International Journal of Current Research in Science and Technology
Volume 3, Issue 10 (2017), 1–10.
ISSN: 2394-5745
Available Online: http://ijcrst.in/

International Journal of Current Research in Science and Technology

On Contra-λ_B-Continuous Functions

Research Article

T. Kavitha*

Department of Mathematics, RVS College of Engineering and Technology, Dindigul, Tamil Nadu, India.

Abstract: In [5] Dontchev introduced and investigated a new notion of continuity called contra-continuity. Jafari and Noiri [7–9] introduced new generalizations of contra-continuity called contra-α-continuity, contra-super-continuity, contra-precontinuity. The objective of this paper is to introduce and study the properties of new class of contra continuous functions via the new sets called λ_B-closed sets.

MSC: 54A05, 54D10.

Keywords: Λ_B-sets, λ_B-open set, contra-B-continuous function, contra-λ_B-continuous function, contra-Bg-continuous function, contra-BRC-continuous function.

1. Introduction

Maki [11] in 1986 introduced the notion of Λ-sets in topological spaces. A Λ-set is a set A which is equal to its kernel (= *saturated set*) i.e. to the intersection of all open supersets of A. Arenas et al. [1] introduced and investigated the notion of λ-closed sets by involving Λ-sets and closed sets. This enabled them to obtain some nice results. Caldas et al. [4] introduced the notion of λ-closure of a set by utilizing the notion of λ-open sets defined in [1]. Jafari and Noiri introduced and investigated the notions of contra-precontinuity [7], contra-α-continuity [8] and contra-super-continuity [9] as a continuation of research done by Dontchev and Noiri [5, 6] on the interesting notions of contra-continuity and contra-semi-continuity respectively. Caldas and Jafari [3] introduced and investigated the notion of contra-β-continuous functions in topological spaces.

In [5] Dontchev introduced and investigated a new notion of continuity called contra-continuity. Jafari and Noiri [7–9] introduced new generalizations of contra-continuity called contra-α-continuity, contra-super-continuity, contra-precontinuity. The objective of this chapter is to introduce and study the properties of new class of contra continuous functions via the new sets called λ_B-closed sets and has as purpose to investigate some properties of contra-λ_B-continuous functions, contra-BRC-continuous functions, contra-Bg-continuous functions by using λ_B-open sets.

2. Preliminaries

Throughout this paper, $(X, \tau(B_X))$, $(Y, \sigma(B_Y))$ *and* $(Z, \eta(B_Z))$(briefly X, Y and Z) will denote simply extended topological spaces.

* *E-mail: kavisakthi1983@gmail.com*

Definition 2.1. *Levine [10] in 1964 defined $\tau(B) = \{O \cup (\acute{O} \cap B) : O, \acute{O} \in \tau\}$ and called it simple extension of τ by B, where $B \notin \tau$. The sets in $\tau(B)$ are called B-open sets. And the complement of B-open set is called B-closed.*

Definition 2.2 ([10])**.** *Let S be a subset of a simply extended topological space X. Then*

(1). The B-closure of S, denoted by Bcl(S), is defined as $\cap \{F : S \subseteq F$ and F is B-closed$\}$;

(2). The B-interior of S, denoted by Bint(S), is defined as $\cup \{F : F \subseteq S$ and F is B-open$\}$.

Definition 2.3 ([12])**.** *Let $(X, \tau(B))$ be a SETS and $A \subseteq X$. Then A is said to be*

(1). B-semiopen if $A \subseteq Bcl(Bint(A))$;

(2). B-preopen if $A \subseteq Bint(Bcl(A))$;

(3). B-α-open if $A \subseteq Bint(Bcl(Bint(A)))$;

(4). Bβ-open if $A \subseteq Bcl(Bint(Bcl(A)))$.

The complement of B-semiopen (resp. B-preopen, B-α-open, Bβ-open) is said to be B-semiclosed (resp. B-preclosed, B-α-closed, Bβ-closed).

In this chapter, let us denote by $\sigma(\tau(B))$ (or σ) the class of all B-semiopen sets on X, by $\pi(\tau(B))$ (or π) the class of all B-preopen sets on X, by $\alpha(\tau(B))$ (or α) the class of all B-α-open sets on X, by $\beta(\tau(B))$ (or β) the class of all Bβ-open sets on X.

Definition 2.4 ([14])**.** *A subset S of X is called regular B-open if $S = Bint(Bcl(S))$. The complement of regular B-open set is called regular B-closed. The B-semi-closure of a subset A of X, denoted by B-scl(A), is the intersection of all B-semi-closed sets of X containing A. The Bβ-closure of a subset A of X, denoted by Bβ-cl(A), is the intersection of all Bβ-closed sets of X containing A. The B-semi-interior of a subset A of X, denoted by B-sint(A), is defined to be the union of all B-semi-open sets contained in A.*

Definition 2.5 ([12])**.** *A function $f : (X, \tau(B_X)) \to (Y, \sigma(B_Y))$ is called*

(1). B-continuous if $f^{-1}(V)$ is B-open in X, for every B-open set V of Y.

(2). B-precontinuous if $f^{-1}(V) \in BPO(X)$, for every B-open set V of Y.

(3). B-α-continuous if $f^{-1}(V) \in B\alpha O(X)$, for every B-open set V of Y.

(4). B-semi-continuous if $f^{-1}(V) \in BSO(X)$, for every B-open set V of Y.

(5). Bβ-continuous if $f^{-1}(V) \in B\beta O(X)$, for every B-open set V of Y.

Definition 2.6. *A subset A of a space $(X, \tau(B_X))$ is called B-g-closed set [2] if $Bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X. The complement of B-g-closed set is called B-g-open set.*

Definition 2.7 ([13])**.** *A subset S of a simply extended topological space $(X, \tau(B_X))$ is called locally B-closed if $A = P \cap Q$ where P is B-open and Q is B-closed.*

3. Contra-λ_B-continuous Functions

Definition 3.1. *A function $f : (X, \tau(B_X)) \to (Y, \sigma(B_Y))$ is called*

(1). contra-BRC-continuous if $f^{-1}(V)$ is regular B-closed in X for every B-open set V of Y

(2). contra-B-continuous if $f^{-1}(V)$ is B-closed in X, for every B-open set V of Y.

(3). contra-B-precontinuous if $f^{-1}(V)$ is B-preclosed in X, for every B-open set V of Y.

(4). contra-B-semi-continuous if $f^{-1}(V)$ is B-semi-closed in X, for every B-open set V of Y.

(5). contra-Bα-continuous if $f^{-1}(V)$ is Bα-closed in X, for every B-open set V of Y.

(6). contra-Bβ-continuous if $f^{-1}(V)$ is Bβ-closed in X, for every B-open set V of Y.

Definition 3.2. *A Λ_B-set is a set A which is equal to the intersection of all B-open supersets of A.*

Definition 3.3. *A subset A of X is called λ_B-closed if $A = L \cap D$, where L is a Λ_B-set and D is a B-open in X. The complement of a λ_B-closed set is called λ_B-open. We denote the collection of all λ_B-open sets (resp. λ_B-closed sets) by $\lambda_B O(X)$ (resp. λ_B-C(X)). We set $\lambda_B O(X, x) = \{U : x \in U \in \lambda_B O(X)\}$ and $\lambda_B C(X, x) = \{U : x \in U \in \lambda_B C(X)\}$.*

Definition 3.4. *The collection of all B-open (resp. B-closed, B-clopen) subsets of X will be denoted by BO(X) (resp. BC(X), BCO(X)). We set BC(X, x)=$\{V \in BC(X) : x \in V\}$ for $x \in X$. we define similarly BCO(X, x).*

Definition 3.5. *A function $f : (X, \tau(B_X)) \to (Y, \sigma(B_Y))$ is said to be*

(1). λ_B-continuous if the inverse image of every B-closed set in Y is λ_B-closed in X.

(2). BLC-continuous if the inverse image of every B-open set in Y is locally B-closed in X.

Definition 3.6. *A function $f : (X, \tau(B_X)) \to (Y, \sigma(B_Y))$ is called*

(1). contra-λ_B-continuous if $f^{-1}(V)$ is λ_B-closed in X, for every B-open set V of Y.

(2). contra-Bg-continuous if $f^{-1}(V)$ is Bg-closed in X, for every B-open set V of Y.

Definition 3.7. *A point x in a simply extended topological space $(X, \tau(B_X))$ is called a λ_B-cluster point of A if every λ_B-open set U of X containing x such that $A \cap U \neq \phi$. The set of all λ_B-cluster points is called the λ_B-closure of A and is denoted by B-$cl_\lambda(A)$.*

Definition 3.8. *A subset A_x of a simply extended topological space X is said to be λ_B-neighborhood of a point $x \in X$ if there exists a λ_B-open set U such that $x \in U \subseteq A_x$.*

Definition 3.9. *A simply extended topological space $(X, \tau(B_X))$ is called a B-T_2 space if every Bg-closed subset of X is B-closed.*

Lemma 3.10. *Let A, C and A_i $(i \in I)$ be subsets of a simply extended topological space $(X, \tau(B_X))$. The following properties hold:*

(1). A is λ_B-closed if and only if $A =$ B-$cl_\lambda(A)$.

(2). $A \subset$B-$cl_\lambda(A)$.

(3). If $A \subset C$, then B-cl$_\lambda(A) \subset$ B-cl$_\lambda(C)$.

(4). B-cl$_\lambda(A))$ is λ_B-closed.

Definition 3.11. *A simply extended topological space $(X, \tau(B_X))$ is said to be*

(1). λ_B-$T_{1/2}$ if every singleton is λ_B-open or λ_B closed.

(2). λ_B-T_2 if for any distinct pair of points x and y in X, there exist $U \in \lambda_B\, O(X,\, x)$ and $V \in \lambda_B\, O(X,\, y)$ such that $U \cap\, V = \sigma$.

(3). B-Ultra Hausdorff if for each pair of distinct points x and y in X there exist $U \in BCO(X, x)$ and $V \in BCO(X,\, y)$ such that $U{\cap}V = \sigma$.

Definition 3.12. *Let A be a subset of a space $(X, \tau(B_X))$. The set $\cap\{U \in BO(X) : A \subseteq U\}$ is called the B-kernel of A and is denoted by B-ker(A).*

Lemma 3.13. *The following properties hold for the subsets A, B of a space X:*

(1). $x \in$ B-ker(A) if and only if $A \cap F \neq \phi$, for any $F \in$ BC(X, x).

(2). $A \subset$ B-ker(A) and $A =$ B-ker(A) if A is B-open in X.

(3). If $A \subset B$, then B-ker(A) $\subset$ B-ker(B).

Theorem 3.14. *Let $f : (X,\, \tau(B_X)) \to (Y,\, \sigma(B_Y))$ be a function from a simply extended topological space X into a simply extended topological space Y. The following statements are equivalent.*

(1). f is contra-λ_B-continuous;

(2). the inverse image of each B-closed set in Y is λ_B-open in X;

(3). for each point x in X and each B-closed set V in Y with $f(x) \in V$, there exists a λ_B-open set U in X such that $x \in U$, $f(U) \subset V$;

(4). for every subset A of X, f(B-cl$_\lambda(A)) \subset$ B-ker(f(A)) ;

(5). for each subset B of Y, B-cl$_\lambda(f^{-1}(B)) \subset f^{-1}$(B-ker(B)) ;

Proof.
(1) $\Leftrightarrow$ (2) By Definition 3.3.
(2) $\Rightarrow$ (3) Let $x \in X$ and V be a B-closed set containing f(x). By (2), $U = f^{-1}(V)$ *is* λ_B-open set containing x such that $f(U) \subset V$. It follows from the fact that the union of any family of λ_B-open sets is λ_B-open.
(3) $\Rightarrow$ (4) Let A be any subset of X. Suppose that $y \notin$ B-ker(f(A)). Then by Lemma 3.13, there exists $V \in BC(Y, y)$ such that $f(A) \cap V = \phi$. For any $x \in f^{-1}(V)$, by (3) there exists $U_x \in \lambda_B O(X, x)$ such that $f(U_x) \subset V$. Hence $f(A \cap U_x) \subset f(A) \cap f(U_x) \subset f(A) \cap V = \phi$ and $A \cap U_x = \phi$. This shows that $x \notin$ B-cl$_\lambda(A)$ for any $x \in f^{-1}(V)$. Therefore, $f^{-1}(V)\cap$ B-cl$_\lambda(A) = \phi$ and hence $V\cap$ f(B-cl$_\lambda$(A)) $= \phi$. Thus $y \notin$ f(B-cl$_\lambda(A)$). Consequently, we obtain f(B-cl$_\lambda(A)$) $\subset$ B-ker(f(A)).
(4)$\Rightarrow$ (5) Let B be any subset of Y. By (4) and Lemma 3.13, we have f(B-cl$_\lambda$(f^{-1}(B))) $\subset$ B-ker($f(f^{-1}$(B))) $\subset$ B-ker(B) and B-cl$_\lambda(f^{-1}(B)) \subset f^{-1}$(B-ker(B)).
(5) $\Rightarrow$ (1) Let V be any B-open set of Y. Then by Lemma 3.13, we have B-cl$_\lambda(f^{-1}(V)) \subset f^{-1}$(B-ker(V)) $= f^{-1}(V)$ and B-cl$_\lambda(f^{-1}(V)) = f^{-1}(V)$. This shows that $f^{-1}(V)$ is λ_B-closed in X. ∎

We have the following implications:

contra-λ_B-continuity
↑
contra-B-continuity ⟶ contra-Bα-continuity ⟶ contra-B pre-continuity
↓ ↓
contra-B semi-continuity ⟶ contra-B β-continuity

Remark 3.15.

(1). The following Examples 3.16 and 3.17 show that λ_B-continuity and contra-λ_B-continuity are independent concepts.

(2). The following Examples 3.17 and 3.21 show that contra-λ_B-continuity and contra-Bg-continuity are independent concepts.

Example 3.16. *Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}\}$ and $B_X = \{a, b\}$. Then $\tau(B_X) = \{\phi, X, \{a\}, \{a, b\}\}$. Let $\sigma = \{\phi, Y\}$ and $B_Y = \{a, b\}$ then $\sigma(B_Y) = \{\phi, Y, \{a, c\}\}$. Let f: $(X, \tau(B_X)) \to (Y, \sigma(B_Y))$ be an identity map. Then f is λ_B-continuous but not contra-λ_B-continuous.*

Example 3.17. *Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}\}$ and $B_X = \{a, b\}$. Then $\tau(B_X) = \{\phi, X, \{a\}, \{a, b\}\}$. Let $\sigma = \{\phi, Y\}$ and $B_Y = \{a,b\}$ then $\sigma(B_Y) = \{\phi, Y, \{a, c\}\}$. Clearly $\lambda_B O(X) = \{\phi, X, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$. Let f: $(X, \tau(B_X)) \to (Y, \sigma(B_Y))$ be an identity map. Then f is contra-λ_B-continuous but not λ_B-continuous and also it is not contra-Bg-continuous.*

Remark 3.18. *It should be mentioned that every contra-B continuous function is contra-Bg-continuous and none of implications in the above diagram are reversible as shown by the following Examples.*

Example 3.19. *Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}\}$ and $B_X = \{a, c\}$. Then $\tau(B_X) = \{\phi, X, \{a\}, \{a, c\}\}$. Let $\sigma = \{\phi, Y\}$ and $B_Y = \{a\}$ then $\sigma(B_Y) = \{\phi, Y, \{a\}\}$. Let f: $(X, \tau(B_X)) \to (Y, \sigma(B_Y))$ be an identity map. Then f is contra-λ_B-continuous but not contra-B-continuous.*

Example 3.20. *Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, X\}$ and $B_X = \{a\}$. Then $\tau(B_X) = \{\phi, X, \{a\}\}$. Let $\sigma = \{\phi, Y, \{b\}, \{b, c\}\}$ and $B_Y = \{c\}$ then $\sigma(B_Y) = \{\phi, Y, \{b\}, \{c\}, \{b, c\}\}$. Let f: $(X, \tau(B_X)) \to (Y, \sigma(B_Y))$ be an identity map. Then f is contra-Bα-continuous but not contra-B-continuous.*

Example 3.21. *Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}\}$ and $B_X = \{b\}$. Then $\tau(B_X) = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$. Let $\sigma = \{\phi, Y\}$ and $B_Y = \{a\}$ then $\sigma(B_Y) = \{\phi, Y, \{a\}\}$. Let f: $(X, \tau(B_X)) \to (Y, \sigma(B_Y))$ be an identity map. Then f is contra-B semi-continuous but not contra-B α-continuous.*

Example 3.22. *Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, X\}$ and $B_X = \{a, b\}$. Then $\tau(B_X) = \{\phi, X, \{a, b\}\}$. Let $\sigma = \{\phi, Y\}$ and $B_Y = \{a, c\}$ then $\sigma(B_Y) = \{\phi, Y, \{a, c\}\}$. Let f: $(X, \tau(B_X)) \to (Y, \sigma(B_Y))$ be an identity map. Then f is contra-B-precontinuous but not contra-Bα-continuous.*

Example 3.23. *Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, X\}$ and $B_X = \{a, b\}$. Then $\tau(B_X) = \{\phi, X, \{a, b\}\}$. Let $\sigma = \{\phi, Y\}$ and $B_Y = \{a\}$ then $\sigma(B_Y) = \{\phi, Y, \{a\}\}$. Let f: $(X, \tau(B_X)) \to (Y, \sigma(B_Y))$ be an identity map. Then f is contra-Bβ-continuous but not contra-B semi-continuous.*

Example 3.24. *Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{a, b\}\}$ and $B_X = \{b\}$. Then $\tau(B_X) = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$. Let $\sigma = \{\phi, Y, \{a\}\}$ and $B_Y = \{c\}$ then $\sigma(B_Y) = \{\phi, Y, \{a\}, \{c\}, \{a, c\}\}$. Let f: $(X, \tau(B_X)) \to (Y, \sigma(B_Y))$ be an identity map. Then f is contra-B β precontinuous but not contra-B pre-continuous.*

Theorem 3.25. *For a simply extended topological space $(X, \tau(B))$ the following conditions are equivalent:*

(1). X is a $BT_{1/2}$ space.

(2). Every subset of X is λ_B-closed.

Lemma 3.26. *Let $(X, \tau(B))$ be a $BT_{1/2}$ space and $f : (X, \tau(B_X)) \to (Y, \sigma(B_Y))$. If f is contra-$B\beta$-continuous (resp. contra-B-semi-continuous, contra-B-precontinuous, contra-$B\alpha$-continuous, contra-Bg-continuous), then f is contra-λ_B-continuous.*

Theorem 3.27.

(1). The following statements are equivalent for a function $f : (X, \tau(B_X)) \to (Y, \sigma(B_Y))$.

(i). f is BRC-continuous;

(ii). f is contra-B-precontinuous and B-semi-continuous;

(iii). f is contra-$B\alpha$-continuous and $B\beta$-continuous;

(iv). f is contra-B-continuous and $B\beta$-continuous;

(2). If $f : (X, \tau(B_X)) \to (Y, \sigma(B_Y))$ is BRC-continuous, then f is contra-λ_B-continuous.

Proof.

(1). It is obvious.

(2). Every BRC-continuous function is contra-B-continuous and hence contra-λ_B-continuous. □

4. A New Decomposition of contra-λ_B-continuity

Lemma 4.1.

(1). Every locally B-closed set is λ_B-closed;

(2). Every Λ_B-set is λ_B-closed.

Lemma 4.2. *A subset $A \subseteq (X, \tau(B))$ is Bg-closed if and only if Bcl(A)$\subseteq \Lambda_B$-set.*

Lemma 4.3. *For a subset A of a simply extended topological space $(X, \tau(B))$ the following conditions are equivalent:*

(1). A is B-closed;

(2). A is Bg-closed and locally B-closed;

(3). A is Bg-closed and λ_B-closed.

Proof.
(1)⇒(2) Every B-closed set is both Bg-closed and locally B-closed.
(2)⇒(3) Refer Lemma 4.1.
(3)⇒(1) A is Bg-closed. So by Lemma 4.2 Bcl(A)$\subseteq \Lambda_B$-set. Now A is λ_B-closed, so by definition A = Λ_B-set ∩ Bcl(A). Hence A = Bcl(A), that is A is B-closed. □

Theorem 4.4. *For a function $f : (X, \tau(B_X)) \to (Y, \sigma(B_Y))$, the following conditions are equivalent:*

(1). f is contra-B-continuous;

(2). f is contra-Bg-continuous and BLC-continuous;

(3). f is contra-Bg-continuous and contra-λ_B-continuous.

Definition 4.5. *Suppose that one point sets are closed in X. Then X is said to be regular if for each pair consisting of a point x and a closed set C disjoint from x, there exist disjoint open sets containing x and C, respectively.*

Theorem 4.6. *If a function $f : (X, \tau(B_X)) \to (Y, \sigma(B_Y))$ is contra-λ_B-continuous and Y is B-regular, then f is λ_B-continuous.*

Proof. Let x be an arbitrary point of X and V an B-open set of Y containing f(x). Since Y is B-regular, there exists an B-open set W of Y containing f(x) such that B-cl(W) $\subset V$. Since f is contra-λ_B-continuous, so by Theorem 3.14, there exists $U \in \lambda_B O(X, x)$ such that $f(U) \subset$ B-cl(W). Then $f(U) \subset$ B-cl(W) $\subset V$. Hence f is λ_B-continuous. □

Definition 4.7. *A space $(X, \tau(B))$ is said to be λ_B S-space if every λ_B-open subset of X is B-semi-open in X.*

Definition 4.8. *A subset S of a simply extended topological space $(X, \tau(B))$ is called locally B indiscrete if every B-open set is B-closed.*

Theorem 4.9. *If a function $f : (X, \tau(B_X)) \to (Y, \sigma(B_Y))$ is contra-λ_B-continuous and X is a λ_B S-space (resp. locally B indiscrete), then f is contra-B-semi-continuous (resp. contra-B-continuous, B-continuous).*

Theorem 4.10. *If X is a simply extended topological space and for each pair of distinct points x_1 and x_2 in X, there exists a map f of X into a B-Urysohn space Y such that $f(x_1) \neq f(x_2)$ and f is contra-λ_B-continuous at x_1 and x_2, then X is λ_B-T_2.*

Proof. Let x_1 and x_2 be any distinct points in X. Then by hypothesis, there is a B-Urysohn space Y and a function $f : (X, \tau(B_X)) \to (Y, \sigma(B_Y))$, which satisfies the conditions of the Theorem. Let $y_i = f(x_i)\ for\ i = 1, 2$. Then $y_1 \neq y_2$. Since Y is B-Urysohn space, there exist B-open neighborhoods U_{y_1} and U_{y_2} of y_1 and y_2 respectively, in Y such that $Bcl(U_{y_1}) \cap Bcl(U_{y_2}) = \phi$. Since f is contra-B-continuous at x_i, there exists a λ_B-open neighborhood W_x of x_i in X such that $f(W_{X_i}) \subset$ B-cl(U_{y_i}) for $i = 1, 2$. Hence we get $W_{x_1} \cap W_{x_2} = \phi$ since $Bcl(U_{y_1}) \cap Bcl(U_{y_2}) = \phi$. Hence X is λ_B-T_2. □

Definition 4.11. *Define the λ_B-frontier of A, denoted by $BFr_\lambda(A)$, as B-$Fr_\lambda(A) = Bcl_\lambda(A) \backslash Bint_\lambda(A)$.*
Equivalently $BFr_\lambda(A) = Bcl_\lambda(A) \cap Bcl_\lambda(X \backslash A)$.

Theorem 4.12. *The set of points $x \in X$ which $f : (X, \tau(B_X)) \to (Y, \sigma(B_Y))$ is not contra-λ_B-continuous is identical with the union of λ_B-frontiers of the inverse image of B-closed sets of Y containing f(x).*

Proof. Necessity: Suppose that f is not contra-λ_B-continuous at a point x of X. Then there exists a B-closed set $F \subset Y$ containing f(x) such that f(U) is not a subset of F for every $U \in \lambda_B O(X, x)$. Hence we have $U \cap (X \backslash f^{-1}(F)) \neq \phi$, for every $U \in \lambda_B O(X, x)$. It follows that $x \in Bcl_\lambda(X \backslash f^{-1}(F))$. We also have $x \in f^{-1}(F) \subset Bcl_\lambda(f^{-1}(F))$. This means that $x \in$B-$Fr_\lambda(f^{-1}(F))$.
Sufficiency: Suppose that $x \in$B-$Fr_\lambda(f^{-1}(F))$ for some $F \in BC(Y, f(x))$. Now , we assume that f is contra-λ_B-continuous at $x \in X$. Then there exists $U \in \lambda_B O(X, x)$ such that $f(U) \subset F$. Therefore we have $x \in U \subset f^{-1}(F)$ and hence $x \in Bint_\lambda(f^{-1}(F)) \subset X \backslash$ B-$Fr_\lambda(f^{-1}(F))$. This is a contradiction. This means that f is not contra-λ_B-continuous. □

Definition 4.13. *A topological space X is B-normal if and only if for every pair of distinct B-closed subsets F_1 and F_2 of X and closed interval [a, b] of reals, there exists a B-continuous mapping $f : X \to [a, b]$ such that $f(F_1) = \{a\}$ and $f(F_2) = \{b\}$.*

Corollary 4.14. *If f is a contra-λ_B-continuous injection of a simply extended topological space X into a B-Urysohn space Y, then X is λ_B-T_2.*

Proof. For each pair of distinct points x_1 and x_2 in X, f is a contra-λ_B-continuous function of X into a B-Urysohn spaces space Y such that $f(x_1) \neq f(x_2)$ since f is injective. Hence by Theorem 4.10, X is λ_B-T_2. □

Corollary 4.15. *If f is a contra-λ_B-continuous injection of a simply extended topological space X into an B-Ultra space Y, then X is λ_B-T_2.*

Proof. Let x_1 and x_2 be any distinct points in X . Since f is injective and Y is B-Ultra space, $f(x_1) \neq f(x_2)$, and there exist $V_1, V_2 \in BCO(Y)$ such that $f(x_1) \in V_1, f(x_1) \in V_2$ and $V_1 \cap V_2 = \phi$. Then $X_i \in f^{-1}(Vi) \in \lambda_B O(X)$ for $i = 1, 2$ and $f^{-1}(V_1) \cap f^{-1}(V_2) = \phi$. Thus X is λ_B-T_2.
We say that the product space $X = X_1 \times X_2 \times ... \times X_n$ has property P_λ if A_i a λ_B-open set in a simply extended topological space X_i, $for\ i = 1, 2, 3, ..., n$, then$A_1 \times A_2 \times ... \times A_n$ is also λ_B-open in the product space $X = X_1 \times X_2 \times ... \times X_n$. □

Theorem 4.16. *Let $f : (X_1, \tau(B_X)) \to (Y, \sigma(B_Y))$ and $g : (X_2, \tau(B_X)) \to (Y, \sigma(B_Y))$ be two functions, where*

(1). $X = X_1 \times X_2$ have the property P_λ.

(2). Y is a B-Urysohn space.

(3). f and g are contra-λ_B-continuous.

Then $\{(x_1, x_2) : f(x_1) = g(x_2)\}$ is λ_B-closed in the product space $X = X_1 \times X_2$.

Proof. Let A denote the set $\{(x_1, x_2) : f(x_1) = g(x_2)\}$. In order to show that A is λ_B-closed. We show that $X_1 \times X_2 - A$ is λ_B-open. Let $(x_1, x_2) \notin A$. Then $f(x_1) \neq g(x_2)$. Since Y is B-Urysohn space, there exist open neighbourhood V_1 and V_2 of $f(x_1)$ and $g(x_2)$ such that $Bcl(V_1) \cap Bcl(V_2) = \phi$. Since $f_i(i = 1, 2)$ is contra-λ_B-continuous, $f_i^{-1}(Bcl(V_i))$ is a λ_B-open set containing x_i in $X_i(i = 1, 2)$. Hence by (1), $f^{-1}(Bcl(V_1)) \times g^{-1}(Bcl(V_2))$ is λ_B-open. Furthermore $(x_1, x_2) \in f^{-1}(Bcl(V_1)) \times g^{-1}(Bcl(V_2)) \subset X_1 \times X_2 - A$. It follows that $X_1 \times X_2 - A$ is λ_B-open. Thus A is λ_B-closed in the product space $X = X_1 \times X_2$. □

Theorem 4.17. *Let $f : (X, \tau(B_X)) \to (Y, \sigma(B_Y))$ be a function and $g : X \to X \times Y$ the bigraph function, given by $g(x) = (x, f(x))$ for every $x \in X$. Then f is contra-λ_B-continuous if and only if g is contra-λ_B-continuous.*

Proof. Let $x \in X$ and let W be a closed subset of $X \times Y$ containing g(x). Then $W \cap (\{x\} \times Y)$ is closed in $\{x\} \times Y$ containing g(x). Also $\{x\} \times Y$ is homeomorphic to Y. Hence $\{y \in Y : (x, y) \in W\}$ is a closed subset of Y. Since f is contra-λ_B-continuous. $\cup\{f^{-1}(y) : (x, y) \in W\}$ is a λ_B-open subset of X. Further $x \in \cup\{f^{-1}(y) : (x, y) \in W\} \subset g^{-1}(W)$. Hence $g^{-1}(W)$ is λ_B-open. Then g is contra-λ_B-continuous.
Conversely, let F be a closed subset of Y. Then $X \times F$ is a closed subset of $X \times Y$. Since g is contra-λ_B-continuos, $g^{-1}(X \times F)$ is a λ_B-open subset of X. Also $g^{-1}(X \times F) = f^{-1}(F)$. Hence f is contra-$\lambda_B$-continuous. □

Theorem 4.18. *If $f : (X, \tau(B_X)) \to (Y, \sigma(B_Y))$ is a contra-λ_B-continuous function and $g : Y \to Z$ is a B-continuous function, then $g \circ f : X \to Z$ is contra-λ_B-continuous.*

Definition 4.19. *A simply extended topological space X is said to be*

(1). λ_B-compact if every λ_B-open cover of X has a finite subcover. (resp. $A \subset X$ is λ_B-compact relative to X if every cover of X by λ_B-open sets of X has a finite subcover).

(2). strongly-BS-closed if every B-closed cover of X has a finite subcover. (resp. $A \subset X$ is strongly-BS-closed if the subspace A is strongly-B-S-closed.

(3). mildly-λ_B-compact if every λ_B-clopen cover of X has a finite subcover.

Recall that for a function $f : (X, \tau(B_X)) \to (Y, \sigma(B_Y))$, the subset $\{(x, f(x)) : x \in X\} \subset X \times Y$ is called the bigraph of f and is denoted by BG(f).

Definition 4.20. *A bigraph BG(f) of a function $f : (X, \tau(B_X)) \to (Y, \sigma(B_Y))$ is said to be contra-λ_B-closed if for each $(x, y) \in (X \times Y)\backslash BG(f)$, there exist $U \in \lambda_B O(X)$ containing x and $V \in BC(Y)$ containing y such that $(U \times V) \cap BG(f) = \phi$.*

Lemma 4.21. *BG(f) is contra-λ_B-closed in $X \times Y$ if and only if for each $(x, y) \in (X \times Y)\backslash BG(f)$, there exist $U \in \lambda_B O(X)$ containing x and $V \in BC(Y)$ containing y such that $f(U) \cap V = \phi$.*

Theorem 4.22. *If $f : (X, \tau(B_X)) \to (Y, \sigma(B_Y))$ is contra-λ_B-continuous and Y is B-Urysohn space, then BG(f) is contra-Bclosed in $X \times Y$.*

Proof. Let $(x, y) \in (X \times Y)\backslash BG(f)$, then $f(x) \neq y$ and there exist B-open sets V, W such that $f(x) \in V$, $y \in U$ and B-cl(V)$\cap$ B-cl(W) $= \phi$. Since f is contra-λ_B-continuous, there exists $U \in \lambda_B O(X, x)$ such that $f(U) \cap Bcl(V)$. Therefore, we obtain $f(U)\cap$ B-cl(W) $= \phi$. This shows that BG(f) is contra-λ_B-closed in $X \times Y$. □

Theorem 4.23. *Let X be a λ_B-space. If $f : (X, \tau(B_X)) \to (Y, \sigma(B_Y))$ has a contra-λ_B-closed graph, then the inverse image of a strongly-Bs-closed set K of Y is B-closed in X.*

Proof. Assume that K is a strongly-BS-closed set of Y and $x \notin f^{-1}(k)$. For each $k \in K, (x, k) \notin BG(f)$. By Lemma 4.21, there exist $U_k \in \lambda_B O(X, x)$ and $V_k \in BC(Y, k)$ such that $f(U_k) \cap V_k = \phi$. Since $\{K \cap V_k : k \in K\}$ is closed cover of the subspace K, there exists a finite subset $K_0 \in K$ such that $K \subset \{V_k : k \in K_0\}$. Set $U = \cap\{U_k : k \in K_0\}$, then U is open since X is λ_B-space. Therefore $f(U) \cap K = \phi$ and $U \cap f^{-1}(k) = \phi$. This shows that $f^{-1}(k)$ is B-closed in X. □

Theorem 4.24. *If $f : (X, \tau(B_X)) \to (Y, \sigma(B_Y))$ is contra-λ_B-continuous and K is λ_B-compact relative to X, then f(k) is strongly-BS-closed in Y.*

Proof. Let $\{H_\alpha : \alpha \in I\}$ be any cover of f(k) by closed sets of the subspace f(k). For each $\alpha \in I$, there exists a closed set K_α of Y such that $H_\alpha = K_\alpha \cap f(k)$. For each $x \in K$, there exist $\alpha(x) \in I$ such that $f(x) \in K_\alpha(x)$. By Theorem 4.22 there exists $U_x \in \lambda_B O(X, x)$ such that $f(U_x) \subset K_\alpha(x)$. Since the family $\{U_x : x \in K\}$ is a λ_B-open cover of K, there exists a finite subset K_0 of K such that $K \subset \cup\{f(U_x) : x \in K_0\}$. Therefore, we obtain $f(K) \subset \cup\{K_\alpha(x) : \alpha \in K_0\}$. Thus $f(k) = \{H_\alpha(x) : x \in K_0\}$ and hence f(k) is strongly-BS-closed in Y. □

Theorem 4.25. *If $f : (X, \tau(B_X)) \to (Y, \sigma(B_Y))$ is contra-λ_B-continuous and λ_B-continuous surjective and X is mildly-λ_B-compact, then Y is compact.*

Proof. Let $\{V_\alpha : \alpha \in I\}$ be an open cover of Y. Since f is contra-Bλ-continuous and λ_B-continuous, we have that $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is λ_B-open cover of X. Since X is mildly-λ_B-compact, there exists a finite subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Since f is surjective $Y = \cup\{V_\alpha : \alpha \in I_0\}$ and therefore Y is compact. □

Theorem 4.26. *Let $f : (X, \tau(B_X)) \to (Y, \sigma(B_Y))$ be a surjective B-preclosed contra-λ_B-continuous function. If X is a λ_B-space, then Y is locally B indiscrete.*

Proof. Let V be any open set of Y. By hypothesis, f is contra-λ_B-continuous and therefore $f^{-1}(V) = U$ is λ_B-closed in X. Since X is a λ_B-space, the set U is B-closed in X. Since f is B-preclosed, then V is also B-preclosed in Y. Now we have $Bcl(V) = Bcl(Bint(V)) \subset V$. This means that V is B-closed and hence Y is locally B indiscrete. □

Theorem 4.27. *Let $f : (X, \tau(B_X)) \to (Y, \sigma(B_Y))$ be a contra-λ_B-continuous function. If X is λ_B-space, then f is contra-B-continuous.*

References

[1] F.G.Arenas, J.Dontchev and M.Ganster, *On λ-sets and dual of generated continuity*, Questions and answers Gen. Topology, 159(1997), 3-13.

[2] M.E.Abd El-Monsef, A.M.Kozae and R.A.Abu-Gdairi, *New approaches for generalized continuous functions*, Int. Journal of Math. Analysis, 4(27)(2010), 1329-1339.

[3] M.Caldas and S.Jafari, *Some properties of contra-β-continuous functions*, Mem. Fac. Sci. Kochi Univ. Ser. A Math., 22(2001), 19-28.

[4] M.Caldas, S.Jafari, G.Navalagi and M.Simoes, *More on λ-closed sets in topological spaces*, (submitted).

[5] J.Dontchev, *Contra-continuous functions and strongly S-closed spaces*, Internat. J. Math. Math. Sci., 19(2)(1996), 303-310.

[6] J.Dontchev and T.Noiri, *Contra semi-continuous functions*, Math. Pannon, 10(2)(1999), 159-168.

[7] S.Jafari and T.Noiri, *On contra-precontinuous functions*, Bull. Malays. Math. Sci. Soc., (2)25(2)(2002), 115-128.

[8] S.Jafari and T.Noiri, *Contra α-continuous functions between topological spaces*, Iran. J. Sci., 2(2)(2001), 153-167.

[9] S.Jafari and T.Noiri, *Contra-super-continuous functions*, Annal. Univ. Sci. Budapest, 42(1999), 27-34.

[10] N.Levine, *Simple extension of topologies*, Amer. Math. Monthly, 71(1964), 22-25.

[11] H.Maki, *Generalised Λ-sets and the associated closure operator*, The special issue in Commemoration of Prof. Kazusada IKEDA Retirement, (1986), 139-146.

[12] M.Murugalingam, O.Ravi and S.Nagarani, *New generalized continuous functions*, International Journal of Mathematics And its Applications 3(3B)(2015), 5562.

[13] S.Pious Missier, O.Ravi and S.Jeyashri, *On Λ_{Bg}-closed sets in topological spaces*, Int. Journal Mathematical Archive, (2011), 870-875.

[14] A.Vadivel, R.Vijayalakshmi and D.Krishnaswamy, *B-Generalized regular and B-generalized normal spaces*, International Mathematical Forum, 5(54)(2010), 2699-2706.