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On Contra-pre-B-semi Continuous Functions

Research Article

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Abstract: In this paper, We introduce and investigate contra-pre-B-semi-continous function. This new class is a super class of the class of contra-B-β-continuous functions and contra-B-pre continuous function.

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1. Introduction

Dontchev [2] introduced the notion of contra-continuity and some results concerning compactness, S-closedness and strong S-closedness in 1996. Dontchev and Noiri [3] introduced and investigated contra-semi-continuous functions and RC-continuous functions between topological spaces in 1999. Jafari and Noiri [4] introduced contra-precontinuous functions and obtained their basic properties. Jafari and Noiri [5] introduced contra-α-continuous functions between topological spaces. Veera Kumar [12] introduced the class of contra-ψ-continuous functions. The same Veera Kumar [11, 13] introduced pre-semi-closed sets and contra-pre-semi-continuous functions for topological spaces. In this chapter, we introduce and investigate pre-B-semi-closed sets and contra-pre-B-semi-continuous function in simply extended topological spaces. This new class is the super class of the class of contra-B-β-continuous functions and contra-B-pre continuous functions.

2. Preliminaries

Throughout this paper, $(X,\, \tau(B_X))$, $(Y,\, \sigma(B_Y))$ *and* $(Z,\, \eta(B_Z))$(briefly X, Y and Z) will denote simply extended topological spaces.

Definition 2.1. *Levine [6] in 1964 defined $\tau(B) = \{O \cup (\acute{O} \cap B) : O, \acute{O} \in \tau\}$ and called it simple extension of τ by B, where $B \notin \tau$. The sets in $\tau(B)$ are called B-open sets. And the complement of B-open set is called B-closed.*

Definition 2.2 ([6])**.** *Let S be a subset of a simply extended topological space X. Then*

(1). The B-closure of S, denoted by Bcl(S), is defined as $\cap\, \{F : S \subseteq F\, and$
$F\, is\, B\text{-}closed\}$;

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(2). The B-interior of S, denoted by Bint(S), is defined as $\cup\{F : F \subseteq S \, \text{and}$
$F \, \text{is} \, B\text{-open}\}$.

Definition 2.3 ([6]). *A subset A of a topological space* $(X, \tau(B))$ *is said to be*

(1). B-semi-open if $A \subseteq Bcl\,(Bint(A))$,

(2). B-preopen if $A \subseteq Bint\,(Bcl\,(A))$,

(3). B-α-open if $A \subseteq Bint\,(Bcl\,(Bint(A)))$,

(4). B-β-open or B-semi-preopen if $A \subseteq Bcl\,(Bint\,(Bcl(A)))$,

(5). B-b-open if $A \subseteq Bcl\,(Bint(A)) \cup Bint(Bcl(A))$.

The family of all B-open (resp. B-semi-open , B-preopen, B-α-open, B-β-open, B-b-open) sets in a topological space $(X, \tau(B))$ *is denoted by B(X) (resp. BSO(X) BPO(X), Bα(X), BβO(X), BbO(X)).*

Definition 2.4. *A subset S of X is called B-regular open [9] if S = Bint(Bcl(S)). The complement of B-regular open set is called B-regular closed. The B-semi-closure of a subset A of X, denoted by Bscl(A), is the intersection of all B-semi-closed sets of X containing A. The B-β-closure of a subset A of X, denoted by Bβcl(A), is the intersection of all B-β-closed sets of X containing A. The B-semi-interior of a subset A of X, denoted by Bsint(A), is defined to be the union of all B-semi-open sets contained in A.*

Definition 2.5. *A subset A of a space* $(X, \tau(B))$ *is called Bg-closed set [1] if* $Bcl(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is open in X. The complement of Bg-closed set is called Bg-open set.*

Definition 2.6. *A subset A of a space* $(X, \tau(B))$ *is called Bsg-closed set [8] if* $Bscl(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is B-semi-open in X. The complement of Bsg-closed set is called Bsg-open set.*

Definition 2.7. *A subset A of a space* (X, τ) *is called Ψ-closed set [10] if* $scl(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is sg-open set of X. The complement of Ψ-closed set is called Ψ-open.*

Definition 2.8 ([10]). *A subset A of a space* (X, τ) *is called Generalized semi-preclosed (briefly gsp-closed) set if* $\beta cl(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is open set of X.*

Definition 2.9 ([7]). *A function* $f : (X, \tau(B_X)) \to (Y, \sigma(B_Y))$ *is called*

(1). B-continuous if $f^{-1}(V)$ *is B-open in X, for every B-open set V of Y.*

(2). B-pre continuous if $f^{-1}(V) \in BPO(X)$*, for every B-open set V of Y.*

(3). B-α-continuous if $f^{-1}(V) \in B\alpha O(X)$*, for every B-open set V of Y.*

(4). B-semi-continuous if $f^{-1}(V) \in BSO(X)$*, for every B-open set V of Y.*

(5). B-β-continuous if $f^{-1}(V) \in B\beta O(X)$*, for every B-open set V of Y.*

3. Pre-B-semi-closed Sets

Definition 3.1. *A subset A of a space $(X, \tau(B))$ is called $B\Psi$-closed set if $Bscl(A) \subseteq U$ whenever $A \subseteq U$ and U is Bsg-open set of X. The complement of $B\Psi$-closed set is called $B\Psi$-open.*

Definition 3.2. *A subset A of a space $(X, \tau(B))$ is called B-generalized semi-preclosed (briefly Bgsp-closed) set if $B\beta cl(A) \subseteq U$ whenever $A \subseteq U$ and U is B-open set of X.*

Definition 3.3. *A subset A of a space $(X, \tau(B))$ is called pre-B-semi-closed set if $B\beta cl(A) \subseteq U$ whenever $A \subseteq U$ and U is B-g-open set of X. The complement of pre-B-semi-closed set is called pre-B-semi-open.*

Definition 3.4. *A function $f : (X, \tau(B_X)) \to (Y, \sigma(B_Y))$ is called*

(1). perfectly B-continuous if $f^{-1}(V)$ is B-clopen in X for every B-open set V of Y.

(2). B-RC-continuous if $f^{-1}(V)$ is B-regular open in X for every B-open set V of Y.

(3). contra-B-continuous if $f^{-1}(V)$ is B-closed in X for every B-open set V of Y.

(4). contra-B-pre continuous if $f^{-1}(V)$ is B-preclosed in X for every B-open set V of Y.

(5). contra-B-semi-continuous if $f^{-1}(V)$ is B-semi-closed in X for every B-open set V of Y.

(6). contra-B-α-continuous if $f^{-1}(V)$ is B-α-closed in X for every B-open set V of Y.

(7). contra-B-β-continuous if $f^{-1}(V)$ is B-β-closed in X for every B-open set V of Y.

(8). contra-B-Ψ-continuous if $f^{-1}(V)$ is $B\Psi$-closed in X for every B-open set V of Y.

Definition 3.5. *A simply extended topological space $(X, \tau(B))$ is called*

(1). Bsemi-T_0 space if to each pair of distinct points x,y of X, there exist a B-semi-open set containing one but not the other.

(2). Bsemi-generalized-T_0(briefly Bsg-T_0 space if to each pair of distinct points x,y of X, there exist a Bsg-open set containing one but not the other.

Definition 3.6. *A simply extended topological space $(X, \tau(B))$ is called*

(1). pre-Bsemi-$T_{1/2}$ space if every pre-B-semi-closed set in it is B-β-closed.

(2). pre-Bsemi-T_b space if every pre-B-semi-closed set in it is B-semi-closed.

(3). pre-Bsemi-$T_{3/4}$ space if every pre-B-semi-closed set in it is B-pre-closed.

(4). Bsemi-pre-$T_{1/2}$ space if every Bgsp-closed set in it is B-β-closed.

Theorem 3.7. *Every B-β-closed set is a pre-B-semi-closed set.*

Proof. Follows from the fact that Bβcl(A)=A for any B-β-closed set. □

The following Example shows that the implication in the above Theorem is not reversible.

Example 3.8. *Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}\}$ and $B = \{a, c\}$. Then the sets in $\{\phi, X, \{a\}, \{a, c\}\}$ are called B-open. Let $A = \{a, b\}$. Then A is a pre-B-semi-closed set but not a B-β-closed set.*

Thus the class of pre-B-semi-closed sets properly contains the class of B-β-closed sets.

Remark 3.9. *Union of two pre-B-semi-closed sets need not be pre-B-semi-closed. Let* $X = \{a,\ b,\ c\}$, $\tau = \{\phi,\ X\}$ *and* $B = \{a, b\}$. *Then the sets in* $\{\phi, X, \{a, b\}\}$ *are called B-open. Let* $A = \{a\}$ *and* $C = \{b\}$. *Then A and C are pre-B-semi-closed sets. But* $A \cup B = \{a, b\}$ *is not a pre-B-semi-closed set.*

Theorem 3.10. *If A is a pre-B-semi-closed set of* $(X, \tau(B))$, *then* $B\beta cl(A){-}A$ *does not contain any non-empty B-g-closed set.*

Proof. Let F be a B-g-closed set of $(X, \tau(B))$ such that F $\subseteq$ Bβcl(A)$-A$. Then A $\subseteq X - F$. Since A is pre-B-semi-closed and $X - F$ is B-g-open, then Bβcl(A)$\subseteq X - F$. This implies F$\subseteq X - B\beta$cl(A). So F$\subseteq (X - B\beta$cl(A))$\cap$ (Bβcl(A)$-A) \subseteq$ $X - B\beta$cl(A)$\cap$ Bβcl(A)=ϕ. Thus F=ϕ. □

Result 3.11.

(1). Every B-open set is B-g-open set but not conversely.

(2). Every B-preclosed set is B-β-closed set but not conversely.

Proposition 3.12. *Every pre-B-semi-closed set is Bgsp-closed set but not conversely.*

The converse of the above Proposition is not true in general as can be seen from the following Example.

Example 3.13. *Let* $X = \{a,\ b,\ c\}$, $\tau = \{\phi,\ X\}$ *and* $B = \{b\}$. *Then the sets in* $\{\phi,\ X,\ \{b\}\}$ *are B-open. Then* $\{a,\ b\}$ *is Bgsp-closed but not pre-B-semi-closed set.*

Theorem 3.14. *Every B-semi-pre-*$T_{1/2}$ *space is a pre-B-semi-*$T_{1/2}$ *space.*

The converse of the above Theorem is not true in general as can be seen from the following Example.

Example 3.15. *Let* $X = \{a,\ b,\ c\}$, $\tau = \{\phi,\ X\}$ *and* $B = \{a\}$ *then* $\tau(B) = \{\phi,\ X,\ \{a\}\}$. *Then X is a B-semi-pre-*$T_{1/2}$ *space since* $\{a,\ b\}$ *is a Bgsp-closed set but not a B-β-closed in X. However X is a pre-B-semi-*$T_{1/2}$ *space.*

Thus the class of pre-B-semi-$T_{1/2}$ space properly contains the class of Bsemi-pre-$T_{1/2}$ space.

Lemma 3.16. *For a subset A of a space X, the following are equivalent.*

(1). A is regular B-closed;

(2). A is B-preclosed and B-semi-open;

(3). A is B-α-closed and B-β-open.

Proof.
(1)$\Rightarrow$(2) Let A be regular B-closed. Then A= Bcl(Bint(A)). Since every regular B-closed is B-closed and hence B-preclosed, A is B-preclosed and B-semi-open.
(2)$\Rightarrow$(3) Let A be B-preclosed and B-semi-open. Then Bcl(Bint(A)) $\subset$ A and A $\subset$ Bcl(Bint(A)). Therefore, we have Bcl(Bint(Bcl(A)))$\subset$ Bcl(Bint(Bcl(Bint(A)))) = Bcl(Bint(A))$\subset$ A. This shows that A is B-α-closed. Since BSO(X)$\subset$ BβO(X), it is obvious that A is B-β-open.
(3)$\Rightarrow$(1) Let A be B-α-closed and B-β-open. Then $A = Bcl(Bint(Bcl(A)))$ and hence $Bcl(Bint(A)) = Bcl(Bint(Bcl(Bint(Bcl(A))))) = Bcl(Bint(Bcl(A))) = A$. Therefore A is regular B-closed. □

As a consequence of the above Lemma, we have the following Result.

Theorem 3.17. *The following statements are equivalent for a function f: X → Y:*

(1). f is B-RC-continuous;

(2). f is a contra-B-pre-continuous and B-semi-continuous;

(3). f is contra-B-α-continuous and B-β-continuous.

Theorem 3.18. *For a set $A \subseteq (X, \tau(B))$ the following conditions are equivalent:*

(1). A is B-clopen;

(2). A is B-α-open and B-closed;

(3). A is B-preopen and B-closed.

Proof.
(1)⇒(2) and (2)⇒(3) are obvious from the fact that every B-open set is B-α-open and hence B-preopen.
(3)⇒(1) Since A is B-preopen, then A⊆ Bint(Bcl(A)). Since A is B-closed, then A⊆ Bint(Bcl(A))=Bint(A) or equivalently A is B-open and hence B-clopen. □

As a consequences of the above decompositions of B-clopen sets we have the following decomposition of perfect B-continuity.

Theorem 3.19. *For a function $f : (X, \tau(B_X)) \to (Y, \sigma(B_Y))$ the following conditions are equivalent:*

(1). f is perfectly B-continuous;

(2). f is B-continuous and contra-B-continuous;

(3). f is B-α-continuous and contra-B-continuous;

(4). f is B-pre-continuous and contra-B-continuous.

Theorem 3.20. *The following statements are equivalent for a function f: X → Y:*

(1). f is contra-B-α-continuous;

(2). f is a contra-B-pre-continuous and contra-B-semi-continuous.

Proof. It follows from the fact that BαO(X)=BSO(X)∩BPO(X). □

Proposition 3.21. *Every B-semi-open set is Bψ-open.*

The converse of the above Proposition is not true in general as can be seen from the following Example.

Example 3.22. *Let $X = \{a, b, c\}$, $\tau = \{\phi, X\}$ and $B = \{a, b\}$. Then the sets in $\{\phi, X, \{a, b\}\}$ are called B-open. Here $\{a\}$ is Bψ-open set but not B-semi-open set.*

Definition 3.23. *A space X is called Bψ-T_0 if and only if to each pair of distinct points x,y of X, there exists a Bψ-open set containing one but not the other.*

Theorem 3.24. *Every Bsemi-T_0 space is Bψ-T_0 space.*

Proposition 3.25. *Every $B\psi$-open set is Bsg-open set.*

The converse of the above Proposition is not true in general as can be seen from the following Example.

Example 3.26. *Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{b\}\}$ and $B = \{a, c\}$. Then the sets in $\{\phi, X, \{b\}, \{a,c\}\}$ are called B-open. Then $\{a\}$ is Bsg-open but not $B\psi$-open set.*

Theorem 3.27. *Every B-closed (resp. B-α-closed, B-semi-closed, B-preclosed and Bsg-closed) set is pre-B-semi-closed set but the converses are not true.*

Proof. Follows from the above Example 3.8 and the fact that every B-closed (resp. B-α-closed, B-semi-closed, B-preclosed and Bsg-closed) set is B-α-closed (resp. B-semi-closed, B-β-closed, B-β-closed and B-β-closed) set. Thus the class of pre-B-semi-closed sets properly contain the classes of B-closed sets, B-α-closed sets, B-semi-closed sets and Bsg-closed sets. □

Theorem 3.28. *If A is B-g-open and pre-B-semi-closed, then A is B-β-closed.*

Theorem 3.29. *If A is pre-B-semi-closed set of $(X, \tau(B))$ such that $A \subseteq C \subseteq B\beta cl(A)$, then C is also a pre-B-semi-closed set of $(X, \tau(B))$.*

Proof. Let U be a Bg-open set of $(X, \tau(B))$ such that $B \subseteq U$. Then $A \subseteq U$. Since A is pre-B-semi-closed, Bβcl(A)$\subseteq$ U. Now, Bβcl(C)$\subseteq$ Bβcl(Bβcl(A))=Bβcl(A)$\subseteq$U. Therefore B is also a pre-B-semi-closed set. □

Definition 3.30. *A subset A of simply extended topological space X is said to be B-nowhere dense if Bint(Bcl(A)) $= \phi$.*

Theorem 3.31. *For a space $(X, \tau(B))$ the following are equivalent:*

(1). X is a pre-B-semi-$T_{1/2}$ space.

(2). Every singleton of X is B-g-closed or B-β-open.

Proof.
(1)⇒(2) Suppose that $\{x\}$ is not B-g-closed for some $x \in X$. Then $X - \{x\}$ is not B-g-open. So X is the only B-g-open set containing $X - \{x\}$ and hence $X - \{x\}$ is trivially a pre-B-semi-closed set of $(X, \tau(B))$. By (1), $X - \{x\}$ is B-β-closed set or equivalently $\{x\}$ is B-β-open.
(2)⇒(1) Suppose that $\{x\}$ is not B-preopen for some $x \in X$. Since every singleton is either B-preopen or B-nowhere dense, $\{x\}$ is a B-nowhere dense. Hence $\{x\} \notin$ Bcl(Bint(Bcl($\{x\}$))) $= \phi$. Therefore $\{x\}$ is not B-β-open. By (2), $\{x\}$ is a B-g-closed set of $(X, \tau(B))$. □

Theorem 3.32. *If $(X, \tau(B))$ is a pre-B-semi-T_b space, then for each $x \in X$, $\{x\}$ is either B-g-closed or B-semi-open.*

Proof. Suppose that $\{x\}$ is not a B-g-closed set of pre-B-semi-T_b space $(X, \tau(B))$. Then X is the only B-g-open set containing $X - \{x\}$ and hence $X - \{x\}$ is a pre-B-semi-closed set. Since $(X, \tau(B))$ is a pre-B-semi-T_b space, $X - \{x\}$ is B-semi-closed or equivalently $\{x\}$ is B-semi-open. □

4. Contra-pre-B-semi-continuous Functions

Definition 4.1. *A function $f : (X, \tau(B_X)) \to (Y, \sigma(B_Y))$ is called contra-pre-B-semi-continuous if $f^{-1}(V)$ is pre-B-semi-closed in X, for every B-open set V of Y.*

Theorem 4.2. *Every contra-B-β-continuous function is contra-pre-B-semi-continuous.*

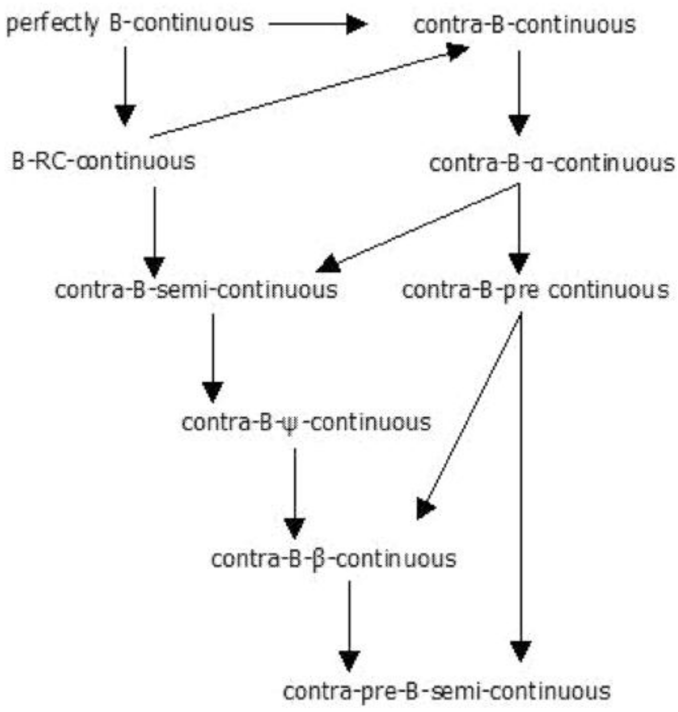
Proof. It follows from the fact that every B-β-closed set is pre-B-semi-closed. □

Example 4.3. *A contra-pre-B-semi-continuous function need not be contra-B-β-continuous. Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}\}$ and $B_X = \{a, c\}$. Then the sets in $\{\phi, X, \{a\}, \{a, c\}\}$ are called B_X-open. Let $\sigma = \{\phi, Y\}$ and $B_Y = \{a\}$. Then the sets in $\{\phi, Y, \{a\}\}$ are called B_Y-open. Let $f : (X, \tau(B_X)) \to (Y, \sigma(B_Y))$ be defined by f(a)=a, f(b)=a and f(c)=b. Then f is contra-pre-B-semi-continuous but not contra-Bβ-continuous. Then f is contra-pre-B-semi-continuous but not contra-Bβ-continuous. Thus the class of contra-pre-B-semi-continuous functions properly contain the class of contra-Bβ-continuous functions.*

Theorem 4.4. *Every contra-B-pre continuous function is contra-pre-B-semi-continuous.*

Proof. It follows from the fact that every B-preclosed set is pre-B-semi-closed. □

Example 4.5. *A contra-pre-B-semi-continuous function need not be contra-B-pre continuous. Let $X=Y=\{a, b, c\}$, $\tau = \{\phi, X, \{a\}\}$ and $B_X = \{b\}$. Then the sets in $\{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ are called B_X-open. Let $\sigma=\{\phi, Y\}$ and $B_Y = \{a\}$. Then the sets in $\{\phi, Y, \{a\}\}$ are called B_Y-open. Let $g : (X, \tau(B_X)) \to (Y, \sigma(B_Y))$ be defined by g(a)=a, g(b)=b and g(c)=c. Then g is contra-pre-B-semi-continuous but not contra-B-precontinuous. Thus the class of contra-pre-B-semi-continuous functions properly contain the class of contra-B-pre continuous functions. Thus we have the following diagram for the functions we defined:*



In the above diagram, $A \to B$ denotes A implies B but not conversely.

Example 4.6. *Let $X = Y = \{a,\ b,\ c\}$, $\tau = \{\phi,\ X,\ \{c\}\}$ and $B_X = \{b,\ c\}$ then $\tau(B_X) = \{\phi,\ X,\ \{c\},\ \{b,\ c\}\}$. Let $\sigma = \{\phi,\ Y\}$ and $B_Y = \{a\}$ then $\sigma(B_Y) = \{\phi,\ Y,\ \{a\}\}$. Let $f : (X, \tau(B_X)) \to (Y, \sigma(B_Y))$ be an identity map. Then f is contra-B-continuous but not perfectly B-continuous.*

Example 4.7. *Let $X = Y = \{a,\ b,\ c\}$, $\tau = \{\phi,\ X,\ \{b\},\ \{b,\ c\}\}$ and $B_X = \{c\}$ then $\tau(B_X) = \{\phi,\ X,\ \{b\},\ \{c\},\ \{b,\ c\}\}$. Let $\sigma = \{\phi,\ Y\}$ and $B_Y = \{a,\ b\}$ then $\sigma(B_Y) = \{\phi,\ Y,\ \{a,\ b\}\}$. Let $f : (X, \tau(B_X)) \to (Y, \sigma(B_Y))$ be an identity map. Then f is B-RC-continuous but not perfectly B-continuous.*

Example 4.8. *Let* $X = Y = \{a,\ b,\ c\}$, $\tau = \{\phi,\ X,\ \{c\}\}$ *and* $B_X = \{b,\ c\}$ *then* $\tau(B_X) = \{\phi,\ X,\ \{c\},\ \{b,\ c\}\}$. *Let* $\sigma = \{\phi,\ Y\}$ *and* $B_Y = \{a\}$ *then* $\sigma(B_Y) = \{\phi,\ Y,\ \{a\}\}$. *Let* $f : (X,\ \tau(B_X)) \rightarrow (Y,\ \sigma(B_Y))$ *be an identity map. Then f is contra-B-continuous but not B-RC-continuous.*

Example 4.9. *Let* $X = Y = \{a,\ b,\ c\}$, $\tau = \{\phi,\ X,\ \{c\}\}$ *and* $B_X = \{a\}$ *then* $\tau(B_X) = \{\phi,\ X,\ \{a\},\ \{a,\ c\}\}$. *Let* $\sigma = \{\phi,\ Y\}$ *and* $B_Y = \{c\}$ *then* $\sigma(B_Y) = \{\phi,\ Y,\ \{c\}\}$. *Let* $f : (X,\ \tau(B_X)) \rightarrow (Y,\ \sigma(B_Y))$ *be an identity map. Then f is contra-B-*α*-continuous but not contra-B-continuous.*

Example 4.10. *Let* $X = Y = \{a,\ b,\ c\}$, $\tau = \{\phi,\ X,\ \{c\}\}$ *and* $B_X = \{a\}$ *then* $\tau(B_X) = \{\phi,\ X,\ \{a\},\ \{a,\ c\}\}$. *Let* $\sigma = \{\phi,\ Y\}$ *and* $B_Y = \{c\}$ *then* $\sigma(B_Y) = \{\phi,\ Y,\ \{c\}\}$. *Let* $f : (X,\ \tau(B_X)) \rightarrow (Y,\ \sigma(B_Y))$ *be an identity map. Then f is contra-B-semi-continuous but not B-RC-continuous.*

Example 4.11. *Let* $X = Y = \{a,\ b,\ c\}$, $\tau = \{\phi,\ X,\ \{a\},\ \{a,\ b\}\}$ *and* $B_X = \{b\}$ *then* $\tau(B_X) = \{\phi,\ X,\ \{a\},\ \{b\},\ \{a,\ b\}\}$. *Let* $\sigma = \{\phi,\ Y\}$ *and* $B_Y = \{b\}$ *then* $\sigma(B_Y) = \{\phi,\ Y,\ \{b\}\}$. *Let* $f : (X,\ \tau(B_X)) \rightarrow (Y,\ \sigma(B_Y))$ *be an identity map. Then f is contra-B-semi-continuous but not contra-B-*α*-continuous.*

Example 4.12. *Let* $X = Y = \{a,\ b,\ c\}$, $\tau = \{\phi,\ X,\ \{a\}\}$ *and* $B_X = \{b,\ c\}$ *then* $\tau(B_X) = \{\phi,\ X,\ \{a\},\ \{b,\ c\}\}$. *Let* $\sigma = \{\phi,\ Y\}$ *and* $B_Y = \{c\}$ *then* $\sigma(B_Y) = \{\phi,\ Y,\ \{c\}\}$. *Let* $f : (X,\ \tau(B_X)) \rightarrow (Y,\ \sigma(B_Y))$ *be an identity map. Then f is contra-B-pre-continuous but not contra-B-*α*-continuous.*

Example 4.13. *Let* $X = Y = \{a,\ b,\ c\}$, $\tau = \{\phi,\ X,\ \{a\},\ \{a,\ b\}\}$ *and* $B_X = \{b\}$ *then* $\tau(B_X) = \{\phi,\ X,\ \{a\},\ \{b\},\ \{a,\ b\}\}$. *Let* $\sigma = \{\phi,\ Y\}$ *and* $B_Y = \{a\}$ *then* $\sigma(B_Y) = \{\phi,\ Y,\ \{a\}\}$. *Let* $f : (X,\ \tau(B_X)) \rightarrow (Y,\ \sigma(B_Y))$ *be an identity map. Then f is contra-B-*β*-continuous but not contra-B-pre-continuous.*

Example 4.14. *Let* $X = Y = \{a,\ b,\ c\}$, $\tau = \{\phi,\ X\}$ *and* $B_X = \{a,\ b\}$ *then* $\tau(B_X) = \{\phi,\ X,\ \{a,\ b\}\}$. *Let* $\sigma = \{\phi,\ Y\}$ *and* $B_Y = \{a,\ c\}$ *then* $\sigma(B_Y) = \{\phi,\ Y,\ \{a,\ c\}\}$. *Let* $f : (X,\ \tau(B_X)) \rightarrow (Y,\ \sigma(B_Y))$ *be an identity map. Then f is contra-pre-B-semi-continuous but not contra-B-pre-continuous.*

Example 4.15. *Let* $X = Y = \{a,\ b,\ c\}$, $\tau = \{\phi,\ X\}$ *and* $B_X = \{a,\ b\}$ *then* $\tau(B_X) = \{\phi,\ X,\ \{a,\ b\}\}$. *Let* $\sigma = \{\phi,\ Y\}$ *and* $B_Y = \{a,\ c\}$ *then* $\sigma(B_Y) = \{\phi,\ Y,\ \{a,\ c\}\}$. *Let* $f : (X,\ \tau(B_X)) \rightarrow (Y,\ \sigma(B_Y))$ *be an identity map. Then f is contra-pre-B-semi-continuous but not contra-B-*β*-continuous.*

Example 4.16. *Let* $X = Y = \{a,\ b,\ c\}$, $\tau = \{\phi,\ X\}$ *and* $B_X = \{a,\ b\}$ *then* $\tau(B_X) = \{\phi,\ X,\ \{a,\ b\}\}$. *Let* $\sigma = \{\phi,\ Y\}$ *and* $B_Y = \{a,\ c\}$ *then* $\sigma(B_Y) = \{\phi,\ Y,\ \{a,\ c\}\}$. *Let* $f : (X,\ \tau(B_X)) \rightarrow (Y,\ \sigma(B_Y))$ *be an identity map. Then f is contra-B-*ψ*-continuous but not contra-B-semi-continuous.*

Example 4.17. *Let* $X = Y = \{a,\ b,\ c\}$, $\tau = \{\phi,\ X\}$ *and* $B_X = \{a,\ b\}$ *then* $\tau(B_X) = \{\phi,\ X,\ \{a,\ b\}\}$. *Let* $\sigma = \{\phi,\ Y\}$ *and* $B_Y = \{b\}$ *then* $\sigma(B_Y) = \{\phi,\ Y,\ \{b\}\}$. *Let* $f : (X,\ \tau(B_X)) \rightarrow (Y,\ \sigma(B_Y))$ *be an identity map. Then f is contra-B-*β*-continuous but not contra-B-*ψ*-continuous.*

Definition 4.18. *A space* $(X,\ \tau(B))$ *is called pre-B-semi-locally indiscrete if every pre-B-semi-open set in it is B-closed.*

Example 4.19. *Let* $X = \{a, b\}$, $\tau = \{\phi, X, \{a\}$ *and* $B = \{b\}$. *Then the sets in* $\{\phi, X, \{a\}, \{b\}\}$ *are called B-open. Then the sets in* $\{\phi, X, \{a\}, \{b\}\}$ *are called B-closed. Then the sets in* $\{\phi, X, \{a\}, \{b\}\}$ *are called pre-B-semi-open sets. Therefore* $(X, \tau(B))$ *is a pre-B-semi-locally indiscrete space. The space* $(X, \tau(B))$ *in Example 4.5 is not a pre-B-semi-locally indiscrete space since* $\{a\}$ *is a pre-B-semi-open set but it not B-closed.*

Theorem 4.20. *If a function* $f : (X, \tau(B_X)) \rightarrow (Y, \sigma(B_Y))$ *is pre-B-semi-continuous and* $(X, \tau(B_X))$ *is a pre-B-semi-locally indiscrete, then f is contra-B-continuous.*

Proof. Let V be an B_Y-open. Then $f^{-1}(V)$ is pre-B-semi-open in X, since f is pre-B-semi-continuous. Since $(X, \tau(B_X))$ is pre-B-semi-locally indiscrete, $f^{-1}(V)$ is B_X-closed. Therefore f is contra-B-continuous. □

Theorem 4.21. *If a function $f : (X, \tau(B_X)) \to (Y, \sigma(B_Y))$ is contra-pre-B-semi-continuous and $(X, \tau(B_X))$ is a pre-B-semi-$T_{1/2}$ space, then f is contra-B-β-continuous.*

Proof. Let V be an B_Y-open. Then $f^{-1}(V)$ is pre-B-semi-closed in X, since f is contra-pre-B-semi-continuous. Since $(X, \tau(B_X))$ is pre-B-semi-$T_{1/2}$, $f^{-1}(V)$ is B-β-closed in X. Therefore f is contra-B-β-continuous. □

Theorem 4.22. *If a function $f : (X, \tau(B_X)) \to (Y, \sigma(B_Y))$ is contra-pre-B-semi-continuous and $(X, \tau(B_X))$ is a pre-B-semi-T_b space, then f is contra-B-semi-continuous.*

Proof. Let V be an B_Y-open set. Then $f^{-1}(V)$ is pre-B-semi-closed in X, since f is contra-pre-B-semi-continuous. Since $(X, \tau(B_X))$ is pre-B-semi-T_b, $f^{-1}(V)$ is B-semi-closed in X. Therefore f is contra-B-semi-continuous. □

Theorem 4.23. *If a function $f : (X, \tau(B_X)) \to (Y, \sigma(B_Y))$ is contra-pre-B-semi-continuous and $(X, \tau(B_X))$ is a pre-B-semi-$T_{3/4}$ space, then f is contra-B-pre continuous.*

Proof. Let V be an B_Y-open. Then $f^{-1}(V)$ is pre-B-semi-closed in X, since f is contra-pre-B-semi-continuous. Since $(X, \tau(B_X))$ is pre-B-semi-$T_{3/4}$, $f^{-1}(V)$ is B-preclosed in X. Therefore f is contra-B-pre continuous. □

References

[1] M.E.Abd El-Monsef, A.M.Kozae and R.A.Abu-Gdairi, *New approaches for generalized continuous functions*, Int. Journal of Math. Analysis, 4(27)(2010), 1329-1339.

[2] J.Dontchev, *Contra-continuous functions and strongly S-closed spaces*, Internat. J. Math. Math. Sci., 19(2)(1996), 303-310.

[3] J.Dontchev and T.Noiri, *Contra semi-continuous functions*, Math. Pannon, 10(2)(1999), 159-168.

[4] S.Jafari and T.Noiri, *On contra-precontinuous functions*, Bull. Malays. Math. Sci. Soc., (2)25(2)(2002), 115-128.

[5] S.Jafari and T.Noiri, *Contra α-continuous functions between topological spaces*, Iran. J. Sci., 2(2)(2001), 153-167.

[6] N.Levine, *Simple extension of topologies*, Amer. Math. Monthly, 71(1964), 22-25.

[7] M.Murugalingam, O.Ravi and S.Nagarani, *New generalized continuous functions*, International Journal of Mathematics And its Applications, 3(3B)(2015), 5562.

[8] O.Ravi, M.Murugalingam and S.Nagarani, *On Decompositions of continuity in simply extended topological spaces*, (submitted).

[9] A.Vadivel, R.Vijayalakshmi and D.Krishnaswamy, *B-Generalized regular and B-generalized normal spaces*, International Mathematical Forum, 5(54)(2010), 2699-2706.

[10] M.K.R.S.Veera Kumar, *Between semi-closed sets and semi-pre-closed sets*, Rend. Istit. Mat. Univ. Trieste., 32(1-2)(2000), 25-41.

[11] M.K.R.S.Veera Kumar, *Pre-semi-closed sets*, Indian J. Math., 44(2)(2002), 165-181.

[12] M.K.R.S.Veera Kumar, *Contra-ψ-continuous functions*, preprint.

[13] M.K.R.S.Veera Kumar, *Contra-Pre-semi-continuous functions*, Bull. Malays. Math. Sci. Soc., (2)28(1)(2005), 67-71.