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Quick Switching Variables Sampling System Indexed by Six Sigma AQL and Six Sigma AOQL

Research Article

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Abstract: Quick Switching Sampling System was originally proposed by Dodge (1967) further Romboski (1969) studied the QSS-1 with the single sampling plan as a reference plan. Six Sigma Quick Switching Variables Sampling System, designated as SSQSVSS $(n_{\sigma}; k_{T_{\sigma}}, k_{N_{\sigma}})$ was studied by Senthilkumar and Esha raffie (2012) and the probability of acceptance of the lot is $1-3.4\times10^{-6}$, where (n, k_N) and (n, k_T) , $k_T > k_N$ are respectively the normal and tightened single sampling plans. The method and procedure for designing the Six Sigma Quick Switching Variables Sampling System based on the given Six Sigma Acceptable Quality Level and Six Sigma Limiting Quality Level. In this paper, the procedure and tables for the selection of Six Sigma Quick Switching Variables Sampling System (SSQSVSS $(n_{\sigma}; k_{T_{\sigma}}, k_{N_{\sigma}})$) indexed by Six Sigma AQL and Six Sigma AOQL are given, whenever rejected lots are 100% inspected for replacement of nonconforming units. The tables are constructed for easy selection of the sampling system.

Keywords: Quick Switching Sampling System, Six Sigma, AOQ, AOQL, Six Sigma AQL and Six Sigma AOQL. c JS Publication.

1. Introduction and Preliminaries

The quick switching systems consist of two inspections, sampling plan follows along with a set of switching rules between them. The first sampling plan called normal inspection plan proposed for using periods of good quality. It has a smaller sample size to reduce inspection costs. The second sampling plan called tightened inspection plan proposed for use when problems are encountered. It gives a high level of protection. The switching rules make sure that the correct plan is used. A lot- by – lot rectifying inspection scheme for a series of lots calls for 100% inspection of rejected lots under application of a sampling plan. If one prefers to use a single sampling plan for variables under a rectification inspection system, the quality indicator for the selection of the sampling plan will be the average outgoing quality limit (AOQL), which is the worst average quality the consumer will receive in the long run, no matter what the incoming quality is. Rejected lots are often a nuisance to the producer as they result in extra work and cost. If too many lots are rejected, this will damage the reputation of the producer or supplier. From the producer point of view, he would prefer fixing an acceptable quality level (AQL) and designing sampling plan so that if the incoming product quality is maintained at AQL, most of his lot (say 99.9%) will be accepted during the sampling inspection stage itself. Thus, designing sampling inspection plan indexed by SSAQL and SSAOQL satisfies both the producer and consumer whenever rectifying inspection is necessary. Soundararajan (1981) has developed procedures and tables for the selection of single sampling plans for attributes for given AQL and AOQL. Govindaraju (1990) has developed procedures and tables for the selection of single sampling plans for variables indexed by

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AQL and AOQL. Later Soundarajan and Palanivel (2000) have developed procedures and tables for the selection of quick switching single sampling variables systems indexed by AQL and AOQL. In this section, tables and procedures for selection of six sigma quick switching variables sampling system indexed by SSAQL and SSAOQL are given. This concept can be extended to variable quality characteristics of the study, the resulting plan would be designated as SSQSVSS and would be applied under the following conditions:

- The production is steady, so that results on current and preceding lots are broadly indicative of a continuous process.
- Lots are submitted substantially in the order of production.
- Inspection is by variables, with the quality being defined as the fraction of non-conforming.
- The sample units are selected from a large lot and production is continuous.
- The production process should depend on automation and human handling is very negligible.
- The industry may adopt system method with decision makers having an experience in adopting the six sigma quality initiatives.

2. Basic Assumptions

- The quality characteristic is represented by a random variable X measurable on a continuous scale.
- Distribution of X is normal with mean and standard deviation.
- An upper limit U, has been specified and a product is qualified as defective when $X > U$. [when the lower limit L is Specified, the product is a defective one if $X < L$.
- The Purpose of inspection is to control the fraction defective, p in the lot inspected.

When the conditions listed above are satisfied the fraction defective in a lot will be defined by $p = 1 - F(v) = F(-v)$ with $v = \frac{(U-\mu)}{\sigma}$ and

$$
F(y) = \int_{-\infty}^{y} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz
$$
 (1)

where $z \sim N(0, 1)$. Here the decision criterion for the σ-method variables plan is to accept the lot if $\bar{X} + k\sigma \leq U$, where U is the upper specification limit or if $\bar{X} + k\sigma \geq L$, where L is the lower specification limit.

3. SSQSVSS $(n_{\sigma}; k_N, k_T)$ with Known σ Variables Plan as the Reference Plan

The Six Sigma Quick Switching Variables Sampling System with known σ variables plan as the reference plan has following Operating Procedure

Operating Procedure

Step 1: Start with normal inspection using the single sampling plan with a sample of size n_{σ} from the lot, inspect and record the measurement of the quality characteristic for each unit of the sample. Compute the sample mean \bar{X} .

Step 2: If $X + k_{N_{\sigma}}$; $\sigma \leq U$ or $X + k_{N_{\sigma}}$; $\sigma \geq L$ accept the lot and repeat Step 1 otherwise, reject the lot and follow Step 3.

Step 3: Inspect under tightened inspection, using sample of size n_{σ} from the next lot inspect and record the measurement of the quality characteristic for each unit of the sample. Compute the sample mean X .

Step 4: If $\bar{X} + k_{T\sigma}$; $\sigma \le U$ or $\bar{X} + k_{T\sigma}$; $\sigma \ge L$ accept the lot and repeat Step 1 otherwise, reject the lot and repeat Step 3. where $k_{N\sigma}$ and $k_{T\sigma}$ are the acceptance criterion of a normal and tightened single sampling variable plans respectively under σ-method. \bar{X} and σ are the average quality characteristic and standard deviation respectively.

4. Variable Sampling Plan and SSAOQL Procedures

The OC function of the SSQSVSS, according to Romboski (1969),

$$
P_a(p) = \frac{P_T}{1 - P_N + P_T} \tag{2}
$$

The fraction nonconforming in a given lot will be

$$
p = F(-v) \tag{3}
$$

with $v = \frac{(U-\mu)}{\sigma}$ and its probability of acceptance will be

$$
P_a(p) = F(w_N, w_T) \tag{4}
$$

with

$$
w_N = (v - k_{N\sigma})\sqrt{n_{\sigma}}\tag{5}
$$

and

$$
w_T = (v - k_{T\sigma})\sqrt{n_{\sigma}}
$$
\n(6)

If the quality of the accepted lot is p and all defective units found in the rejected lots are replaced by non-defective units in a rectifying inspection plan, the Six Sigma average outgoing quality (SSAOQ) can be approximated as

$$
SSAOQ = pP_a(p) \tag{7}
$$

If p_m is the proportion nonconforming items at which SSAOQ is maximum, one has

$$
SSAOQL = p_m P_a(p_m) \tag{8}
$$

If SSAQL (p_1) is prescribed, then the corresponding value of v_{SSAQL} or v_1 will be fixed and if $P_a(p)$ is fixed at 99.99966%, that is $(1-\alpha)$. Where, $\alpha = 0.0000034 \times 10^{-6}$. Hence we have $P_a(p_1) = (1-\alpha)$. So that for given values of n_{σ} , w_N , w_T and SSAQL, $k_{N\sigma}, k_{T\sigma}$ are determined.

5. Selection of Known σ SSQSVSS $(n_{\sigma}; k_{T\sigma}, k_{N\sigma})$ for Given SSAQL and SSAOQL

Table 1 is used for selection of σ -method SSQS variables system. For example, if the SSAQL is fixed at $p_1 = 0.000002$ and the SSAOQL is fixed at $p_2 = 0.000003$, Table 1 yields $n_{\sigma} = 5589$, $k_{T_{\sigma}} = 4.416$ and $k_{N_{\sigma}} = 4.266$, which is associated with 4.7 sigma level of SSQSVSS $(n_{\sigma}; k_{T_{\sigma}}, k_{N_{\sigma}})$. The user of Table 1 should understand the limitations of plans indexed by SSAOQL.

Sampling with rectifying of rejected lots on the one hand reduces the average percentage of nonconforming items in the lots, but on the other hand introduces non-homogeneity in the series of lots finally accepted. That is, any particular lot will have a quality of p% or 0% nonconforming depending on whether the lot is accepted or rectified. Thus the assumption underlying the SSAOQL principle is that the homogeneity in the qualities of individual lots is unimportant and only the average quality matters. For plans listed in Table 1, if the individual lot quality happens to be the product quality p_m at which SSAOQL occurs, then the associated probability of acceptance will be poor. Table 2 gives $P_a(p_m)$ values of plans given in Table 1. For example, for SSAQL of $p_1 = 0.000003$ and SSAOQL of $p_2 = 0.00005$, Table 2 gives $P_a(p_m) = 0.34$. Then $p_m = SSAOQL/P_a(p_m) = 0.00014$. In order to avoid such inconvenience, the producer should maintain the process quality more or less at the SSAQL. The higher rate of rejection of lots at $p = p_m$ will also indirectly put pressure on the producer to improve the submitted quality.

6. Selection of Unknown σ SSQSVSS for Given SSAQL and SSLQL

If the population standard deviation σ is unknown, then it is estimated from the sample standard deviation S $(n-1)$ as the divisor). If the sample size of the unknown sigma variables system (S-method) is n_s and the acceptance parameters are k_N and k_T , then the operating procedure is as follows.

The steps involved in this procedure are as follows

Step 1: Draw a sample of size n_s from the lot, inspect and record the measurement of the quality characteristic for each unit of the sample. Compute the sample mean \bar{X} and sample standard deviation S.

Step 2: If $\bar{X} + k_{Ns}$; $S \le U$ or $\bar{X} + k_{Ns}$; $S \ge L$ accept the lot and repeat Step 1 otherwise, reject the lot and follow Step 3.

Step 3: Draw a sample of size n_s from the next lot inspect and record the measurement of the quality characteristic for each unit of the sample. Compute the sample mean \bar{X} and sample standard deviation S.

Step 4: If $\bar{X} + k_T$; $S \le U$ or $\bar{X} + k_T$, $S \ge L$ accept the lot and repeat Step 1 otherwise, reject the lot and repeat Step 3. where X and S are the average and the standard deviation of quality characteristic respectively from the sample. Under the assumptions for a Six Sigma Quick Switching System stated, the probability of acceptance $P_a(p)$ of a lot is given in the equation [\(4\)](#page-2-0) and [\(5\)](#page-2-1), P_T and P_N respectively are

$$
P_T = \int_{-\infty}^{w_T} \frac{1}{\sqrt{2\pi}} e^{\frac{-z^2}{2}} dz \text{ and } P_N = \int_{-\infty}^{w_N} \frac{1}{\sqrt{2\pi}} e^{\frac{-z^2}{2}} dz
$$

with

$$
w_N = \frac{U - k_N - \mu}{S} \frac{1}{\sqrt{\left(\frac{1}{n_s} + \frac{k_N^2}{2n_s}\right)}} \text{ and } w_T = \frac{U - k_T - \mu}{S} \frac{1}{\sqrt{\left(\frac{1}{n_s} + \frac{k_T^2}{2n_s}\right)}}
$$

The values of n_S , k_{NS} , and k_{Ts} for given p_1 , p_2 , α and β can be determined and should satisfy the following equations

$$
P_a(p_1) = \frac{P_{T1}}{1 - P_{N1} + P_{T1}} = 1 - \alpha
$$
\n(9)

$$
P_a(p_2) = \frac{P_{T2}}{1 - P_{N2} + P_{T2}} = \beta \tag{10}
$$

where

$$
P_{T_1} = \Pr\left[\frac{\overline{X} - \overline{X}_{p_1}}{S\sqrt{\frac{1}{n_S} + \frac{k_T^2}{2n_s}}} \ge \frac{(k_T - z_{p_1})}{\sqrt{\frac{1}{n_S} + \frac{k_T^2}{2n_s}}} \right]; \quad P_{N_1} = \Pr\left[\frac{\overline{X} - \overline{X}_{p_1}}{S\sqrt{\frac{1}{n_S} + \frac{k_N^2}{2n_s}}} \ge \frac{(k_N - z_{p_1})}{\sqrt{\frac{1}{n_S} + \frac{k_N^2}{2n_s}}} \right]
$$

and

$$
P_{T_2} = \Pr\left[\frac{\overline{X} - \overline{X}_{p_2}}{S\sqrt{\frac{1}{n_S} + \frac{k_T^2}{2n_s}}} \ge \frac{(k_T - z_{p2})}{\sqrt{\frac{1}{n_S} + \frac{k_T^2}{2n_s}}} \right]; \quad P_{N_2} = \Pr\left[\frac{\overline{X} - \overline{X}_{p_2}}{S\sqrt{\frac{1}{n_S} + \frac{k_N^2}{2n_s}}} \ge \frac{(k_N - z_{p2})}{\sqrt{\frac{1}{n_S} + \frac{k_N^2}{2n_s}}} \right]
$$

7. Selection of Unknown SSQSVSS $(n_s; kT_s, kN_s)$ for Given SSAQL and SSAOQL

Table 1 also gives such matched S-method plan. For example, for given SSAQL of $p_1 = 0.00001$ and SSAOQL of $p_2 = 0.00005$, one obtains the parameters of the s-method plan from Table 1 to be $n_s = 4524$, $k_{T\sigma} = 3.263$ and $k_{N\sigma} = 3.113$, which is associated with 4.7 sigma level of SSQSVSS $(n; k_N, k_T)$.

8. Construction of Table 1

For constructing Table 1, a trial value of p_m is assumed and the probability of acceptance at p_m is found using [\(8\)](#page-2-2) as

$$
P_a(p_m) = \frac{SSAOQL}{p_m}
$$

The auxiliary variables v_m , w_{Nm} and w_{Tm} corresponding to the values of p_m and $P_a(p_m)$ respectively, are found using P_N , P_T , equations [\(3\)](#page-2-3) and [\(4\)](#page-2-0). For given values of p_1 , determine the values of v_1 , w_N and w_T using the approximation (Abramwitz and Stegun (1972)) for the ordinate of the cumulative normal distribution. With the values of v_m , w_{Nm} and wr_m , the following equation is used for calculating n_{σ} .

$$
\sqrt{n_{\sigma}} = \left[\frac{AOQL}{\left(p_m^2(1 - P_N + P_T)\sqrt{(\exp(v_m^2 - w_T^2))}\right) - \frac{P_T\left(\sqrt{(\exp(v_m^2 - w_N^2)}) - \sqrt{(\exp(v_m^2 - w_T^2))}\right)}{(1 - P_N + P_T)^2}}\right]
$$
(11)

where $P_N = F(w_N) = pr[(U - \bar{x})/\sigma > k_{N\sigma}]$ and $P_T = F(w_T) = pr[(U - \bar{x})/\sigma > k_{T\sigma}]$. Equation [\(11\)](#page-4-0) is the formula for finding the sample size of a known σ SSQSVSS. With the values of n obtained from [\(11\)](#page-4-0), it is then checked to see whether the assumed value of p_m corresponds to the proportion non-conforming at which the SSAOQL occurs or not. That is, it is checked to see whether or not the trial value of p_m satisfies the following condition.

$$
AOQL - \left(p_m^2 (1 - P_N + P_T) \sqrt{\left(\exp(v_m^2 - w_T^2) \right)} \right) - \frac{P_T \left(\sqrt{\left(\exp(v_m^2 - w_N^2) \right)} - \sqrt{\left(\exp(v_m^2 - w_T^2) \right)} \right)}{\left(1 - P_N + P_T \right)^2} = 0 \tag{12}
$$

Equation [\(11\)](#page-4-0) was obtained from the following relation

$$
\frac{d(SSAOQ)}{dp} = P_a(p) + p \frac{dP_a(p)}{dp} = 0\tag{13}
$$

in which

$$
\frac{dP_a(P)}{dp} = (1 - P_N + P_T) \sqrt{(n_\sigma \exp(v_m^2 - w_T^2))} - \frac{P_T \left(\sqrt{(n_\sigma \exp(v_m^2 - w_N^2))} - \sqrt{(n_\sigma \exp(v_m^2 - w_T^2))}\right)}{(1 - P_N + P_T)^2}
$$
(14)

If the assumed value of p_m does not satisfy [\(10\)](#page-3-0), then another trial value of p_m is obtained from [\(13\)](#page-4-1) by numerical methods. The methods of successive substation are often found to give good results and [\(13\)](#page-4-1) is rewritten for this purpose as

$$
p_m = \frac{AOQL}{\left(p_m(1 - P_N + P_T)\sqrt{(n_\sigma \exp(v_m^2 - w_T^2))} - \frac{P_T\left((\sqrt{(n_\sigma \exp(v_m^2 - w_N^2))} - \sqrt{(n_\sigma \exp(v_m^2 - w_T^2))}\right)}{(1 - P_N + P_T)^2}\right)}
$$
(15)

✷✺

After determining the next trial value of p_m , again the values of v_m , w_{Nm} , w_{Nm} , and n_{σ} are found and the condition [\(12\)](#page-4-2) rechecked. This iterative procedure continues until the convergence of p_m is achieved. Then the value of $k_{N\sigma}$ and kit_{σ} are obtained from [\(5\)](#page-2-1) and [\(6\)](#page-2-4). For obtaining the values of v_1 , w_N and w_T , the approximation for the ordinate of the cumulative normal distribution available in Abramowitz and Stegun (1972) was used. The S-method plans matching the σ-method plans were obtained using computer search routine through C++ programme. For selected combinations of SSAQL and SSAOQL, Table 1 was constructed following the above iterative procedure.

References

- [1] M.Abramowitz and I.A.Stegun, Handbook of Mathematical functions, Dover Publications, New York, (1972).
- [2] H.F.Dodge, A New Dual System of Acceptance Sampling Technical Report, No.16, The Statistics Center, Rutgers-The State University, New Brunswick, NJ, (1967).
- [3] K.Govindaraju, Single sampling Plans for Variables Indexed by AQL and AOQL, Journal of Quality Technology, 22(4)(1990), 310-313.
- [4] L.D.Romboski, An Investigation of Quick Switching Acceptance Sampling Systems, Doctoral Dissertation, Rutgers the State University, New Brunswick, New Jersey, (1969).
- [5] D.Senthilkumar and B.Esha Raffie, Six Sigma Quick Switching Variables Sampling System Indexed by Six Sigma Quality Levels, International Journal of Computer Science & Engineering Technology, 3(12)(2012), 565-576.
- [6] D.J.Sommers, Two-point double variables sampling plans, Journal of Quality Technology, 13(1981).
- [7] V.Soundarajan and M.Palanivel, Quick Switching Variables Single Sampling (QSVSS) System indexed by AQL and AOQL, Journal of Applied Statistics, 27(7)(2000), 771-778.
- [8] V.Soundararajan, Single Sampling Attributes Plans Indexed by AQL and AOQL, Journal of Quality Technology, 13(3)(1981), 195-200.

Table 1: SSQSVSS with known and unknown σ indexed by SSAQL and SSAOQL

Table 2: $P_a(p_m)$ values of known σ plans

SSAOQL (p_2)	$\overline{\text{SSAQ}}$ L (p_1)						
	0.000001		0.000002 0.000003 0.000004 0.000005 0.00001				0.00005
0.000002	0.95						
0.000003	0.94	0.96					
0.000004	0.92	0.94	0.95				
0.000005	0.85	0.92	0.93	0.94			
0.000006	0.74	0.86	0.86	0.87	0.89		
0.000007	0.50	0.74	0.75	0.76	0.78		
0.000008	0.49	0.51	0.52	0.53	0.54		
0.000009	0.45	0.49	0.50	0.51	0.53		
0.00001	0.43	0.46	0.47	0.48	0.49		
0.00002	0.38	0.44	0.45	0.45	0.47	0.51	
0.00003	0.35	0.39	0.40	0.41	0.42	0.49	
0.00004	0.32	0.36	0.37	0.38	0.39	0.44	
0.00005	0.21	0.33	0.34	0.35	0.36	0.41	
0.00006	0.17	0.22	0.23	0.24	0.25	0.38	0.40
0.00007	0.14	0.18	0.19	0.19	0.21	0.27	0.29
0.00008		0.15	0.16	0.17	0.19	0.23	0.24
0.00009			0.15	0.16	0.18	0.20	0.22
0.0001				0.14	0.15	0.17	0.21
0.0002						0.15	0.18
0.0003							0.15
0.0004							0.13
0.0005							0.12