\frac{2\sqrt{1 \times 1}}{1 + 1}\right)(9n^2 - 33n + 30) \\
&= 9n^2 + \left(6\sqrt{3} - 27\right)n + \left(\frac{48\sqrt{3}}{7} + \frac{24}{5} + 12\right).
\end{aligned}
$$

$\square$

In the following theorem, we compute the sum connectivity reverse index of hexagonal networks.

Theorem 3.2. *The sum connectivity reverse index of hexgonal networks is given by*

$$SC(HX_n) = \frac{9}{\sqrt{2}}n^2 + \left(\sqrt{6} + 6 - \frac{33}{\sqrt{2}}\right)n + \left(\frac{12}{\sqrt{7}} + \frac{6}{\sqrt{5}} - 3\sqrt{6} - 12 + \frac{30}{\sqrt{2}}\right).$$

Proof. By definition, we have

$$SC(H) = \sum_{uv \in E(H)} \frac{1}{\sqrt{c_u + c_v}}.$$

Thus

$$
\begin{aligned}
SC(HX_n) &= \sum_{CE_{43}} \frac{1}{\sqrt{c_u + c_v}} + \sum_{CE_{41}} \frac{1}{\sqrt{c_u + c_v}} + \sum_{CE_{33}} \frac{1}{\sqrt{c_u + c_v}} + \sum_{CE_{31}} \frac{1}{\sqrt{c_u + c_v}} + \sum_{CE_{11}} \frac{1}{\sqrt{c_u + c_v}} \\
&= \left(\frac{1}{\sqrt{4 + 3}}\right) 12 + \left(\frac{1}{\sqrt{4 + 1}}\right) 6 + \left(\frac{1}{\sqrt{3 + 3}}\right)(6n - 18) \\
&\quad + \left(\frac{1}{\sqrt{3 + 1}}\right)(12n - 24) + \left(\frac{1}{\sqrt{1 + 1}}\right)(9n^2 - 33n + 30) \\
&= \frac{9}{\sqrt{2}}n^2 + \left(\sqrt{6} + 6 - \frac{33}{\sqrt{3}}\right)n + \left(\frac{12}{\sqrt{7}} + \frac{6}{\sqrt{5}} - 3\sqrt{6} - 12 + \frac{30}{\sqrt{2}}\right).
\end{aligned}
$$

$\square$

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